Personalized prices and uncertainty in monopsony*

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Abstract

We analyze personalized pricing by a monopsonist facing a finite number of *ex ante* identical, capacity constrained suppliers with privately known costs. When the distribution of costs is sufficiently smooth and regular, the buyer chooses to make the same offer to all suppliers, leading to a posted price. This price is lower than the classical monopsony price if the demand function is concave, and higher if the demand is convex. In the limit as the seller capacities tend to zero we obtain the classical monopsony price. Therefore, our model provides a decentralized micro-foundation for monopsony.

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1 Introduction

The classical model of monopsony postulates a single buyer who faces a deterministic supply curve resulting from the aggregation of suppliers' (marginal) costs. It is well-known that the optimal linear (posted) price equates the "mark-down"[1] to the reciprocal of this supply's elasticity. Even if the monopsonist could offer different prices to different suppliers, or not to make offers to some of them, this linear price would still be optimal as long as the monopsonist does not know where in the supply function each supplier is located. Indeed, a deterministic supply guarantees that the monopsonist knows with certainty the quantity purchased at each price.

The situation is different when the monopsonist faces uncertainty about supply, say, due to asymmetric information about costs. Even if suppliers are indistinguishable and even if (cost) uncertainty is independent across suppliers, in that case the monopsonist faces (aggregate) uncertainty about the total supply it receives for any given price vector. To deal with this uncertainty, there is a potential role both for different price offers to different (*ex ante* identical) suppliers and for excluding some suppliers from the offers.

In this paper we study the optimal discriminatory price policy for a monopsonist that faces this type of aggregate uncertainty. We obtain conditions under which linear prices, that is, a common price offer for all available suppliers, is still optimal, and compare this optimal offer to the corresponding classical monopsonist price under certainty. We also study the limiting properties of personalized prices as the aggregate uncertainty vanishes.

While – to the best of our knowledge – this question has never been posed before, Bulow and Roberts (1989) have shown that the mathematical problem of setting the optimal monopsony price is the same as setting the optimal reserve price in an auction, independently of the number of bidders. In other words, the optimal auction for *ex ante* identical suppliers involves a (common) reserve price, which equals the optimal take-it-or-leave-it offer to a single supplier: the monopsony price. Since Bulow and Roberts allow for supply uncertainty and obtain uniform (reserve) prices, one might consider that posted

[1] This is the eqivalent of the Lerner index for monopsony, the difference between the buyer's valuation of the marginal unit and the price as a proportion of the price: $\frac{P(S(p))-p}{p}$.

prices continue to be optimal.[2] However, setting (possibly personalized) prices and setting reserve prices in an auction are conceptually different mechanisms. First, even though the optimal "auction" treats suppliers symmetrically, the realized price(s) will often fall short of the reserve price, the competition among suppliers reduces their information rent; second, an "auction" is a centralized mechanism.[3] That is, the terms of trade between the monopsonist and any individual seller depend on the trades with other suppliers. Putting it differently, the buyer commits for any given supplier not only to the way in which he will use the information *she* reveals, but also to how he will use the information revealed by all the other suppliers. Instead, we wish to investigate monopsony pricing when the buyer is not able/willing to commit to a centralized mechanism.[4]

Thus, we consider the same problem of a monopsonist with multi-unit demand who is faced with a finite number of *ex ante* symmetric, capacity constrained suppliers of privately known costs, but maintaining decentralized interaction: the monopsonist may offer prices personalized to each supplier – committing to trade at those prices if accepted – but this is all he can do. That is, agreements/commitments are bilateral: the terms of trade with each supplier is independent of the terms of trade with other suppliers. As we discuss in the conclusions, this personalized-pricing procedure is also a useful ingredient in models of price competition, where it leads to novel insights.

Our first result is to show that, under "mild" conditions on the distribution function of costs, when suppliers are *ex ante* identical the personalized prices are all the same

[2] Although one has to be careful with extending the equivalence between setting optimal prices and reserve prices beyond the environment of Bulow and Roberts (1989). For example, Burguet and Sákovics (1999) show that identical competing sellers will not set reserve prices equal to marginal cost in their auctions despite what happens in Bertrand competition.

[3] And not even optimal, at that. As argued in Bulow and Roberts (1989), for a monopsonist with full commitment power, in the optimal mechanism the monopsonist announces a demand curve and solicits ask prices by sellers. The resulting aggregate supply schedule together with the announced demand is used to establish the market clearing price at which all the suppliers with ask prices below it trade. Of course, in order to reduce the sellers' information rents, the announced demand schedule is distorted relative to the monopsonist's true demand function.

[4] For empirical evidence of firms using second-best organizational form (and thus pricing) see, for example, Thomas (2011) and references therein. See also McElheran (2014) on delegation. For a theoretical overview of decentralization see Mookherjee (2006).

and as a result, posted, i.e., linear, prices are constrained optimal. The condition we identify is a strengthening of the traditional "regularity" condition in problems of trade under asymmetric information, that the virtual cost[5] of an arbitrary supplier has to be increasing. Basically, we need a steeper slope for the virtual cost the more inelastic the demand is and the fewer suppliers there are. Additionally, the range of possible costs comes into play. When there is a gap between the lowest possible cost and the lowest marginal valuation, the monopsonist may prefer to make fewer (serious) offers than there are traders (and demand). If on the other hand, there is a gap between the highest possible cost and the highest marginal valuation, the monopsonist may prefer to make some offers that are surely accepted in addition to different offers at lower prices. Notably, however, if the slope condition is met, all interior prices (that is, prices that will be refused and accepted both with positive probability) are the same. It is only if the cost distribution of suppliers is particularly "bumpy" that we observe heterogeneous interior prices offered to homogeneous suppliers.

We also show that even though the buyer offers a posted price, this price can be lower or higher than the corresponding classical monopsony price (roughly depending on whether the demand function is "concave" or "convex", respectively). The reason stems from the fact that the monopsony price is determined by a point elasticity, while the personalized price is optimized by taking expectations over the aggregate uncertainty.

Next, we introduce heterogeneity in cost distributions and show that, contrary to the classic result, it is not necessarily the case that the less elastic market is offered the lower price. We find conditions for that to be the case. Just as the ones for the optimality of posted prices, these conditions are related to the slope of the monopsonist's demand function. Finally, we establish our convergence result: as supply is broken up into more and more suppliers, the outcome of our mechanism converges to the textbook monopsony pricing against a continuous supply function and the conditions for posted prices to be optimal are eventually always satisfied.

[5]If costs are random draws from the distribution function $F(c)$, the virtual cost function is given by $c + F(c)/f(c)$ (c.f. Myerson, 1981).

1.1 A brief review of the literature

The literature on optimal trading mechanisms[6] is not directly relevant, as our interest here is in a second best. Another strand of the literature makes pairwise comparisons between bargaining, auctions and posted prices.[7] Again, this is very different from our approach, where we stay with the standard pricing mechanism and investigate the benefits of discrimination in a hitherto unexplored context. Let us discuss some of the papers that are more closely related to our proposed mechanism.

Riley and Zeckhauser (1983) consider a seller with commitment power who is visited by buyers in sequence until she sells her unit. They show that the optimal strategy is a common take-it-or-leave-it price. Of course, due to the sequential resolution (and the unique item on offer) the aggregate uncertainty is minimal in this model.

Winter (2004) also obtains that offering different prices to identical agents is useful. However, in his case the principal is using (some of the) prices as a coordination device in a multiple-equilibrium scenario.

Alonso et al. (2008) also look at the possibility of decentralized organizational structure but they assume that the monopsonist is constrained to name a single price. Therefore, the issue that determines whether a centrally set price or delegation to one of the local managers is optimal is how local managers are willing to report their private information about demand. As it turns out, when they are expected to widely disagree, decentralization is optimal.

Chen and Ishida (2013) consider the benefits of personalized pricing in a dynamic context. They show that price discrimination can increase a seller's expected profit if she can commit to dynamic price schedules. Otherwise, the ability to price discriminate not only is useless but can even harm the seller.

Finally, the logic of calculating expected marginal valuations is reminiscent of the analysis of Martin and Pindyck (2015) of the benefit of averting one catastrophe of several

[6]Harris and Raviv (1981) is the classical study of the best mechanism of a single price setter faced with asymmetric information.

[7]Notable early contributions are Bester (1993) and Wang (1993, 1995).

impending ones.

2 The set-up: personalized pricing

Consider a risk neutral monopsonist with (marginal) willingness to pay $v_l \in [0,1]$ for the l^{th} unit of a homogeneous good, $l \in \{1, 2, \ldots, Q\}$, with $v_1 = 1$ and $v_l \geq v_{l+1}$. There are Q unit-supply sellers[8]. Each seller has a unit capacity. Seller i's cost (reservation price) is c_i and it is i's private information. From the monopsonist's – and the other sellers' – point of view, c_i (Seller i's "type") is the realization of an independent random variable, with – strictly increasing and common knowledge – distribution function $F_i(.)$ and (differentiable) density function $f_i(.)$ on $[\underline{c}_i, \bar{c}_i]$, where $0 \leq \underline{c}_i < \bar{c}_i \leq 1$. To retain simplicity and focus, we assume that $F_i(.)$ is *regular*: $c_i + \frac{F_i(c_i)}{f_i(c_i)}$ is monotone increasing.[9]

We study monopsony pricing as implemented by a simultaneous personalized offer to each seller, with full commitment. Note that, since there is aggregate uncertainty, and as with a posted price, the buyer risks having to acquire too many (or too few) units: he cannot adjust the prices and quantities *ex post*. This is an additional ingredient to the usual trade-off under complete information, between paying a low price and increasing the amount bought, that the monopsonist needs to take into account.

For clarity's sake, we first consider *ex ante* identical sellers, where each seller's cost is independently drawn from the same $F(.)$. We will later relax the symmetry assumption (c.f. Section 5).

Before continuing with our analysis, we first specify the benchmark case of classical monopsony and relate it to our personalized pricing model.

[8] As we allow for $v_i = 0$ and as the demand for a higher number of units than there are available sellers would never be satisfied, it is without loss of generality to assume that both maximum aggregate supply and demand are Q units.

[9] This is the assumption that ensures that the first-order conditions imply the second-order conditions in the standard auction design problem (c.f. Myerson, 1981).

2.1 The benchmark: classical monopsony

The classical monopsony model postulates a buyer with a continuous,[10] weakly decreasing (inverse) demand function, $V(.)$ that we can normalize, so that $V(0)=1$ and $V(Q)=0$. To fit with our discrete set-up, we assume that $V(.)$ is a left continuous step function, with its steps at integer values: $V(x)\equiv v_n$ for $x\in(n-1,n]$. The buyer faces a differentiable, increasing (inverse) supply function $S(.)$. On the supply side, our set-up also reduces to the classical model if we remove the uncertainty about the costs, so that the total supply at price p coincides with the expected quantity that the Q suppliers are willing to sell at that price: $S^{-1}(p)=QF(p)$.

With this analogue, the optimal monopsony quantity, q^M, calculated by equating marginal valuation with marginal expenditure, would be the solution[11] to

$$V(q^M)=\frac{dq^M S(q^M)}{dq^M}=S(q^M)+q^M S'(q^M). \tag{1}$$

The optimal monopsony price would then be $p^M=S(q^M)$, which gives us a translation of (1) into prices:

$$V(S^{-1}(p^M))=p^M+S^{-1}(p^M)S'(S^{-1}(p^M))=p^M+\frac{S^{-1}(p^M)}{\left(S^{-1}\right)'(p^M)}. \tag{2}$$

Substituting $QF(p^M)$ for $S^{-1}(p^M)$, we obtain[12]

$$p^M+\frac{F(p^M)}{f(p^M)}=V(QF(p^M)). \tag{3}$$

That is, the monopsonist posts a price that equates his marginal valuation with what in our model is the virtual cost of an arbitrary seller. Equation (3) can also be written as equality between the inverse of the supply's elasticity and the markdown.

[10] To highlight the consequences of indivisibilities in our model, we assume – as the textbooks – that the underlying supply and demand are continuous. The consequences of discontinuities in the classical context are standard.

[11] The discontinuity is resolves by taking the infimum of quantities that lead to inverse (virtual) supply (the RHS) higher than inverse demand (the LHS).

[12] Given regularity, this equation has a unique solution.

3 The super-regular case

The first question is, then, whether personalized pricing in the presence of supply uncertainty will also result in identical individual price offers. As we will argue, the answer is affirmative if the probability distribution of costs satisfies an assumption that is stronger than regularity. In Section 4, we discuss what might happen when the assumption is not satisfied.

Unlike in the case of regularity, where the restriction on the allowable cost (i.e., supply) distribution is exogenous, in our definition the constraint depends on the demand function as well.[13]

Definition 1 *The distribution of costs, $F(.)$, is* super-regular *relative to $\{v_l\}_{l=1,2,...,Q}$ if: i)*[14] $c + \frac{F(c)}{f(c)} - F(c) \max_{l \in \{1,2,...,Q\}} \{v_l - v_{l+1}\}$ *is strictly increasing in c, and ii) $F(.)$'s support includes that of the demand ($\underline{c} = 0$, $\bar{c} = 1$).*

The slope restriction is stronger than regularity as $-F(c)$ is strictly decreasing. It serves to ensure that it is suboptimal for the buyer to target different parts of the supply separately – *à la* third-degree price discrimination (e.g. in case of a multi-peaked supply density). The support restriction makes sure that the probability that any positive offer is accepted is positive and that no offer below the maximum valuation will be accepted for certain. For $l \in \{0, 1, ..., Q-1\}$, let

$$\chi_l(x) = \binom{Q-1}{l} F(x)^l (1 - F(x))^{Q-l-1},$$

the probability that l out of $Q-1$ (independent) draws from the distribution F are below x.

Proposition 1 *When the cost distribution is super-regular relative to the buyer's marginal valuation of each unit, the optimal personalized pricing strategy is a unique (posted)*

[13] It is straightforward to strengthen the assumption to be independent of $V(.)$: just substitute 1 for $\max_{l \in \{1,2,...,Q\}} \{v_l - v_{l+1}\}$.

[14] We let $v_{Q+1} = 0$.

price p^D satisfying

$$p^D + \frac{F(p^D)}{f(p^D)} = \sum_{l=0}^{Q-1} \chi_l(p^D) v_{l+1}. \tag{4}$$

Proof. See the Appendix. ■

In other words, under super-regularity, the buyer does not (ab)use his ability to price discriminate: he offers to buy at the same price from all sellers. Thus, even under (aggregate) uncertainty of supply, our model offers a well-founded, game-theoretic foundation for "posted prices". This qualitative feature coincides with what is an assumption in the classical monopsony model. It is of particular note that this is not a convergence result: the one price result holds for any number of sellers.

The optimality of committing to buy from all comers may be somewhat surprising. Consider, for example, the extreme case when the buyer is looking for a single unit (and so $v_l = 0$ for all $l > 1$) from a large number Q of suppliers. The intuition for making a serious offer to each seller even in this situation is, nonetheless, simple. If the buyer did not make a serious offer to some seller then his profit made on her would be zero. On the other hand, as long as the expected marginal valuation of the unit offered by this seller – conditional on the offers made to the other sellers – is positive, by making an offer below this value, the monopsonist would receive a positive expected net marginal payoff. The expected marginal valuation of that seller's unit could be zero only if the entire demand is satisfied with probability one with the offers to the other sellers. However, in equilibrium that cannot happen. It would entail making an offer of 1 to (at least) one seller, leading to non-positive profits (on that seller). As a result, the optimal policy for the monopsonist must include serious offers to all sellers. Note that this intuition only relies on the support restriction in super-regularity. The constraint on the slopes of the cost distribution and the demand ensures that the system of first-order conditions has a unique, uniform solution.

3.1 Posted prices and uncertainty

The fact that, under super-regularity, the buyer names the same price for all sellers does not imply that this price coincides with the classical monopsony price. The decentralized posted price, p^D in (4), differs from the classical monopsony price, p^M in (3), construed as the optimal posted price when supply is (deterministic and) smooth and equals $F(p)$. The left-hand side of (3) and (4), the marginal expenditure (or virtual cost), is common to both expressions. However, the optimal price in the classic monopsony problem equates this marginal expenditure to the (marginal) willingness to pay at the optimal quantity. On the contrary, the optimal monopsony price under uncertainty, p^D, equals that marginal expenditure to the expectation of the marginal willingness to pay. That is, p^M depends only on the demand function evaluated at the optimal quantity (the trade-off that determines it is local), whereas p^D in (4) depends on the entire demand function.

Not surprisingly, there is no general ranking of these prices. The following example illustrates.

Example 1 *Assume $F(x) \equiv x$ and there are three available sellers. Consider the following family of demand functions:*[15] $v_1 = 1$, $v_2 = y$, *and* $v_3 = 0$. *Using (1) it is straightforward to verify that for* $y \leq 2/3$, $p^M = 1/3$. *On the other hand, (**??**) becomes* $2c = (1-c)^2 + 2(1-c)cy$, *leading to* $c^2(1-2y) - (4-2y)c + 1 = 0$. *It is straightforward to check that for* $y > .5$ *this leads to* $p^D > 1/3$, *and for* $y < .5$ *it leads to* $p^D < 1/3$.

Note that in the above example the threshold value of $y = .5$ corresponds to $V(.)$ being "linear". This is not a coincidence. We can show in general that if $V(.)$ is "concave" then $p^M \geq p^D$. This is a useful result as in most cases marginal valuations are – at least weakly – decreasing. Unfortunately the step-function nature of demand means that to make the result precise we need a few definitions.

Definition 2 *For a step function f, denote by $\underline{x}_f$ the highest value of f that is no greater than x.*

[15] For simplicity we work with discontinuous demand functions, it is trivial to see that the results would hold with arbitrarily close continuous approximations.

This is a rounding device, just like the integer value function, but instead of the set of integers it uses the values of the step function. Next, we need to modify the definition of concavity for step functions (that are obviously not concave in the standard sense):

Definition 3 *We say that the left-continuous step function $f(.)$ with steps at $i \in \{1, ..., Q\}$, is* step-concave *if and only if for all $(i, j) \in \{1, ..., Q-1\} \times \{2, ..., Q\}$ with $i < j$ and $a \in (0,1)$: $\underline{af(i) + (1-a)f(j)}_f \leq f(ai + (1-a)j)$.*

We can now extend Jensen's inequality to step functions:

Theorem 2 *(Jensen's inequality) Let f be a left-continuous step function with steps at $i \in \{1, ..., Q\}$ and $(\mathbf{p}, \mathbf{x})$ a lottery with x values in $\{1, ..., Q\}$. If f is step-concave then $\underline{\sum p_i f(x_i)}_f \leq f\left(\sum p_i x_i\right)$.*

We can now state our result:

Proposition 2 *If $V(.)$ is step-concave then $p^M \geq \underline{p^D}_V$.*

In other words, when the cost distribution is super-regular and the demand function is step-concave, the Bulow-Roberts intuition[16] holds in our model: the price under uncertainty is lower than in the classical model. However, when either of these conditions is violated, the situation can change: we can have multiple prices and/or the price(s) offered can exceed p^M (c.f. Example 1).

4 The possible consequences of prescinding from super-regularity

While super-regularity is a reasonable assumption, it is clearly not always satisfied, in particular when the number of suppliers is not large. It is therefore pertinent to investigate

[16]Recall that they say that the reserve price is same as the monopsony price and thus the actual price is (weakly) lower.

the consequences of the failure of its components. The proof of Proposition 1 hints at what we might expect. Here we discuss these factors in some detail and provide examples (with a given number of available sellers) to illustrate them. In Subsection 6 we will show that, nonetheless, all the complications that might result from the failure of super-regularity disappear in the limit as the number of supplier gets large (and their capacities decrease to zero).

4.1 Uniform pricing depends on the slope restriction

When the slope restriction in super-regularity fails, the monopsonist may optimally price discriminate between otherwise symmetric sellers, even if "classical" regularity is maintained. Note that this discrimination is different from third degree price discrimination in that all suppliers are still assumed ex-ante identical. That is, the endogenous price discrimination does not depend on any exogenous characteristic of suppliers.

The expected value of the marginal lth unit that the buyer acquires increases by the step size multiplied by the probability of trade with the last inframarginal trader. This needs to be factored into the "regularity" of the virtual cost. When v_l may be significantly larger than v_{l+1}, then given that a high offer is made to a seller, and so the probability that the lth unit is acquired is high, the optimal offer to another seller may be low, and *vice versa*: if the offer made to the former is low, the best offer to the latter may be high. Consider the following example:

Example 3 *Assume that* $v_1 = 1$ *and* $v_2 = 0$*, and that there are only two (identical) sellers. Let* $F(x) = x^{10}$ *for* $x \in [0, 1]$*. Note that* $\frac{d(x+F(x)/f(x))}{dx} = 1.1 > 0$ *but* $\frac{d(x+F(x)/f(x)-F(x))}{dx} = 1.1 - 10x^9 < 0$ *for* $x > .783$*. With the help of Mathematica it is immediate to see that there are three real solutions to the system of first-order conditions. A symmetric one with* $b = .8051$ *and two asymmetric ones with* $b_i = .5714$ *and* $b_j = .9057$*. Substituting them into the objective function, the first leads to an expected buyer profit of* $.0315$*, while the latter(s) to* $.0352$*. Thus, the optimal price vector is asymmetric.*

This result is an interesting parallel with Kotowski (2018), who has shown that when the type distribution of *ex ante* symmetric bidders is not regular, asymmetric reserve prices might be optimal for the bid-taker.

4.2 The number of surely accepted offers in a market depends on $\bar{c}$

Let us turn to the option of making surely accepted offers. For the monopsonist, these have the obvious advantage of reducing uncertainty on the extensive margin. These offers practically remove the highest valued units from the demand and a corresponding number of suppliers from the supply, so that the "posted price" result holds only for the residual market.

The intuition here is also reminiscent of the setting of a reserve price in a standard auction, where the lowest buyer valuation is much higher than the seller's. In that case, a sale for the lowest possible valuation is so valuable that the marginal gain in price does not compensate for risking to lose the sale. In the procurement context, we have the same scenario: since the good can be bought for certain for a price that is a fraction of its valuation, the expected gain from a more aggressive price offer cannot outweigh the expected loss from possibly not buying it. The key factor therefore is the expected valuation of the unit minus the highest possible cost. When this difference is sufficiently large, it is optimal to make an offer that cannot be refused.

The following example illustrates.

Example 4 *Assume that* v_1 $=1$, $v_2 = 0.2$, *there are only two sellers, and* $F(x)$ $=4x$ *with support* $[0, .25]$. *(4) becomes* $.8p + 1 - 4p = 2p$, *with solution* $p = 5/26 \approx .192$ *and corresponding profit* $\pi = 10/13 \approx .769$. *If instead, the buyer sets one price equal to* $.25$ *(which is accepted for certain) and the optimal price of* $.1$ *for the other seller, his expected profit is* $.75 + .1\frac{.1}{.25} = .79$. *Note that the optimal monopsony price without uncertainty of costs, as in Subsection 2.1, would be* $.125$, *buying one unit and expecting a profit of* $.875$.

4.3 The number of serious offers in a market depends on $\underline{c}$

When $\underline{c} > 0$, any serious offer entails a price bounded away from zero. That is, if the buyer makes a number of offers above the number of units for which he has positive willingness to pay, then he risks incurring a loss that is also bounded away from zero. As the expected value of the marginal unit is not bounded away from zero, it may be optimal to make offers to only some of the potential sellers. The following example illustrates this possibility.

Example 5 *Assume that* $v_1 = 1$ *and* $v_2 = 0$*, and that there are only two sellers. Let* $F(x) = \frac{x-\underline{c}}{1-\underline{c}}$ *with support* $[\underline{c}, 1]$*. If only one serious offer is made then* $\Pi^1(b) = (1-b)F(b)$*, and the first-order condition is* $1 = F(b)/f(b) + b$*. Substituting in for* F*, we obtain* $b = \frac{1+\underline{c}}{2}$ *and thus* $\Pi^1 = \frac{1-\underline{c}}{4}$*. If two (equal)*[17] *serious offers are made then the expected profit is* $\Pi^2(b) = 1 - (1 - F(b))^2 - 2F(b)b$*, leading to the first-order condition* $1 - F(b) = F(b)/f(b) + b$*. Substituting in for* F*, we obtain* $b = \frac{1+\underline{c}-\underline{c}^2}{3-2\underline{c}}$ *and thus* $\Pi^2 = \frac{(1-\underline{c})^2}{3-2\underline{c}}$*. It is straightforward to see that* $\Pi^1 > \Pi^2$ *if (and only if)* $\underline{c} > 1/2$*.*

5 Third-degree price discrimination

Aggregate uncertainty might also affect the direction of third-degree price discrimination. Recall that, according to the classical multi-market monopsony model,[18] the buyer should optimally offer a higher price to the market with the higher price elasticity of supply. This result need not hold in our model with uncertainty.

Indeed, let us reintroduce *ex ante* (observable) asymmetry among sellers. To consider the simplest case, suppose there are two "markets" with Q^1 and Q^2 sellers, their cost distributions being $F(.)$ and $G(.)$, respectively. In order to calculate the expected marginal value we first need to calculate the probability that l items are sold when the buyer offers p^1 to Q^1 sellers in market 1 and p^2 to Q^2 sellers in market 2. First, let us denote by $\chi_l^i(x; K)$ the value of $\chi_l(x)$ when $Q = K$ and the distribution function is the one characterizing

[17] Recall that uniform pricing is driven by the slope restriction in super-regularity.

[18] See, for example, Tirole (1988) page 137.

suppliers in market i, for $i = 1, 2$. Also, and to save in notation, let $\chi_l^i(x; K) = 0$ whenever $l \geq K$. We should also introduce an additional piece of notation:

$$\psi_l(p^1, p^2; K^1, K^2) = \sum_{k=0}^{l} \chi_k^1(p^1; K^1)\chi_{l-k}^2(p_2; K^2).$$

ψ_l represents the probability that l offers are accepted when K^i are made to sellers in market i, each with a price of p^i. We can now write the corresponding system of equations for (4) as

$$\sum_{j=1}^{Q^1+Q^2} \psi_{j-1}(p^1, p^2; Q^1 - 1, Q^2)v_j = \frac{F(p^1)}{f(p^1)} + p^1 \tag{5}$$

$$\sum_{j=1}^{Q^1+Q^2} \psi_{j-1}(p^1, p^2; Q^1, Q^2 - 1)v_j = \frac{G(p^2)}{g(p^2)} + p^2. \tag{6}$$

It is now straightforward to generalize Proposition 1 to two (or more) classes of sellers.

Corollary 1 *When both cost distributions are super-regular relative to the buyer's marginal valuation of each unit, the optimal personalized pricing strategy is a posted price in each market, satisfying (5)-(6).*

We are not particularly interested in the uniqueness of the pairs of (uniform) prices solving the first-order conditions. (In case there are several, the buyer simply chooses the pair maximizing his expected utility.)

Returning to third-degree price discrimination, note that in the solution to (5), the monopsonist again equals the marginal expenditure in market 1 (the right hand side) to the expected willingness to pay for the marginal unit, the left hand side. The subtle point here is that this expectation is taken conditional on *all* offers made in market 2, and *all but one* made in market 1. Similarly, the solution to (6) depends on the same expectation but conditional on *all but one* offers in market 2 and *all* offers in market 1.

Note that, for $K^1 + K^2 - 1 \geq l \geq 1$,

$$\begin{aligned} & \psi_l(p^1, p^2; K^1, K^2) \\ = \; & (1 - F(p^1))\psi_l(p^1, p^2; K^1 - 1, K^2) + F(p^1)\psi_{l-1}(p^1, p^2; K^1 - 1, K^2) \\ = \; & (1 - G(p^2))\psi_l(p^1, p^2; K^1, K^2 - 1) + G(p^2)\psi_{l-1}(p^1, p^2; K^1, K^2 - 1). \end{aligned} \tag{7}$$

Indeed, the second line above simply separates one supplier from the first market, computes the probability that l units are obtained from the $K^1 - 1$ and K^2 other suppliers, and the probability that $l - 1$ units are obtained from them. Then multiplies these probabilities by the probability of obtaining no unit or one unit from the separated supplier, respectively. The third line is a similar exercise with a separated supplier from market 2. Using (7), we can write the left-hand side of (5) as

$$\begin{aligned}
& \sum_{j=1}^{Q^1+Q^2} \psi_{j-1}(p^1, p^2; Q^1 - 1, Q^2) v_j \\
= \; & (1 - G(p^2)) \sum_{j=1}^{Q^1+Q^2-1} \psi_{j-1}(p^1, p^2; Q^1 - 1, Q^2 - 1) v_j + \\
& G(p^2) \sum_{j=1}^{Q^1+Q^2-1} \psi_{j-1}(p^1, p^2; Q^1 - 1, Q^2 - 1) v_{j+1} \\
= \; & \sum_{j=1}^{Q^1+Q^2-1} \psi_{j-1}(p^1, p^2; Q^1 - 1, Q^2 - 1) v_j \\
& -G(p^2) \sum_{j=1}^{Q^1+Q^2-1} \psi_{j-1}(p^1, p^2; Q^1 - 1, Q^2 - 1)(v_j - v_{j+1}),
\end{aligned}$$

where we have also used the fact that

$$\begin{aligned}
\psi_0(p^1, p^2; Q^1 - 1, Q^2) &= (1 - G(p^2))\psi_0(p^1, p^2; Q^1 - 1, Q^2 - 1), \\
\psi_{Q^1+Q^2-1}(p^1, p^2; Q^1 - 1, Q^2) &= G(p^2)\psi_{Q^1+Q^2-2}(p^1, p^2; Q^1 - 1, Q^2 - 1)
\end{aligned}$$

Similarly for the left hand side of (6). Thus, we can write (5)-(6) as

$$\begin{aligned}
E_{p^1,p^2} v_j &= \frac{F(p^1)}{f(p^1)} + p^1 + G(p^2) E_{p^1,p^2} \Delta v_j, \\
E_{p^1,p^2} v_j &= \frac{G(p^2)}{g(p^2)} + p^2 + F(p^1) E_{p^1,p^2} \Delta v_j,
\end{aligned}$$

where $E_{p^1,p^2} v_j = \sum_{j=1}^{Q^1+Q^2-1} \psi_{j-1}(p^1, p^2; Q^1 - 1, Q^2 - 1) v_j$ is the expected (marginal) value of a unit bought from a supplier, given the offers made to $Q^1 - 1$ suppliers in market 1 and to $Q^2 - 1$ suppliers in market 2; and $E_{p^1,p^2} \Delta v_j$ is the expectation of the marginal increase in this value relative to the case where one more unit is bought from the other sellers, $\Delta v_j = v_j - v_{j+1}$, $E_{p^1,p^2} \sum_{j=1}^{Q^1+Q^2-1} \psi_{j-1}(p^1, p^2; Q^1 - 1, Q^2 - 1)(v_j - v_{j+1})$.

Now recall that the price elasticity of supply in market 1 is

$$\varepsilon_1^s = \left(\frac{dF(p)}{dp}\frac{p}{F(p)}\right)^{-1} = \left(\frac{f(p)}{F(p)}p\right)^{-1}.$$

Thus, the supply elasticity of a market at any price is proportional to the (inverse) hazard rate. The intuition becomes clear if we write the optimal pricing formula in terms of the hazard rate

$$\begin{aligned} E_{p^1,p^2}v_j - p^1 &= \frac{F(p^1)}{f(p^1)} + G(p^2)E_{p^1,p^2}\Delta v_j \\ E_{p^1,p^2}v_j - p^2 &= \frac{G(p^2)}{g(p^2)} + F(p^1)E_{p^1,p^2}\Delta v_j. \end{aligned} \tag{8}$$

The inverse (reverse) hazard rate may be higher, yet the optimal price in that market may be lower.

Note that, for all prices, $E_{p^1,p^2}\Delta v_j \leq \max_j \Delta v_j$. Thus, once again, aggregate uncertainty may results in changes that are related to the the additional term $F(x)\max_j \Delta v_j$. Combining the two equations in (8) we obtain

$$\frac{G(p^2)}{g(p^2)} - \frac{F(p^1)}{f(p^1)} + \left(F(p^1) - G(p^2)\right) E_{p^1,p^2}\Delta v_j = p^1 - p^2.$$

Recall that reverse-hazard-rate dominance implies first order stochastic dominance. Then, the following proposition is immediate:

Proposition 3 *Suppose that both F and G are super-regular relative to $\{v_l\}$, and for any p, $\frac{F(x)}{f(x)} < \frac{G(x)}{g(x)}$ (F reverse-hazard-rate dominates G; i.e., market 1's supply is more elastic than market 2's). A sufficient condition for the optimal monopsony price in market 1 to be larger than that in market 2 is that for all x*

$$\frac{F(x)}{f(x)} - F(x)\max_j \Delta v_j \leq \frac{G(x)}{g(x)} - G(x)\max_j \Delta v_j.$$

Thus, just as a strengthening of regularity guarantees that the posted prices are indeed optimal for a monopsonist, strengthening of inverse hazard rate dominance along the same lines guarantees that prices for a third-degree price discriminating monopsonist follow the same pattern as in the classical model of monopsony.

As an illustration that when the sufficient condition is not satisfied we can indeed obtain the "wrong" price ordering, consider the following example:[19]

Example 6 *Let , $Q^1 = Q^2 = 1$ $v_1 = 1$ and $v_2 = 0$, $F(x) = x$ and $G(x) = x^2$ if $x \leq .5$ and $G(x) = 1.5x - .5$ for $x > .5$.*[20] *Then (5)-(6) become*

$$\left.\begin{array}{ll} 1-\left(p^2\right)^2 & \textit{if} \quad p^2 \leq .5 \\ 1.5(1-p^2) & \textit{if} \quad p^2 > .5 \end{array}\right\} = 2p^1 \tag{9}$$

$$1 - p^1 = \left\{\begin{array}{ll} 1.5p^2 & \textit{if} \quad p^2 \leq .5 \\ 2p^2 - 1/3 & \textit{if} \quad p^2 > .5 \end{array}\right. .$$

Solving, we obtain $p^2 = \frac{3-\sqrt{5}}{2} < .5 < p^1 = \frac{\sqrt{5}}{2} \cdot \frac{3-\sqrt{5}}{2}$, while the price elasticities are $p^1 \cdot \frac{f(p^1)}{F(p^1)} \equiv 1 < p^2 \cdot \frac{g(p^2)}{G(p^2)} = 2$.

Once more, when the number of suppliers is large (and their capacity small, with respect to the size of the market), $\max_j \Delta v_j$ is small and (given regularity) the sufficient condition in Proposition 3 is satisfied. Consequently, the expected values in the left hand side of (5) and (6) for similar values of p^1 and p^2 approach, and then we recover the predictions of the classical monopsony model. We now undertake to prove formally this and previous convergence results. That is, to obtain the classical monopsony model as the limit of our personalized price monopsonist.

6 Large markets and convergence

One of the goals of this paper is to provide a micro-foundation for the classical monopsony model that relies on the buyer being able to make personal commitments to individual suppliers, but without having the ability to make these commitments contingent on dealings with other sellers. Thus, we now show that indeed, in the limit, where the buyer faces

[19] In fact, Example 3 could suffice, if we consider each seller as a different market, since elasticity is (constant and) equal in both markets in that case, yet prices are different.

[20] We cannot use $G(x) = x^2$ as it is not super-regular, so we could not appeal to the corollary.

a large number of (capacity constrained) small suppliers, his optimal price vector reduces to classical monopsony pricing (c.f. Section 2.1) without any additional assumption.

Let's fix an integer t and let $\delta = \frac{1}{t}$. Also, to save in notation without losing any generality, let $Q = 1$. Suppose that each seller has an indivisible supply of δ units to sell with probability α, and let s_i be the per-unit price the monopsonist offers supplier i for her supply. When $t = 1$, this is the model analyzed in the previous section, for $Q = 1$. As t gets large, both the demand and the supply become a closer approximation of the underlying continuous functions ($V(.)$ and $F(.)$): $\frac{v_q}{\delta} = \int_{q-\delta}^{q} \frac{1}{\delta} V(x)dx \rightarrow_{t\rightarrow\infty} V(q)$ and the realization of tQ draws from $F(.)$ converges to $tQF(.)$ a.s. (by the Strong Law of Large Numbers). In other words, as t increases without bound, our set-up converges to the classical monopsony set-up. The question is whether our predictions converge as well.

The answer is affirmative, and we will show it in two steps. First we will show that, under symmetric pricing, (4) converges to (3). Next, we will argue that the optimal solution to (4) near the limit must be symmetric, that is, a posted price, as long as $F(.)$ is regular, even if it is not super-regular.

Lemma 1 *For any posted-price p, $\sum_{j=1}^{t} v_j \chi_{j-1}(p) \rightarrow_{t\rightarrow\infty} V(F(p))$.*

Proof. For each posted price p and given the total number of sellers t, the number j of sellers (other than i) that accept the offer, n is a random variable with probability distribution, $\chi_n(p)$, a binomial with parameters $(t-1, F(c))$. Also, by the Strong Law of Large Numbers, these sellers' average supply converges a.s. to $QF(p)$ as $t-1 \rightarrow \infty$. That is, taking into account that each seller sells $\delta = \frac{1}{t}$ when accepting the offer, and that $\frac{t-1}{t} \rightarrow 1$, total supply of these $t-1$ sellers converges a.s. to $F(c)$. That is,

$$\Pr\left[|\delta n - F(c)| < \epsilon\right] \rightarrow 1, \qquad \forall \epsilon > 0.$$

Therefore, $\sum_{j/t \notin (F(p)-\epsilon, F(p)+\epsilon)} \chi_{j-1}(p) \rightarrow 0$ as $t \rightarrow \infty$, for all ϵ, and the result follows. ■

Lemma 2 *For t sufficiently large, the optimal personalized pricing scheme is a posted price.*

Proof. That interior prices must be uniform for high enough t follows immediately from the proof of Proposition 1. We only need to observe that, as $V(.)$ is continuous, $\max_l \{v_l - v_{l+1}\}$ converges to zero as $t \to \infty$, and so $H(b_{-(j,k)})$ (an expectation of these values) does too. Consequently, regularity is sufficient for a unique interior solution.

To show that for high enough t no extreme offer will be made, we will first prove that in equilibrium the marginal valuation must eventually be strictly above $\underline{c}$. It then follows that it is in the buyer's interest to make serious offers.

Take a price which is strictly above $\underline{c}$ (there must be at least one since $\underline{c} < 1 = v_1$). The marginal valuation for this unit must be at least as much as the price. Now take another price which is not serious. As t increases, the difference between the marginal valuations of these two units converges to zero, so the second unit is also worth a serious price.

Next, note that unless the classical monopsony price equals $\bar{c}$ – which happens if the lowest marginal valuation is above the highest virtual cost – it must be the case that, for t large enough, the marginal valuation is less than the highest virtual cost, implying that (4) has an interior solution. ■

Putting these two lemmas together, we have proved our main convergence result:

Proposition 4 *For t sufficiently large, the buyer-optimal personalized price vector converges to the classical monopsonist's posted price.*

It is also a straightforward corollary from Proposition 3 that the direction of third-degree price discrimination also fixes itself when sellers are small:

Corollary 2 *For t sufficiently large, the market with higher price elasticity is offered the higher price.*

The assumption that the underlying (inverse) demand $V(.)$ function is continuous greatly simplifies the proof of these convergence results. We conjecture that it is possible to extend the argument to an exogenously discontinuous demand, and to demonstrate that our convergence results do not hinge on the continuity of $V(.)$.

7 Conclusion

In this paper, we delve into the micro-structure of monopsony and provide a "decentralized" mechanism, whose limit is the standard model. We show that there is no need for an "invisible hand": under mild conditions, optimal pricing with personalized commitment leads to a posted price even far away from the limit.

Our one-seller-one-unit set-up can be easily extended to multiunit sellers, as long as they have constant marginal costs. While, our procedure would allow the buyer to make a different price offer for each unit of a seller, optimally he would set a constant price for all. Increasing (decreasing) marginal costs would introduce the usual incentives towards distributing (concentrating) procurement over suppliers and would take us away from the classical model.

We have dealt with bilateral commitment in static games. In dynamic games and without dynamic (multilateral) commitment, the monopsonist could make their future decisions depend on past realizations of trade. That dynamic monopoly problem is an interesting extension of this paper.

The restriction to static mechanisms imposed by our main goal of microfounding monopsony, makes it impractical to think about our model in a mechanism design context: the sequential resolution of uncertainty would clearly be beneficial. Nonetheless, it is of note that our personalized pricing scheme is the best mechanism the buyer can devise subject to bilateral commitment in the static context. Insisting on bilateral commitment is the alternative approach to designing credible mechanisms where the principal does not cheat because the incentives are set right, as in Li (2017) and Akbarpour and Li (2018).

Finally, it is important to point out that the personalized price setting mechanism that we analyze in this paper can be usefully adapted to the context of competing price setters. Burguet and Sákovics (2017a, 2017b, 2019) are witnesses to this. In the first paper, personalized pricing leads to a model of simultaneous price competition without the need for rationing (in case, given prices, demand exceeds supply) or demand sharing (in case, given prices, supply exceeds demand) as these are determined endogenously by the equilibrium bid vectors. The equilibrium is unique even when marginal cost are increasing: the price is competitive with positive profits.

In the second paper there is competition for input between two firms that also compete in the product market. Here, personalized pricing allows firms to strategically target their offers at the suppliers of their competitors. The "competitive foreclosure" that ensues leads to higher aggregate input (and, therefore output and efficiency), contrary to the usual foreclosure logic, which tends to lead to inefficiency.

The third paper extends the previous study to the case where the product market is collusive, as in the competition for talent in a sports league. It provides micro-foundations for some classical invariance theorems in the literature.

8 Appendix

8.1 Proof of Proposition 1

First, we show that there can be no two different interior prices. Take any two interior prices, b^j, b^k. Given the rest of the prices, we can compute the probabilities that the buyer obtains $l \in \{0, 1, .., Q-2\}$ units from these other sellers. Let those probabilities be denoted by $\tilde{\Phi}_l(b_{-(j,k)})$. Then, the buyer's expected profit can be written as

$$\sum_{l=0}^{Q-2} \tilde{\Phi}_l(b_{-(j,k)}) \left\{ \left[F(b^k) + (1 - F(b^k))F(b^j)\right] v_{l+1} + F(b^j)F(b^k)v_{l+2} - F(b^j)b^j - F(b^k)b^k \right\}.$$

To see this, note that, from the two sellers considered, the buyer will buy at least one unit if either he buys from the k^{th} seller (and either buys or not from the j^{th} one) or if he does not buy from the k^{th} but buys from the j^{th} seller. He will get a second unit if and only if he buys from both. Finally, he pays each seller if and only if he buys from them.

Thus, the first-order condition for b^j is

$$\sum_{l=0}^{Q-2} \tilde{\Phi}_l(b_{-(j,k)}) \left\{ f(b^j) \left[F(b^k)v_{l+2} + \left(1 - F(b^k)\right) v_{l+1} - b^j\right] - F(b^j) \right\} = 0,$$

and similarly for b^k. As $\sum_{l=0}^{Q-2} \tilde{\Phi}_l(b_{-(j,k)}) = 1$, we can write this first-order condition in the familiar way (c.f. (3)),

$$\frac{F(b^j)}{f(b^j)} + b^j = \widehat{v}^j \quad ,$$

where

$$\widehat{v}^j = \sum_{l=0}^{Q-2} \tilde{\Phi}_l(b_{-(j,k)}) \left\{ F(b^k) v_{l+2} + \left(1 - F(b^k)\right) v_{l+1} \right\}$$

is the expected value of the unit potentially bought from seller j.[21] We can rewrite

$$\begin{aligned} \widehat{v}^j &= \sum_{l=0}^{Q-2} \tilde{\Phi}_l(b_{-(j,k)}) v_{l+1} - F(b^k) \sum_{l=0}^{Q-2} \tilde{\Phi}_l(b_{-(j,k)}) \left(v_{l+1} - v_{l+2}\right) \\ &= \sum_{l=0}^{Q-2} \tilde{\Phi}_l(b_{-(j,k)}) v_{l+1} - F(b^k) H(b_{-(j,k)}), \end{aligned}$$

where $H(b_{-(j,k)}) = \sum_{l=0}^{Q-2} \tilde{\Phi}_l(b_{-(j,k)}) \left(v_{l+1} - v_{l+2}\right)$. Then, the first-order conditions with respect to b^j and b^k imply

$$\begin{aligned} &\frac{F(b^j)}{f(b^j)} + b^j - F(b^j) H(b_{-(j,k)}) = \frac{F(b^k)}{f(b^k)} + b^k - F(b^k) H(b_{-(j,k)}) = \\ &\sum_{l=0}^{Q-2} \tilde{\Phi}_l(b_{-(j,k)}) v_{l+1} - \left[F(b^k) + F(b^j)\right] H(b_{-(j,k)}) \quad . \end{aligned}$$

Therefore, if

$$b + \frac{F(b)}{f(b)} - F(b) H(b_{-(j,k)}) \tag{10}$$

is strictly monotone, we must have $b^j = b^k$. Finally, observe that, since $H(b_{-(j,k)}) = \sum_{l=0}^{Q-2} \tilde{\Phi}_l(b_{-(j,k)}) \left(v_{l+1} - v_{l+2}\right) \leq \max_l \{v_l - v_{l+1}\}$, the strict monotonicity of $b + \frac{F(b)}{f(b)} - F(b) \max_l \{v_l - v_{l+1}\}$ implies strict monotonicity of (10) – and super-regularity implies the former. Repeating this argument for every pair of sellers, we obtain that all interior prices must be equal. Note that the above argument is independent of the support of $F(.)$.

We now prove that no corner prices – 0 or 1 – can be charged in equilibrium, so a posted interior price is indeed optimal. Assume that it is optimal to offer a price of 1 – note that, given $\bar{c} \geq 1$, that is the lowest offer possibly accepted for certain – to a seller. As the marginal value of this unit is bounded by 1, the maximum profit on this transaction is zero, attained only when the expected marginal valuation is indeed 1. But

[21] Note that, by regularity, the second-order condition is satisfied.

then, as the probability that a price $p < 1$ is accepted is $F(p) > 0$, offering a lower price would lead to a positive profit.

Next, assume that it is optimal to offer a price of 0 to a seller. Obviously, that would lead to no profit on that seller. However, as long as there is a positive expected marginal valuation for that "last" unit, an offer to buy for a price above it will be accepted with positive probability – note that, given $\underline{c} = 0$, there are always seller types below any positive value – and thus lead to positive marginal profit. The only way not to have a positive expected marginal valuation would be if the seller made at least one offer that is certainly accepted. However, we have just shown that such an offer is never made.

Finally, to obtain (4) just note that $\sum_{l=0}^{Q-2} \tilde{\Phi}_l(p^D) \left\{ F(p^D) v_{l+2} + \left(1 - F(p^D)\right) v_{l+1} \right\} = \sum_{l=0}^{Q-1} \chi_l(p^D) v_{l+1}$.

8.2 Proof of Theorem 2

We prove by induction. Let us first treat the trivial case where the lottery has two possible prizes. As f is concave, we directly have that $\underline{p_1 f(x_1) + p_2 f(x_2)}_f \leq f(p_1 x_1 + p_2 x_2)$. Now assume the result holds for n prizes. Now assume that we have $n+1$ prizes. Write (note the square brackets denote the integer part of an expression)

$$f\left(\sum_{i=1}^{n+1} p'_i x'_i\right) \equiv f\left((1 - p'_{n+1}) \sum_{i=1}^{n} \frac{p'_i x'_i}{1 - p'_{n+1}} + p'_{n+1} x'_{n+1}\right) \tag{11}$$

$$= f\left((1 - p'_{n+1}) \left(\left[\sum_{i=1}^{n} \frac{p'_i x'_i}{1 - p'_{n+1}}\right] + 1\right) + p'_{n+1} x'_{n+1}\right) \tag{12}$$

$$\geq \underline{(1 - p'_{n+1}) f\left(\left[\sum_{i=1}^{n} \frac{p'_i x'_i}{1 - p'_{n+1}}\right] + 1\right) + p'_{n+1} f\left(x'_{n+1}\right)}_f\ , \tag{13}$$

where the second equality follows from f being a step function with steps at integer values, while the inequality follows from the concavity of f. Applying the result for n prizes we obtain

$$\underbrace{(1-p'_{n+1})f\left(\left[\sum_{i=1}^{n}\frac{p'_i x'_i}{1-p'_{n+1}}\right]+1\right)+p'_{n+1}f\left(x'_{n+1}\right)}_{f} = \underbrace{(1-p'_{n+1})f\left(\sum_{i=1}^{n}\frac{p'_i x'_i}{1-p'_{n+1}}\right)+p'_{n+1}f\left(x'_{n+1}\right)}_{f} \geq$$

$$\underbrace{(1-p'_{n+1})\sum_{i=1}^{n}\frac{p'_i f(x'_i)}{1-p'_{n+1}}+p'_{n+1}f\left(x'_{n+1}\right)}_{f} = \underbrace{\sum_{i=1}^{n+1}p'_i f(x'_i)}_{f}$$

completing the proof.

8.3 Proof of Proposition 2

Note that both prices are equating the virtual cost to: i) in the no uncertainty case the demand function evaluated at the expected amount of trade; ii) in case of personalized pricing to the expected value of the marginal valuation (demand). Thus, as the virtual cost is increasing, all we need to show is that

$$\sum_{l=0}^{Q-1}\chi_l(p)v_{l+1} \leq V\left(QF(p)\right).$$

Since $V(.)$ is step-concave, by Theorem 2

$$\sum_{l=0}^{Q-1}\chi_l(p)v_{l+1} = \underbrace{\sum_{l=0}^{Q-1}\chi_l(p)V(l+1)}_{V} \leq V\left(\sum_{l=0}^{Q-1}\chi_l(p)(l+1)\right).$$

It is immediate – by the formula for the mean of the binomial distribution – that the right-hand side equals $V\left((Q-1)F(p)+1\right)$. Therefore, since V is decreasing, it follows that

$$\underbrace{\sum_{l=0}^{Q-1}\chi_l(p)v_{l+1}}_{V} \leq V\left(QF(p)\right).$$

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