Equilibrium Informativeness in Veto-Based Delegation

Dmitry Lubensky* and Eric Schmidbauer[†] September 27, 2016

Abstract

In veto delegation a biased expert recommends an action that an uninformed decision maker can accept or reject for an outside option. The arrangement is ubiquitous in political institutions, corporations, and consumer markets but has seen limited use in applications due to a poor understanding of the equilibrium set and an ensuing debate over selection. We develop a simple algorithm that constructs every veto equilibrium and identify the most informative equilibrium in a setting that spans prior work. We show that Krishna and Morgan's (2001) and Mylovanov's (2008) equilibria are maximally informative in their respective settings and strengthen Dessein's (2002) comparison of full and veto delegation. In an application we study the relationship between a patient and a doctor with a financial incentive to overtreat, and in contrast with existing literature show that the doctor's bias harms the patient *both* through excessive treatment and information loss, that the latter can be the dominant effect, and that insurance benefits both parties by improving communication.

Keywords: veto-based delegation, cheap talk, physician-induced demand, non-compliance

JEL Classification: D82, I10

^{*}Indiana University, Kelley School of Business. dlubensk@indiana.edu.

[†]University of Central Florida, College of Business Administration. eschmidb@ucf.edu.

1 Introduction

When an expert with an upward bias advises an uninformed decision maker, the effect of the bias depends on how the decision maker incorporates the advice. If the decision maker follows the advice exactly (i.e., full delegation), the bias induces higher actions than the decision maker prefers but allows all the expert's information to be transmitted. On the other hand, if the decision maker draws inference from the advice and then acts optimally (i.e., cheap talk), Crawford and Sobel (1982) (hereafter CS) shows that on average the actions are those preferred by the decision maker but communication is noisy.

Alternatively, the decision maker may draw inference but is restricted in his actions, as is the case in veto-based delegation in which the decision maker's only options are to accept the expert's proposal or to reject in lieu of an exogenous outside option. The veto terminology owes to a literature examining the "closed rule" governing legislative committees (Gilligan and Krehbiel, 1987, hereafter GK; Krishna and Morgan, 2001, hereafter KM), in which the full legislature may either accept the committee's bill without amendments or reject it entirely. However the veto-based arrangement is exceedingly common across a variety of settings beyond legislatures. In elections, constituents vote to approve or disapprove particular policies such as the issuance of municipal debt, but cannot for instance write in an alternative monetary amount for the bond; the board of directors of a corporation often holds the power to approve or disapprove proposals but not to unilaterally enact a proposal of their own; and in healthcare, a patient may either accept or reject a doctor's orders but may not self-prescribe treatment.¹

Aside from describing a ubiquitous institutional arrangement, the veto model also captures both of the known inefficiencies associated with biased experts. That is, in contrast to the benchmark CS and full delegation models, veto-based equilibria identified in the literature exhibit both noisy communication and excessive actions (see Figure 1). Yet despite its appeal the veto-based approach has seen limited application in economic modeling due to an ongoing debate, starting with GK and KM, about equilibrium selection. In sender-receiver games it is common to focus on the most informative equilibrium, for instance in the canonical CS environment the equilibrium set is well-understood and the most informative equilibrium is easily identified.² By contrast, the veto-based delegation equilibrium set is yet to be characterized. While KM's equilibrium

¹For additional examples of the veto-based arrangement see Marino (2007) and Mylovanov (2008).

²Chen et. al. (2008) propose instead a perturbation-based criterion and show that it too selects the most informative CS equilibrium.

Figure 1: Modes of Communication

Communication	Excessive	Noisy
Protocol	Action	Communication
Cheap Talk	No	Yes
Full Delegation	Yes	No
Veto-Based Delegation	Yes	Yes

rium is the most informative found to date, they note that "it appears to be difficult to characterize explicitly the most informative [veto-based delegation] equilibrium," (p. 445) and this has remained an open question.

In this paper we describe the full set of veto-based delegation equilibria. Using Melumad and Shibano's (1991) approach to reframe the sender's problem with the revelation principle, we show that every equilibrium is described by an interval partition of states into pooling and separating intervals, and that for each partition there is at most one action profile consistent with equilibrium, up to an initial condition. Then, we show that every equilibrium partition can be constructed with a simple algorithm, starting with the initial condition and sequentially choosing interval endpoints which satisfy constraints that depend only on the previous endpoint. Compared to the equilibrium set in CS, the veto-based delegation set is substantially larger.

We then use insights from constructing the set to identify the most informative equilibrium in the commonly studied uniform quadratic setting. Informativeness is related to how many states are covered by pooling rather than separating intervals, with separation possible only in states in which the status quo is an unattractive option. For example, with an intermediate status quo separation can be supported in both high and low states, but for intermediate states the status quo becomes viable, the sender strategically misreports, and pooling intervals arise. Using our equilibrium set characterization, we describe the minimal region of states over which these pooling intervals must occur, and then use this result to identify the most informative equilibrium for all values of the status quo.

While our main goal is to enable comparative statics in veto environments, by characterizing the equilibrium set and identifying its most informative element we also shed light on several issues in the existing literature. For example, GK and KM compare the informativeness of veto delegation versus pure cheap talk and obtain conflicting

results by focusing on different veto equilibria. The veto equilibrium of GK involves simpler strategies while the veto equilibrium of KM is more complex but also more informative, and the debate about the appropriate selection criterion remains active, with recent experimental work by Battaglini et. al. (2016) providing some support for GK's approach in a veto setting with multiple senders. However the inherent difficulty in this type of analysis is that comparing the GK and KM equilibria omits other equilibria that, for any particular selection criterion, may outperform both. Since we describe the full equilibrium set we help address this concern.

For instance, we demonstrate that the KM equilibrium is not simply more informative than GK's equilibrium and all cheap talk equilibria, but is in fact *the* most informative veto equilibrium. This in turn enables comparisons of outcomes other than informativeness, such as sender and receiver payoffs, between veto delegation and other mechanisms. In particular, we strengthen Dessein's (2002) (hereafter DE) result that the receiver prefers full delegation to veto delegation. DE uses the KM equilibrium since it "is thus far the most [informative] equilibrium identified in the literature" (p. 828), and we prove that there are no more informative equilibria. Our equilibrium construction algorithm can also directly address the apparent discrepancy of DE's result with Marino's (2007), who shows instead that the receiver can prefer veto-based to full delegation. While it has been suggested this discrepancy is due to the status quo being low in Marino and high in DE, we show that in fact it is the distributional assumptions that are crucial, and that even at DE's high status quo his result is easily reversed when the distribution is appropriately adjusted.

Our characterization of the equilibrium set also complements Mylovanov's (2008) finding that from the receiver's perspective veto delegation can outperform not just cheap talk or full delegation but in fact all communication protocols. In particular, an equilibrium of any communication game corresponds to an equilibrium of a constrained delegation game in which the receiver commits to accept a set of actions and then the sender chooses from among this set for each state. Alonso and Matouschek (2008) describes the optimal such delegation set for the receiver, and then Mylovanov shows that this constrained delegation outcome can be implemented without full commitment but rather in an equilibrium of a veto game when the status quo is appropriately chosen. However, with veto implementation "the problem of multiple equilibria is more severe...than in constrained delegation" (Mylovanov, 2008; footnote 2, p. 299) and it is not clear the receiver's preferred equilibrium will result. We show that in the uniform quadratic setting Mylovanov's equilibrium is maximally informative and in

fact Pareto dominant, lending further support for its selection.

To demonstrate some practical implications of our results, we apply the veto model to study the interaction between an informed doctor with a financial incentive to overtreat and an uninformed patient with the option to reject treatment. By focusing on the strategic behavior of both sides of the market we address a divide in the health literature where "papers on insurance and demand tend to view the supply side as competitive and accommodating; papers on supply tend to view patients as passively accepting provider recommendations" (McGuire, 2012; p.339). In contrast to the workhorse physician-induced demand framework (i.e. full delegation) in which the doctor's bias leads only to overtreatment, in the veto model the patient is additionally harmed by the information loss stemming from the doctor's strategic misdiagnosis to forestall rejection. We show the utility loss from the latter communication effect may be larger than that from the effect of a higher expected treatment, and thus that empirical studies focusing only on treatment level substantially underestimate the welfare effect of financial incentives.

We also examine the role of health insurance in which the patient's ex-post cost of treatment is reduced by paying an upfront actuarially fair premium. While a standard approach predicts extra treatment due to the patient's moral hazard, in the veto equilibrium the treatment level is determined solely by the doctor's bias, and thus the sole effect of insurance is to align doctor and patient preferences and improve communication, leading to a Pareto improvement. In this way, even risk neutral patients find insurance valuable as a means to reduce the doctor's incentive to strategically misdiagnose.

Our doctor-patient application is related to the work of Pitchik and Schotter (1987) and De Jaegher and Jegers (2001) who analyze a cheap talk game in which a doctor makes a recommendation to a patient who can obtain any available treatment. In these models the doctor prefers the maximal action regardless of the state, thus departing from the CS paradigm and resulting in substantially different equilibria and comparative statics.

The rest of the paper is organized as follows. In Section 2 we introduce the general model and describe the equilibrium set. In Section 3 we then restrict attention to the uniform quadratic setting and identify the most informative equilibrium across low, intermediate, and high values of the status quo. Finally we apply the model to the healthcare setting in Section 4 and conclude in Section 5.

2 General Model and Equilibrium Set

An uncertain state $\theta \sim F$ on [0,1] is observed by a sender but not by a receiver. The sender proposes an action $m \in \mathbb{R}$ and the receiver then chooses either to accept the proposed action a = m or to reject in lieu of an exogenous outside option $a = \hat{a}$. Sender and receiver preferences $u_s(a,\theta)$ and $u_r(a,\theta)$ are single-peaked at $a_s(\theta)$ and $a_r(\theta)$, with $a_s(\theta) > a_r(\theta)$, $a_s(\theta)$ and $a_r(\theta)$ continuous and increasing for all states θ , and $\lim_{a\to\infty}u_i(a,\theta)=\lim_{a\to-\infty}u_i(a,\theta)=-\infty$ for both i=s,r. Finally to ensure the status quo is relevant assume $\hat{a}\in[a_r(0),a_s(1)]$.

An action profile $a(\theta)$ is a perfect Bayesian equilibrium if (i) it satisfies the informed sender's incentive constraint, (ii) it generates beliefs for each accepted action that make accepting a best response for the receiver, and (iii) there exist beliefs for rejected actions off the equilibrium path that make rejecting a best response for the receiver. We look for the set of profiles $a(\theta)$ that satisfy these three conditions.

For a profile $a(\theta)$ let $A \equiv \{a(\theta), \theta \in [0, 1]\}$ be the set of accepted actions and for each accepted action let $\tau(a) \equiv \{\theta \mid a(\theta) = a\}$ be the set of states for which each action is induced. Also define $a^-(\theta) \equiv \lim_{\delta \to 0} a(\theta - \delta)$ and $a^+(\theta) \equiv \lim_{\delta \to 0} a(\theta + \delta)$. We now establish basic properties of any equilibrium.

Lemma 1 The profile $a(\theta)$ is an equilibrium profile if and only if

- (i) $a(\theta)$ is weakly increasing;
- (ii) if $a(\theta)$ is strictly increasing and continuous on an open interval, then $a(\theta) = a_s(\theta)$ on this interval;
- (iii) if $a(\theta)$ is discontinuous at θ then $u_s(a^-(\theta), \theta) = u_s(a^+(\theta), \theta)$ and

$$a(\theta') = \begin{cases} a^{-}(\theta) & \text{if } \theta' \in \left[a_s^{-1}(a^{-}(\theta)), \theta\right) \\ a^{+}(\theta) & \text{if } \theta' \in \left(\theta, a_s^{-1}(a^{+}(\theta))\right) \end{cases};$$

- (iv) if $\hat{\theta} \equiv a_s^{-1}(\hat{a}) \in [0, 1]$ then $a(\hat{\theta}) = \hat{a}$;
- $(v) \ E_{\tau(a)}[u_r(a,\theta)-u_r(\hat{a},\theta)] \geq 0 \ \ \textit{for all } a \in A.$

Proof The sender's incentive constraint requires that in each state her action is her most preferred in $A \cup \{\hat{a}\}$. A necessary condition is that she cannot improve by mimicking marginally higher or lower types, thus $u_s(a^-(\theta), \theta) = u_s(a^+(\theta), \theta)$ for all θ . The set of

profiles $a(\theta)$ that satisfy this local indifference condition is described in Proposition 1 of Melumad and Shibano (1991), and is restated here in conditions (i)-(iii). In addition, since a deviation to the status quo \hat{a} is always available even if no other types induce it, condition (iv) is required. Then, condition (v) ensures that for the receiver all accepted actions on the equilibrium path are a best response.

Finally for sufficiency it must also be shown that the sender would not deviate to some $a' \notin A$. This is easily accomplished by having the receiver reject every off the path a' and hold off the path beliefs (which are unrestricted) that $\theta = a_r^{-1}(\hat{a})$ for all off the path recommendations.

An implication of Lemma 1 is that all equilibrium profiles partition the set of states into separating and pooling intervals, in which the sender obtains her preferred action in all separating intervals, all pooling intervals include the state for which the pooling action is optimal for the sender, and upward jumps in a make the sender at that state indifferent between the lower and higher action. We now use these observations to demonstrate that every equilibrium can be constructed by a simple algorithm of sequentially choosing the endpoints of intervals that partition the set of states [0,1]. Proposition 1 describes the equilibrium set when $\hat{a} < a_s(0)$, in which case we begin by specifying $a_0 \equiv a(0)$ and intervals are constructed starting with $[0, \theta_1)$ and moving to the right. Proposition 2 then describes the equilibrium set when $\hat{a} \ge a_s(0)$, where condition (iv) of Lemma 1 implies that $a(\hat{\theta}) = \hat{a}$. In this case, which includes that considered in GK and KM, the initial interval (θ_0, θ_1) contains $\hat{\theta}$ and is interior, thus intervals are constructed sequentially both moving to the right and to the left.

First we define several objects of interest. Recalling that the sender's preferences are single-peaked, for $a \le a_s(\theta)$ let $a_s^+(\theta, a)$ be the action above $a_s(\theta)$ that gives the sender the same payoff as a, and conversely if $a \ge a_s(\theta)$ let $a_s^-(\theta, a)$ be the action below $a_s(\theta)$ that gives the sender the same payoff as a. Also, for a pooling interval with left endpoint θ_i and action $a > \hat{a}$ let $\bar{\theta}(a, \theta_i)$ be the smallest right endpoint for which the receiver accepts (i.e., for which $E_{(\theta_i,\bar{\theta})}[u_r(a,\theta)-u_r(\hat{a},\theta)] \ge 0$) and be equal to one if no such endpoint exists. Similarly for a pooling interval with right endpoint θ_{i+1} and action $a < \hat{a}$ let $\underline{\theta}(a,\theta_{i+1})$ be the largest left endpoint for which the receiver accepts and be equal to zero if no such endpoint exists.

Proposition 1 If $\hat{a} < a_s(0)$, then $a(\theta)$ is an equilibrium profile if and only if there exists an increasing (possibly infinite) sequence of interval endpoints $(0 = \theta_0, ..., \theta_I = 1)$ such that

i. on the initial interval $[\theta_0, \theta_1)$ there is a pooling action $a_0 \in [\hat{a}, a_s^+(0, \hat{a})]$,

ii. for any subsequent interval (θ_i, θ_{i+1})

- if $a^-(\theta_i) < a_s(\theta_i)$ then (θ_i, θ_{i+1}) is pooling on action $a_s^+(\theta_i, a^-(\theta_i))$,
- if $a^-(\theta_i) = a_s(\theta_i)$ then (θ_i, θ_{i+1}) is separating on $a_s(\theta)$ whenever (θ_{i-1}, θ_i) was pooling and (θ_i, θ_{i+1}) is pooling on $a_s(\theta_i)$ whenever (θ_{i-1}, θ_i) was separating,

iii. in any interval (θ_i, θ_{i+1}) with a pooling action $a_i \neq \hat{a}$, $\theta_{i+1} \geq \max (a_s^{-1}(a_i), \bar{\theta}(a_i, \theta_i))$.

Proof First we demonstrate that conditions (i), (ii), and (iii) are necessary. For condition (i), if $a_0 < \hat{a}$ or $a_0 > a_s^+(0,\hat{a})$, then at $\theta = 0$ the sender strictly improves by deviating to \hat{a} . Also the initial interval $[0,\theta_1)$ is pooling, else $a_0 = a_s(0)$ by Lemma 1(ii), the receiver perfectly infers $\theta = 0$ and rejects because $a_r(0) \le \hat{a} < a_s(0)$. For condition (ii), if the preceding interval ends on an action below the sender's preferred action, then by Lemma 1(iii) the next interval starts with an upward jump above the sender's preferred action, and thus cannot be separating. If the preceding interval ends with the sender's preferred action then by the local indifference condition the next interval also begins with the sender's preferred action. Then, either the preceding interval is pooling and the interval that follows it separating or vice versa, otherwise the boundary θ_i between the two intervals is not a true boundary since both the pooling or the separating intervals simply continue. Condition (iii) follows from conditions (iii) and (v) of Lemma 1. To establish sufficiency we need only construct off the equilibrium path beliefs so that it is a best response for the receiver to reject, which for example is accomplished with a posterior $\theta = a_r^{-1}(\hat{a})$.

It can be seen that Proposition 1 implies an algorithm by which any equilibrium must be constructed when $\hat{a} < a_s(0)$. First, from condition (i) choose initial action $a_0 \in [\hat{a}, a_s^+(0, \hat{a})]$ and label the initial interval pooling. Next choose the right endpoint θ_1 of the initial interval, observing the constraint in condition (iii) if $a_0 > \hat{a}$. At this point $a(\theta)$ is specified for $\theta \in [0, \theta_1)$. If $\theta_1 = 1$ then we have found an equilibrium described by $(a_0, \theta_1 = 1)$, else we proceed to the next interval (θ_1, θ_2) . Having fixed a_0 and θ_1 , condition (ii) then determines whether (θ_1, θ_2) is pooling or separating and in turn the resulting actions. The next choice is endpoint θ_2 , which must be large enough to satisfy the constraint in condition (iii) if the interval is pooling, and depends only on θ_1 and a_1 . If $\theta_2 = 1$ then we have found an equilibrium described by $(a_0, \theta_1, \theta_2 = 1)$, else we proceed to the next interval (θ_2, θ_3) . We then continue to repeat the previous step until an endpoint $\theta_I = 1$ is selected. By making a sequence of choices $(a_0, \theta_1, ..., \theta_I = 1)$ according to this process, the equilibrium action profile is uniquely identified at all

states except for a measure zero set of boundary points θ_i , at which either $a^-(\theta_i)$ or $a^+(\theta_i)$ is consistent with equilibrium.

The set of all equilibria is thus described by the set of sequences (a_0 , θ_1 , ..., $\theta_I = 1$) that can be constructed using the above process. It is instructive to view this set in terms of nested subclasses; for instance there is a class of equilibria with the same $a_0 = \tilde{a}$, and a subclass of this class with $a_0 = \tilde{a}_0$ and $\theta_1 = \tilde{\theta}_1$, and so forth.

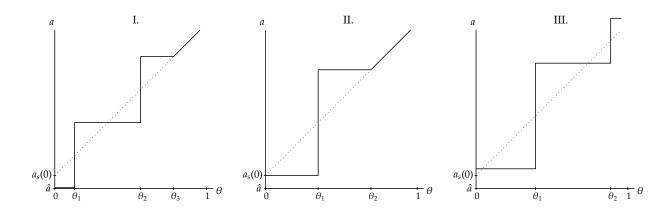


Figure 2: Examples of equilibrium action profiles when $\hat{a} < a_s(0)$. The solid line is action profile $a(\theta)$, the dotted line is the preferred action of the sender $a_s(\theta)$, and the horizontal axis corresponds to the status quo \hat{a} .

Figure 2 depicts several equilibrium profiles, each satisfying the sender's local indifference condition by having separating intervals coincide with $a_s(\theta)$ and by reflecting the action profile above $a_s(\theta)$ at the boundaries of two pooling intervals. Note also that all interior pooling intervals intersect $a_s(\theta)$, i.e. they include the sender for whom that action is optimal. Each of the three profiles starts with a pooling interval $[0, \theta_1)$ according to Proposition 1. The smallest allowable length of the first pooling interval depends on the starting value a_0 , and as a_0 increases in going from I to III, so does the value of θ_1 . The second interval in each example is also pooling, and this will be shown to always be the case, however the third interval may be pooling as in I and III or separating as in II. In addition, note that while the interval $[0, \theta_1)$ is smaller in I than in II, the total number of states covered by pooling intervals is larger in I than in II, and we will demonstrate that sometimes an equilibrium with the structure of II may be more informative than that of type I. That is, constructing an equilibrium by first choosing the smallest allowable θ_1 , and then the smallest allowable θ_2 , and so forth does not necessarily produce the most informative equilibrium.

Next we describe the equilibrium set when $\hat{a} \ge a_s(0)$, the construction of which is very similar to Proposition 1 except that the initial interval is now interior.

Proposition 2 If $\hat{a} \ge a_s(0)$, then $a(\theta)$ is an equilibrium profile if and only if there exists an increasing (possibly infinite) sequence of interval endpoints $(0 = \theta_{-J}, ..., \theta_0, ..., \theta_I = 1)$ such that

- *i.* the initial interval $[\theta_0, \theta_1)$ contains the state $\hat{\theta} \equiv a_s^{-1}(\hat{a})$ and has pooling action $a_0 = \hat{a}$;
- *ii.* for any subsequent $(i \ge 1)$ interval (θ_i, θ_{i+1})
 - if $a^-(\theta_i) < a_s(\theta_i)$ then (θ_i, θ_{i+1}) is pooling on action $a_s^+(\theta_i, a^-(\theta_i))$,
 - if $a^-(\theta_i) = a_s(\theta_i)$ then (θ_i, θ_{i+1}) is separating on $a_s(\theta)$ whenever (θ_{i-1}, θ_i) was pooling and (θ_i, θ_{i+1}) is pooling on $a_s(\theta_i)$ whenever (θ_{i-1}, θ_i) was separating

and for any preceding $(i \le -1)$ interval (θ_i, θ_{i+1})

- if $a^+(\theta_{i+1}) > a_s(\theta_{i+1})$ then (θ_i, θ_{i+1}) is pooling on action $a_s^-(\theta_{i+1}, a^+(\theta_{i+1}))$,
- if $a^+(\theta_{i+1}) = a_s(\theta_{i+1})$ then (θ_i, θ_{i+1}) is separating on $a_s(\theta)$ whenever $(\theta_{i+1}, \theta_{i+2})$ was pooling and (θ_i, θ_{i+1}) is pooling on $a_s(\theta_{i+1})$ whenever $(\theta_{i+1}, \theta_{i+2})$ was separating
- *iii.* in any interval (θ_i, θ_{i+1}) with a pooling action $a_i \neq \hat{a}$,

$$\theta_{i+1} \ge \max \left(a_s^{-1}(a_i), \ \bar{\theta}(a_i, \theta_i)\right) \quad \text{if } i \ge 1, \text{ and}$$

$$\theta_i \le \min \left(a_s^{-1}(a_i), \underline{\theta}(a_i, \theta_{i+1})\right) \quad \text{if } i \le -1.$$

Proof That $a(\hat{\theta}) = \hat{a}$ in condition (i) is given by condition (iv) of Lemma 1. There must also be pooling to the right of $\hat{\theta}$, otherwise $a(\theta) = a_s(\theta)$ at slightly higher states and then by continuity $a_r(\theta) < \hat{a} < a(\theta)$ so the receiver would reject. Conditions (ii) and (iii) are necessary by the same arguments as in Proposition 1. The argument for sufficiency is also similar. The receiver's beliefs are unrestricted for an off the path action a' and again if the posterior is $\theta = a_r^{-1}(\hat{a})$ then the receiver rationally rejects. For the sender, any off the path message induces the status quo action \hat{a} , which is now on the equilibrium path and thus already demonstrated to not constitute an improvement.

Largely the same procedure applies here as in Proposition 1, in that interval endpoints are chosen sequentially, working outward from the initial interval and depend solely on the last chosen endpoint. Importantly, decisions about interval endpoints to the right and left of the initial interval $[\theta_0, \theta_1)$ are independent of one another. That is, suppose interval (θ_2, θ_3) is pooling on a_2 . By condition (iii), in order to induce the receiver to accept, the right endpoint $\theta_3 \geq \bar{\theta}(a_2, \theta_2)$. Thus, it cannot be guaranteed that θ_3 is sufficiently large until θ_2 is chosen and similarly θ_2 cannot be fixed until θ_1 is chosen. However this is where the chain ends, because on the interval $[\theta_0, \theta_1)$ the action \hat{a} cannot be rejected, and thus any $\theta_1 > \hat{\theta}$ is sufficiently large regardless of the value of θ_0 . Thus, if the sequences $(\theta_{-I}, ..., \theta_0)$ and $(\theta_1, ..., \theta_I)$ are chosen independently by the procedure above, they are mutually consistent and constitute an equilibrium.

The equilibrium set is thus a collection of sequences $(\theta_{-J}, ..., \theta_0, ..., \theta_I)$ that can be parsed into classes and subclasses as in Proposition 1. For instance, there is a class of equilibria with $\theta_0 = \tilde{\theta}_0$, and in that class a subclass in which $\theta_0 = \tilde{\theta}_0$ and $\theta_{-1} = \tilde{\theta}_{-1}$, and so forth.

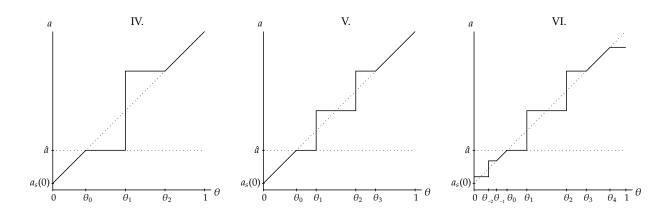


Figure 3: Examples of equilibrium action profiles when $\hat{a} \ge a_s(0)$. The solid line is action profile $a(\theta)$, the dotted line is the preferred action of the sender $a_s(\theta)$, and the horizontal axis corresponds to the status quo action \hat{a} .

In Figure 3, in each of the three examples $\theta_0 = \hat{\theta}$ and all equilibria are anchored at $a(\hat{\theta}) = \hat{a}$ according to Proposition 2. The equilibrium in IV is of the type studied by GK and the equilibrium in V is of the type studied by KM. These two equilibria start and end with separating intervals but differ in the way pooling intervals are constructed for intermediate states. It is visually apparent that V constitutes a finer partition of the states than does IV, and potentially these two equilibria may be easily compared. However, there exist many other equilibria, such as that in VI, which look quite different from IV and V, having many more intervals and not necessarily centered around the sender's preferred actions. Comparisons to these type of equilibria appear less straightforward.

Size of Equilibrium Set versus Crawford and Sobel (1982)

A common feature of sender-receiver games is that there may be more than one equilibrium, for example it is well known that there are multiple equilibria in the canonical CS cheap talk framework. However, with veto-based delegation it is easy to show that the equilibrium set is qualitatively larger than in pure cheap talk. To see this, consider constructing the equilibrium set in pure cheap talk in the following manner. First, choose $\theta_1 > 0$, the right endpoint of initial pooling interval $[0, \theta_1)$. Given θ_1 , the only consistent pooling action on this interval is $a_0 = \arg\max_a E_{[0,\theta_1)}[u_r(a,\theta)]$ since the receiver's actions are unconstrained, and since preferences are single-peaked and monotonic a_0 is unique. Next, with a_0 and θ_1 fixed, the interval (θ_1, θ_2) must be pooling on an action a_1 that makes the sender at θ_1 indifferent, thus the next action is $a_1 = a_s^+(\theta_1, a_0)$. Furthermore, for the receiver to take action a_1 on interval (θ_1, θ_2) , it must be that $a_1 = \arg\max_a E_{(\theta_1, \theta_2)}[u_r(a, \theta)]$, again guaranteed to be unique. Thus, for a given θ_1 , there is at most one endpoint θ_2 that satisfies both of these conditions. Extending this logic, there is at most one value of θ_3 consistent with θ_2 and so forth, and thus there is at most a single equilibrium for any chosen θ_1 .³

By contrast, under veto-based delegation for a chosen endpoint θ_1 there is potentially a continuum of consistent values for θ_2 , and for the pair (θ_1, θ_2) there is potentially a continuum of consistent values for θ_3 , et cetera. Thus, while under CS the equilibrium set can be indexed by a single parameter θ_1 , the set of veto-based delegation equilibria is substantially larger and thus more difficult to classify.

3 Informativeness

The multiplicity of equilibria makes the veto model difficult for use in applications and is at the root of a long-standing debate, starting with GK and KM, about which equilibria should be studied. Since our goal is to make the veto model amenable to comparative statics, welfare, and other similar analyses we now focus on equilibrium selection. At the current level of generality the exercise is intractable and therefore we restrict attention to the constant bias uniform-quadratic specification which is commonly used in the literature (e.g., CS, GK, and KM). Payoffs to the sender and receiver are

$$u_s = -(a - (\theta + b_s))^2$$
 and $u_r = -(a - (\theta - b_r))^2$, $b_s, b_r \ge 0$, (1)

³And as established in CS, there is only a finite set of values of θ_1 that is consistent with an equilibrium.

with preferred actions $a_s(\theta) = \theta + b_s$ and $a_r(\theta) = \theta - b_r$. In a cheap talk environment such as CS, b_r is typically normalized to zero since the only relevant quantity is $b_s + b_r$. This assumption would be restrictive here due to the existence of a status quo action \hat{a} , and we will demonstrate that comparative statics with respect to b_r and b_s differ. To exclude trivial cases we assume $\hat{a} \in [-b_r, 1 + b_s]$ and $b_s + b_r \le \frac{1}{4}$.

Let $z(\theta) \equiv a(\theta) - \theta$ be the realized bias at state θ . Henceforth, when discussing the equilibrium profile, we will refer to either $a(\theta)$ or $z(\theta)$, depending on context.

Definition The informativeness of a profile is $-Var(z(\theta))$.

With some standard algebraic manipulation the ex-ante preferences of the sender and receiver can be expressed as

$$E[u_s] = -Var(z) - (E[z] - b_s)^2$$
 and $E[u_r] = -Var(z) - (E[z] + b_r)^2$, (2)

decomposing into preferences over informativeness and expected bias. All else equal, an increase in informativeness is a Pareto improvement.

The informativeness of an equilibrium is closely related to how many states are covered by pooling rather than separating intervals. For instance, the full delegation profile $a(\theta) = \theta + b_s$ is maximally informative with Var(z) = 0, however it is not an equilibrium since by Propositions 1 and 2 every equilibrium starts with a pooling interval. In fact, as we now demonstrate in the following two lemmas, there is a minimal region of states around where the status quo is viable in which only pooling intervals can be supported. As we will argue, finding the most informative equilibrium can reduce to identifying the most informationally efficient way to cover this pooling region.

To reduce notation define $\hat{\theta} \equiv \hat{a} - b_s$ as the (possibly negative) state at which the sender's preferred action is the status quo.

Lemma 2 If $\hat{a} < b_s$, there is a minimal region T with no separating intervals such that

- (i) if $a(0) = \hat{a}$ then $T = (0, \hat{\theta} + 4(b_s + b_r))$ is covered by at most three pooling intervals;
- (ii) if $a(0) > \hat{a}$ then $T = (0, 2\hat{\theta} + 4(b_s + b_r))$ is covered by at most two pooling intervals.

Lemma 3 If $\hat{a} \ge b_s$, there is a minimal region T with no separating intervals such that

(i) $T = (\hat{\theta}, \hat{\theta} + 4(b_s + b_r))$ is covered by at most three pooling intervals if $\hat{a} \in [b_s, 1 - 3b_s - 4b_r]$;

(ii) $T = (\hat{\theta}, 1)$ is covered by at most three pooling intervals if $\hat{a} \in [1 - 3b_s - 4b_r, 1 - 2b_s - 3b_r)$, at most two if $\hat{a} \in [1 - 2b_s - 3b_r, 1 - b_s - 2b_r)$, and precisely one if $\hat{a} \in [1 - b_s - 2b_r, 1 + b_s]$.

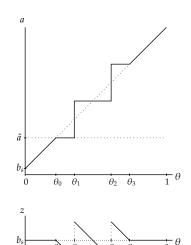
The proofs, both of which can be found in the Appendix, are constructive and follow directly from the equilibrium structure in Propositions 1 and 2. To get some intuition for why a minimal pooling region must exist, suppose there is a separating state $\theta \in (\hat{\theta}, 1]$ so that $a(\theta) = \theta + b_s$. The receiver's best response is to accept only if her preferred action is closer to the prescribed action than to the status quo: $\theta - b_r \ge \frac{1}{2}(\hat{a} + (\theta + b_s)) \Rightarrow \theta \ge \hat{\theta} + 2(b_s + b_r)$. The pooling region must thus extend at least to $\hat{\theta} + 2(b_s + b_r)$ to satisfy the receiver's incentives, and in fact it must extend even farther to also satisfy the sender's incentives. That is, if type $\hat{\theta} + 2(b_s + b_r)$ induced her preferred action then lower types would mimic, reducing the receiver's posterior and causing to receiver to reject. Accounting for this incentive to mimic then further extends the boundary.

We now describe the most informative equilibrium. By considering the full range of statuses quo our work applies to prior investigations of veto-based delegation. For example, the region in the ensuing Proposition 3 corresponds to that studied in GK, KM, and DE, while the lowest values of the status quo, as in Marino (2007), are covered in Proposition 4. The low range is of particular interest for the application to the doctor patient relationship in the following section. Finally, the highest values of the status quo are treated in Propositions 5 and 6, with the latter covering Mylovanov (2008). For convenience, we include a graphical depiction of the action profile $a(\theta)$ and corresponding bias profile $z(\theta) = a(\theta) - \theta$ in each proposition.

Intermediate status quo

Proposition 3 If $\hat{a} \in [b_s, 1 - 3b_s - 4b_r]$ then the strictly most informative equilibrium is separating for high and low states and has three pooling intervals for intermediate states, with partition $(0, \theta_0, \theta_1, \theta_2, \theta_3, 1) = (0, \hat{a} - b_s, \hat{a} + b_r, \hat{a} + 2b_s + 3b_r, \hat{a} + 3b_s + 4b_r, 1)$ and actions:

$$a^{*}(\theta) = \begin{cases} \theta + b_{s} & \text{if } \theta \in [0, \theta_{0}) \\ \hat{a} & \text{if } \theta \in (\theta_{0}, \theta_{1}) \\ \hat{a} + 2(b_{s} + b_{r}) & \text{if } \theta \in (\theta_{1}, \theta_{2}) \\ \hat{a} + 4(b_{s} + b_{r}) & \text{if } \theta \in (\theta_{2}, \theta_{3}) \\ \theta + b_{s} & \text{if } \theta \in (\theta_{3}, 1] \end{cases}$$



with $E[z^*] = b_s$, and $Var[z^*] = \frac{4}{3}(b_s + b_r)^3$.

Proof Sketch Recall that $\hat{\theta} \equiv \hat{a} - b_s$, define $\bar{\theta} \equiv \hat{a} + 3b_s + 4b_r$, and observe that the proposed profile z^* is separating for low states $[0, \hat{\theta})$ and high states $(\bar{\theta}, 1]$. Since in the separating intervals $z^*(\theta) = b_s$ it follows that

$$Var(z^*) = \int_0^1 (z^*(\theta) - b_s)^2 d\theta = \int_{\hat{\theta}}^{\bar{\theta}} (z^*(\theta) - b_s)^2 d\theta,$$

and therefore it suffices to show that z^* outperforms any other profile only over $(\hat{\theta}, \bar{\theta})$.

Meanwhile, in this region every candidate equilibrium profile $z(\theta)$ must satisfy certain properties. In particular, the profile begins with $z(\hat{\theta}) = b_s$ (Proposition 2(i)), is covered by no separating intervals and at most three pooling intervals (Lemma 3), and has at most one state $t \in (\hat{\theta}, \bar{\theta})$ for which $z(t) = b_s$ (proof of Lemma 3). With this in mind, consider the candidate $z(\theta)$ with three pooling intervals depicted in Figure 4. Over the region $(\hat{\theta}, t)$ the realizations of z are uniformly distributed on an interval of length $t - \hat{\theta}$, centered at t = t + t, also centered at t = t + t as depicted in the figure, increasing t = t to bring the lengths of these two intervals closer in size creates a mean-preserving contraction in the distribution of t = t + t and thus increases informativeness. The optimal value is in fact $t = \frac{1}{2}(\hat{\theta} + \bar{\theta})$, which corresponds to the proposed profile t = t + t + t

This argument demonstrates that z^* outperforms any z with three pooling intervals in $(\hat{\theta}, \bar{\theta})$ which has $z(\bar{\theta}) = b_s$. To complete the proof, we must also consider candidate profiles for which $z(\bar{\theta}) \neq b_s$ and profiles that cover $(\hat{\theta}, \bar{\theta})$ with one or two pooling intervals. In these situations, we first construct an auxiliary profile \tilde{z} that has lower

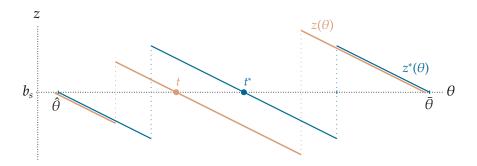


Figure 4: A graph of biases for candidate profile $z(\theta)$ (in tan) and proposed profile $z^*(\theta)$ (in blue) over the range $(\hat{\theta}, \bar{\theta})$. Since z^* is symmetric around t^* it is shown to be a mean-preserving contraction of z over this range, and is thus more informative.

variance than the candidate z and a mean $E[\tilde{z} \mid \theta \in (\hat{\theta}, \bar{\theta})] = b_s$, after which we make a similar mean preserving spread argument with respect to \tilde{z} . The full proof can be found in the Appendix.

The range of intermediate values of the status quo \hat{a} in Proposition 3 includes the range considered by GK, KM, and DE, and the most informative equilibrium z^* matches the equilibrium in KM's Proposition 8, which has thus far been used as a means to compare across different communication games.⁴ By identifying this equilibrium as the *most* informative equilibrium under veto-based delegation we aid these comparisons. For instance, while KM indirectly prove that the most informative veto equilibrium is more informative than that under cheap talk, without identifying this equilibrium they do not know its other properties and thus cannot compare sender or receiver payoffs across mechanisms, which the present work now allows. In addition, while DE finds that full delegation is better for the receiver than KM's particular veto equilibrium, which DE uses since it "is thus far the most [informative] equilibrium identified in the literature" (p. 828), we strengthen the result by showing there are in fact no other more informative equilibria.⁵

⁴GK focuses instead on a different equilibrium with two pooling intervals as depicted in Figure 3(IV), and a broader debate about equilibrium selection in this environment has persisted (Krehbiel, 2001; Battaglini, 2016), in part centered on assumptions for beliefs off the equilibrium path. While we do not make explicit restrictions in the present work, it should be noted that the most informative equilibrium is supported by beliefs surviving several reasonable refinements, including monotonicity and the Cho and Kreps (1987) intuitive criterion.

⁵While the receiver strictly prefers full delegation to the most informative veto-based equilibrium a^* in Proposition 3, it should be noted that a^* is not the best veto-based equilibrium for the receiver. For example, consider the profile a' which mimics a^* for states $(0, 1 - 2(b_s + b_r))$ and pools on action $1 - b_s - 2b_r$

The equilibrium identified in Proposition 3 can also shed light on the discrepancy between DE's result that full delegation outperforms veto-based delegation for the receiver and Marino's (2007) finding that the opposite can hold. It has been suggested in the literature (Marino, 2007; Mylovanov, 2008) that this finding results from the fact that in Marino the status quo is favorable for veto-based delegation and in DE it is not. However, we can show that veto can outperform full delegation for the receiver for any status quo, including that used in DE, if the distribution is appropriately chosen. In particular, using a similar argument to Marino (2007), suppose that θ is uniformly distributed on each interval identified in Proposition 3, but that most of the mass is on the first pooling interval $(\hat{a} - b_s, \hat{a} + b_r)$. It is easy to verify that a^* still constitutes an equilibrium, since the informed sender's incentive constraint does not depend on distributional assumptions while the receiver's constraints in each interval depend only on conditional expectations, which have remained unchanged. Since veto-based delegation outperforms full delegation for the receiver on $(\hat{a} - b_s, \hat{a} + b_r)$, it is the preferred mechanism whenever this interval has sufficient probability mass.

Low status quo

Next we explore values of the status quo that are strictly below the preferred action of even the lowest type of sender ($\hat{a} < b_s$). A low status quo was considered by Marino (2007) and is plausible in many situations. For instance a car owner's outside option may be to perform no repairs while a mechanic, even when observing no necessity for repair, may prefer the owner to pay for a small level of service. Similarly, a patient's outside option may be no treatment at all while a doctor, even if observing a fully healthy patient, may prefer the patient undertake some further costly diagnostics.

In this parameter range much of the logic of equilibrium construction remains the same as previously. The major departure is that now there is no sender type whose preferred action is the status quo, thus as opposed to the previous case in which $a(\hat{\theta}) = \hat{a}$, there is no longer a fixed initial condition from which to start equilibrium construction. Instead, the initial interval is now $[0, \theta_1)$ with associated pooling action a_0 and we examine two families of equilibria, those in which $a_0 = \hat{a}$ and those in which $a_0 > \hat{a}$. We

for states $(1 - 2(b_s + b_r), 1)$. The profile a' is less informative than a^* but has a lower expected action, and it is easily verified that a' constitutes an equilibrium and gives the receiver a strictly higher payoff than a^* . In fact, the receiver's payoff under a' is exactly equal to his payoff under full delegation.

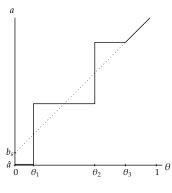
⁶Recall from Proposition 2 that there is no equilibrium with $a_0 < \hat{a}$ because the sender of type 0 would deviate to induce \hat{a} .

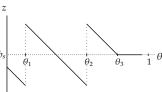
show there is a threshold for \hat{a} above which the most informative equilibrium is of the former type and below which it is of the latter type.

Proposition 4 *If* $\hat{a} \in [-b_r, b_s]$, then there exists an $\underline{a} < b_s$ such that the strictly most informative equilibrium is separating for high states and

(i) if $\hat{a} \in (\underline{a}, b_s]$ has three pooling intervals for all lower states with partition $(0, \theta_1, \theta_2, \theta_3, 1) = (0, \hat{a} + b_r, \hat{a} + 2b_s + 3b_r, a + 3b_s + 4b_r, 1)$ and actions:

$$a_1^*(\theta) = \begin{cases} \hat{a} & \text{if } \theta \in (0, \theta_1) \\ \hat{a} + 2(b_s + b_r) & \text{if } \theta \in (\theta_1, \theta_2) \\ \hat{a} + 4(b_s + b_r) & \text{if } \theta \in (\theta_2, \theta_3) \\ \theta + b_s & \text{if } \theta \in (\theta_3, 1) \end{cases}$$

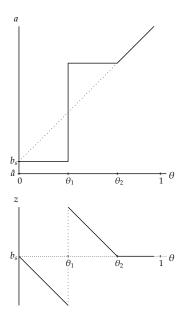




with
$$E[z_1^*] = b_s + \frac{1}{2}(b_s - \hat{a})^2$$
 and $Var(z_1^*) = \frac{4}{3}(b_s + b_r)^3 - \frac{1}{3}(b_s - \hat{a})^3 - \frac{1}{4}(b_s - \hat{a})^4$;

(ii) if $\hat{a} \in [-b_r, \underline{a})$ has two pooling intervals for all lower states with partition $(0, \theta_1, \theta_2, 1) = (0, b_s + 2b_r + \hat{a}, 2b_s + 4b_r + 2\hat{a}, 1)$ and actions:

$$a_2^*(\theta) = \begin{cases} b_s & \text{if } \theta \in (0, \theta_1) \\ 3b_s + 4b_r + 2\hat{a} & \text{if } \theta \in (\theta_1, \theta_2) \\ \theta + b_s & \text{if } \theta \in (\theta_2, 1) \end{cases}$$



with
$$E[z_2^*] = b_s$$
 and $Var(z_2^*) = \frac{2}{3}(b_s + 2b_r + \hat{a})^3$.

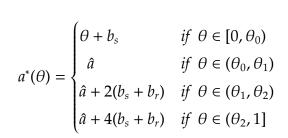
Proof Sketch For part (i), in contrast to Proposition 3 separating intervals now contribute to the variance of z_1^* since in the proposed equilibrium $E[z_1^*] > b_s$, and a different technique is used for the proof. We first fix a candidate profile z and construct an auxiliary profile \tilde{z} that mimics z for low states and uniformly shifts z up for high states, so that $E[\tilde{z}] = E[z_1^*]$. We show that \tilde{z} is more informative than z, and that \tilde{z} is a mean preserving spread of z_1^* . For part (ii) since $E[z_2^*] = b_s$ we follow the same approach as in Proposition 3, showing that it is sufficient to focus on states $(0, \theta_2)$ and that z_2^* is best on this range. Finally, a direct comparison of the expressions for informativeness in parts (i) and (ii) implies the existence of a threshold value for \hat{a} that determines which of the two equilibria is most informative overall.

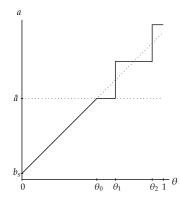
High status quo

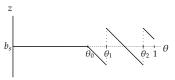
Finally we consider a higher range of values for the status quo. Starting at the right boundary of the intermediate case from Proposition 3, we show that initially the most informative equilibrium can no longer support a separating interval for the highest states but still has three pooling intervals. Then, as the status quo is increased an equilibrium with two pooling intervals becomes more informative. Finally, for the highest non-trivial values of the status quo, including the range studied in Mylovanov (2008), the most informative equilibrium ends in a single interval pooled on the veto \hat{a} .

Proposition 5 If $\hat{a} \in [1-3b_s-4b_r, 1-b_s-2b_r)$ then there exists an $\tilde{a} \in (1-3b_s-4b_r, 1-2b_s-3b_r)$ such that the strictly most informative equilibrium is separating for low states and

(i) if $\hat{a} \in [1 - 3b_s - 4b_r, \tilde{a})$ has three pooling intervals for all higher states with partition $(0, \theta_0, \theta_1, \theta_2, 1) = (0, \hat{a} - b_s, \hat{a} + b_r, \hat{a} + 2b_s + 3b_r, 1)$ and actions:



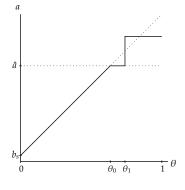


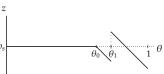


with $E[z^*] = b_s - \frac{1}{2}\delta^2$ and $Var[z^*] = \frac{4}{3}(b_s + b_r)^3 - \frac{1}{3}\delta^3 - \frac{1}{4}\delta^4$, where $\delta = \hat{a} + 3b_s + 4b_r - 1$;

(ii) if $\hat{a} \in (\tilde{a}, 1 - b_s - 2b_r)$ has two pooling intervals for all higher states with partition $(0, \theta_0, \theta_1, 1)$, where $\theta_0 = \hat{a} - b_s$ and θ_1 solves the first order condition from Equation (12) in the Appendix, and actions:

$$a^*(\theta) = \begin{cases} \theta + b_s & \text{if } \theta \in [0, \theta_0) \\ \hat{a} & \text{if } \theta \in (\theta_0, \theta_1) \\ 2(1 - b_s - 2b_r) - \hat{a} & \text{if } \theta \in (\theta_1, 1] \end{cases}$$



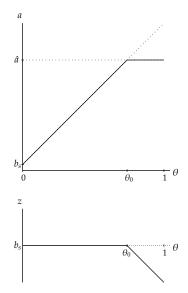


with $E[z^*] = b_s - \frac{1}{2}\eta^2$, $Var[z^*] = \frac{1}{12}(1 - \hat{a} + b_s - \eta)^3 + \frac{1}{3}\eta^3 - \frac{1}{4}\eta^4$, where $\eta = 1 + \hat{a} - b_s - 2\theta_1$.

Proof Sketch This proof is more computational than the ones preceding. It is straightforward to demonstrate that the most informative profile must separate for states $[0, \hat{a} - b_s]$ and from there be covered by either two or three pooling intervals. Then, finding the most informative equilibrium requires an optimization in θ_1 and θ_2 for profiles with three intervals and in θ_1 for profiles with two intervals. A calculation then shows that three pooling intervals are optimal up to some status quo \tilde{a} , and beyond two pooling intervals are optimal.

Proposition 6 If $\hat{a} \in [1 - b_s - 2b_r, 1 + b_s]$ then the strictly most informative equilibrium is separating for low states and pooling for all higher states, with partition $(0, \hat{a} - b_s, 1)$ and actions:

$$a^*(\theta) = \begin{cases} \theta + b_s & \text{if } \theta \in [0, \hat{a} - b_s) \\ \hat{a} & \text{if } \theta \in (\hat{a} - b_s, 1] \end{cases}$$



with
$$E[z^*] = b_s - \frac{1}{2}\gamma^2$$
 and $Var[z^*] = \frac{1}{3}\gamma^3 - \frac{1}{4}\gamma^4$, where $\gamma = 1 - \hat{a} + b_s$.

Proof Sketch In this parameter range \hat{a} is so high that no action above it can be induced, and thus all equilibria pool on \hat{a} for $\theta \geq \hat{\theta}$. We then show that separating up to θ_0 is optimal.

Statuses quo in this region have been of particular interest in the constrained delegation literature. In constrained delegation a receiver commits to a set of actions and then the informed sender chooses from among this set. Any veto equilibrium is thus a constrained delegation equilibrium, since the sender implicitly chooses from among the actions accepted by the receiver in equilibrium, and it has been shown by Mylovanov (2008) that the optimal constrained delegation equilibrium for the receiver is also an equilibrium in a veto game when the status quo action is appropriately chosen. In the present uniform quadratic setting that status quo is $\hat{a} = 1 - b_s - 2b_r$, which falls into the

region covered by Proposition 6. Mylovanov points out that while for this status quo his particular equilibrium implements the receiver's best outcome, there are potentially many other equilibria, and having characterized the set of equilibria we can provide some support for selecting the equilibrium that Mylovanov studies.

Corollary 1 The Mylovanov equilibrium that implements the receiver's optimal constrained delegation outcome has status quo $\hat{a} = 1 - b_s - 2b_r$, and for this \hat{a} is the most informative and Pareto dominant veto equilibrium.

The Pareto dominance result is straightforward to demonstrate. For statuses quo in this highest range every equilibrium must pool on the status quo action for all states $\theta \in [\hat{a} - b_s, 1]$ (by Lemmas 1(iv) and 3(ii)). Thus any other candidate equilibrium differs only on states $\theta \in [0, \hat{a} - b_s)$ and on these states has a bias higher than b_s and a lower informativeness than Mylovanov's equilibrium, with both effects harming the sender and receiver.

4 An application: the doctor-patient interaction

In this section we apply the veto-based delegation model to the doctor-patient relationship, in which information asymmetry problems may naturally arise. Estimates of avoidable clinical care are up to \$700 billion annually in the United States,⁷ and one often-cited cause is the financial incentive of doctors to prescribe more treatment than is medically prudent.⁸ Doctors often exert significant control over treatment since patients cannot self-prescribe, though patients may reject treatment and often do so when they lack trust in their doctor, in particular when suspecting a financial motive.⁹ Veto-based delegation thus closely matches the institutional details of the doctor-patient relationship: an informed and biased expert makes a recommendation to an uninformed but rationally skeptical consumer with the option to reject.

In addition, veto-based delegation predicts outcomes of the doctor-patient relationship better than competing models in the literature, which often focus on only one side of the market while treating the other as a passive participant (McGuire, 2012).

⁷Berwick and Hackbarth (2012) and Institute of Medicine (2010), "The healthcare imperative: lowering costs and improving outcomes: workshop series summary". Washington, D.C.: The National Academies Press.

⁸Emanuel and Fuchs (2008).

⁹See Brownlie et al. (2008) and Chen and Vargas-Bustamante (2013).

For example, a popular approach in the health economics literature is the physician-induced demand framework, in which the patient in effect delegates decision-making to the doctor. Following this approach, communication is unmodeled and a patient's willingness to accept a proposed treatment does not depend on the doctor's financial incentives. Consequently the model predicts overtreatment but cannot explain non-compliance. Alternatively, a pure communication model like CS in which the receiver is unrestricted in his choice of actions allows for non-compliance but does not predict overtreatment. Veto-based delegation, by contrast, results in both.

We now slightly modify the model from Section 2 to accommodate the ensuing comparative statics. An informed doctor observes the patient's health state $\theta \sim U[0,1]$, with higher values corresponding to more serious illnesses, and prescribes a treatment $m \geq 0$. The patient can either accept m or reject in favor of the status quo $\hat{a} = 0$, which represents no treatment. The payoffs for the doctor (sender) and patient (receiver) respectively are

$$u_s = -\frac{1}{2}(\theta - a)^2 + b_s a$$
 and $u_r = -\frac{1}{2}(\theta - a)^2 - b_r a$, $b_s, b_r \ge 0$. (3)

The first term in each payoff reflects the medical prudence of a treatment, on which both the doctor and patient agree. The second term captures financial incentives, whereby the patient pays b_r per unit of treatment while the doctor earns a constant profit margin b_s , with $b_s \le b_r$ to reflect the fact that treatment provision is costly on the margin. The utility functions above allow for a natural interpretation of the bias parameters and are affine transformations of the original specification, thus sharing maximizers.¹¹ The status quo $\hat{a} = 0$ captures the fact that the patient cannot self-prescribe, and corresponds to the patient's lowest feasible preferred treatment.¹²

The current status quo is in the range covered by Proposition 4, which identifies a profile a_1 with three pooling intervals and a profile a_2 with two pooling intervals as the two candidates for the most informative equilibrium. To identify which of the two is

¹⁰The physician-induced demand hypothesis posits that the doctor can change the patient's preferences for treatment and thus induce any prescribed action to be accepted. See Evans (1974), McGuire (2000), and Chandra et al. (2012).

¹¹Specifically, $u_s^{(3)}(a, \theta) = \frac{1}{2}u_s^{(1)}(a, \theta) + \frac{1}{2}b_s(b_s + 2\theta)$ and $u_r^{(3)}(a, \theta) = \frac{1}{2}u_r^{(1)}(a, \theta) - \frac{1}{2}b_r(-b_r + 2\theta)$, with $u^{(1)}$ and $u^{(3)}$ corresponding to the utility functions in lines (1) and (3).

¹²The patient would actually prefer negative treatments for states $(0, b_r)$, but we assume the patient cannot be a net seller of treatment.

more informative, we plug in $\hat{a} = 0$ and obtain

$$Var_1 - Var_2 = \frac{4}{3}(b_s + b_r)^3 - \frac{1}{3}b_s^3 - \frac{1}{4}b_s^4 - \frac{2}{3}(b_s + 2b_r)^3 = -4b_r^3 - 4b_r^2b_s + \frac{1}{3}b_s^3 - \frac{1}{4}b_s^4 < 0,$$

where the inequality follows from $b_s \le b_r$. Thus the equilibrium with three pooling intervals is optimal and is described by profile

$$a(\theta) = \begin{cases} 0 & \text{if } \theta \in (0, b_r) \\ 2(b_s + b_r) & \text{if } \theta \in (b_r, 2b_s + 3b_r) \\ 4(b_s + b_r) & \text{if } \theta \in (2b_s + 3b_r, 3b_s + 4b_r) \end{cases}.$$

$$\theta + b_s & \text{if } \theta \in (3b_s + 4b_r, 1)$$

The patient is untreated for mild illnesses and accepts a single intermediate treatment $a = 2(b_s + b_r)$ and all high treatments starting with $a \ge 4(b_s + b_r)$. When the doctor's preferred treatment falls in the range rejected by the patient, he must choose either to overstate beyond his financial incentive or to understate and induce a smaller treatment. For example, when the doctor prescribes treatment $a = 2(b_s + b_r)$, for illnesses $\theta \in (b_r, b_s + 2b_r)$ it is higher than what he prefers and for illnesses $\theta \in (b_s + 2b_r, 2b_s + 3b_r)$ it is lower than what he prefers. Because of the patient's ability to reject, the doctor is thus prevented from customizing his diagnosis.

Equilibrium Properties

One objective of this application is to compare the predictions made by the present approach which explicitly models communication with the more common physician-induced demand framework, which models the treatment decision as fully delegated to the doctor. For this comparison, we decompose the effect of the divergence in financial incentives into the effect on treatment level and treatment informativeness. Let $\bar{a}(\theta) \equiv \theta + E[z(\theta)]$ be a fully informative profile with the same expected treatment level as the equilibrium profile. Also, let $U_r(a(\theta)) \equiv E[u_r(a(\theta), \theta)]$ be the patient's expected utility from treatment profile $a(\theta)$. Recalling that $a_r(\theta) = \theta - b_r$ is the patient's preferred profile, the patient's utility loss relative to first best can be expressed as

$$\underbrace{U_r(a_r) - U_r(a)}_{\text{Patient welfare loss}} = \underbrace{U_r(a_r) - U_r(\bar{a})}_{\text{Loss from}} + \underbrace{U_r(\bar{a}) - U_r(a)}_{\text{Loss from}}.$$

$$\underbrace{\text{Loss from}}_{\text{treatment level}} = \underbrace{\frac{1}{2}(b_s + \frac{1}{2}b_s^2 + b_r)^2}_{\text{2}} + \underbrace{\frac{1}{2}\left(\frac{4}{3}(b_s + b_r)^3 - \frac{1}{3}b_s^3 - \frac{1}{4}b_s^4\right)}_{\text{2}}.$$

The first term on the right hand side describes the difference in the patient's utility in moving from first best profile a_r to profile \bar{a} which is fully separating but has the same average treatment as the equilibrium profile. In the second line, the particular expression for this term is derived by plugging $E[\bar{z}(\theta)] = E[z(\theta)] = b_s + \frac{1}{2}b_s^2$ (by Proposition 4(i)) into the expected utility decomposition in Equation (2). Note that since $E[z(\theta)] = b_s + \frac{1}{2}b_s^2$, the average treatment level depends only on b_s and not on b_r , an observation that is key in our upcoming discussion of the role of health insurance. Note also that since $E[z(\theta)] > b_s$, the expected level of equilibrium treatment is *higher* than if the decision were fully delegated to the doctor, which is detrimental to both the doctor and patient.

Strategic communication also gives rise to a loss of informativeness, as measured by the move from profile \bar{a} to equilibrium profile a in the second term of the decomposition. Again the particular algebraic expression is derived from the statement of Proposition 4(i) and Equation (2). It is easily verified from the expression that the loss from informativeness grows in both parameters but faster in the doctor's bias than the patient's. Thus we see that both average treatment and treatment informativeness are affected differentially by b_s and b_r . In contrast to standard communication models which tend to normalize one parameter and implicitly focus only on the sum, in veto-based delegation the presence of the outside option \hat{a} makes such a reduction restrictive. The Proposition below follows from this discussion.

Proposition 7 The physician-induced demand framework (equivalently, full delegation) strictly understates the negative effect of financial incentives on patient welfare. Furthermore, the patient's loss from informativeness may be greater than that from treatment level, so that $U_r(\bar{a}) - U_r(a) > U_r(a_r) - U_r(\bar{a})$.

Proposition 7 is important for interpreting the results from empirical studies of doctors' and patients' incentives which often embed the physician-induced demand mechanism (e.g., Clemens and Gottlieb, 2014; Gruber and Owings, 1996; McGuire and Pauly, 1991). Measuring the change in average treatment level alone understates the true impact of financial incentives by ignoring informativeness. In this sense, the estimated welfare loss in a physician-induced demand model is a lower bound for the true welfare loss.

Policies Affecting Financial Incentives

We highlight some comparative statics that correspond to commonly studied policy questions and track their impact on the utilities of the doctor and the patient, the average treatment level, and the informativeness of the treatment plan. Figure 5 summarizes the findings while the results are explored in more detail below.

Policy	U_r	U_s	Treatment Level	Informativeness
Insurance (lower out of pocket costs and higher premium)	1	↑	same	<u> </u>
Increase in reimbursement rate with or without pass-through	\downarrow	↑ or ↓	↑	\downarrow

Figure 5: Effects of Various Policies

Reducing patient out-of-pocket cost through insurance

We consider a simple insurance contract in which a consumer has coinsurance $\gamma \in [0, 1]$ and so pays an out-of-pocket cost γb_r per unit of treatment and an actuarially fair premium $F(\gamma)$ that satisfies the zero expected profit condition $F(\gamma) = (1 - \gamma)b_r E[a(\theta|\gamma)]$.

Proposition 8 When the patient holds more insurance (lower coinsurance γ), informativeness increases and the average treatment level remains unchanged. Therefore, insurance is Pareto improving and full insurance ($\gamma = 0$) is preferred by both the doctor and patient.

Proof As established in Proposition 4(i), the average treatment level is unaffected by the patient's parameter and since the contract is actuarially fair, his total expenditure does not change with γ .¹³ Meanwhile, more insurance reduces the patient's bias parameter, which by the same proposition improves informativeness.

An interesting aspect of this analysis is that the moral hazard which is thought to accompany the purchase of insurance is missing, since the average treatment does not depend on the bias parameter of the patient. Indeed, moral hazard would occur in a

¹³The only possible exception is that when γ becomes small enough, the equilibrium profile a_2^* from Proposition 4(ii) becomes more informative than a_1^* . If such a regime change occurs, informativeness still increases continuously as γ decreases, while at the regime switch E[z] jumps down from $b_s + \frac{1}{2}b_s^2$ to b_s , which is better for both the doctor and patient. Thus, regardless of whether a regime switch occurs, a lower γ is a Pareto improvement.

pure cheap talk model in which the patient is free to choose among all actions. With veto-based delegation, the patient receives a take-it-or-leave-it offer from the doctor, and thus even though his marginal willingness to receive treatment increases, he may not be able to act on this. The invariance of expected treatment with respect to the amount of purchased insurance is consistent with empirical evidence of a rather muted effect of copayments on the quantity of treatment received (Feldstein, 1973; Gibson et al., 2005; Goldman et al., 2007; Manning et al., 1987; McGuire, 2012; Newhouse et al., 1993). The role of insurance in this framework is thus solely to commit the patient to have preferences for treatment closer to the doctor's by reallocating spending from ex-post to ex-ante. This improves informativeness and demonstrates a value for insurance beyond its traditional role of reducing risk.

A change in reimbursement rates

The parameter b_s represents the doctor's profit margin which in particular depends on the reimbursement rates negotiated with insurers,¹⁴ and here we explore how changing these rates affects average treatment level, informativeness, and welfare. First, by Proposition 4(i) higher reimbursement leads to more treatment and less informativeness, and by these two effects patient welfare declines.¹⁵ The doctor is harmed by the loss of informativeness, and also from the increased treatment level, since by Proposition 4(i) the equilibrium treatment exceeds his preferred level by $\frac{1}{2}b_s^2$. In addition, there is a direct effect for the doctor that countervails the previous two. To see this, note that as reimbursement b_s increases, even if the treatment profile $a(\theta)$ remains unchanged, the doctor receives a higher total payment and is thus better off.¹⁶ The net effect for the doctor is ambiguous, and by inspection it can be verified that he is better off as long as

¹⁴The comparative statics similarly apply to other common factors affecting doctors' margins, including equity stakes in hospitals and labs and payments from pharmaceuticals on the revenue side or expenditures related to time, staff, malpractice insurance, technology or other sources on the cost side.

¹⁵It has been suggested that in some contexts doctors reduce treatment when their reimbursement rates increase (see Chandra et al., 2012). This phenomenon, sometimes referred to as income targeting, stems from the idea that when doctors have diminishing marginal utility over money, the income effect of a higher reimbursement rate outweighs the substitution effect. In contrast, the doctor in our model has a constant marginal utility for money and thus no income effect is possible. Our prediction that treatment increases with the markup owes to this construction and not to the effects of communication.

¹⁶Observe that the relationship between the sender's utility function in Equations (1) and (3) is given by $E[u_s^{(3)}] = \frac{1}{2}E[u_s^{(1)}] + \frac{1}{2}b_s(b_s + 1)$. This last term measures the direct effect for the doctor, which does not exist in the original quadratic loss specification.

 b_s and b_r are sufficiently small.

Next, if the zero-profit insurer accounts for the increased reimbursement rate by raising the patient's premium there are no additional effects on total surplus though there is a lump sum transfer of money from the patient to the insurer. If instead the insurer passes through the increased reimbursement with a corresponding increase in the out-of-pocket payment an additional distortion arises. Treatment now becomes less informative with the level remaining unchanged, a Pareto loss.

Proposition 9 An increase in the doctor's profit margin b_s always makes the patient worse off and makes the doctor worse off if and only if b_s and b_r are sufficiently large. In addition, offsetting a higher b_s with higher b_r results in a further Pareto loss.

These findings have relevance for the effectiveness of prospective payment policies that offer doctors a fixed fee based on the classification of a patient or condition with little or no marginal reimbursement, as is found in accountable care organizations and capitation payment systems. Such policies have generally been found to reduce utilization and costs,¹⁷ and this is in line with the prediction of our model. Our approach suggests that reduced marginal reimbursement systems have the additional benefit of improved informativeness.

5 Conclusion

Veto-based delegation is a common institutional arrangement that, in our estimation, has been under-utilized in applications due to a lack of tractability. While several equilibria have been studied in the literature (GK, KM), and the metric of informativeness has been widely accepted in similar communication games, identifying the set of veto-based delegation equilibria and finding the most informative element of this set has remained an unsolved problem. In this paper we describe the equilibrium set and identify the most informative equilibrium in a setting that includes previous work. The key to finding the most informative equilibrium in fact arises from the characterization of the set; namely that there is a subset of states over which any equilibrium profile must have pooling regions, and that the proposed equilibrium is constructed to be optimal over this range.

¹⁷See Christianson and Conrad (2011) and Cutler and Zeckhauser (2000) for literature reviews on provider responses to payment systems.

In finding the most informative equilibrium we contribute to a literature that compares communication protocols. We strengthen the result in DE that for intermediate values of the status quo the receiver prefers full delegation to veto-based delegation. In addition, we extend the work of KM, who showed indirectly that the most informative equilibrium in veto-based delegation is more informative than that in cheap talk. Since we explicitly characterize the most informative veto-based delegation equilibrium, we enable other comparisons with the most informative cheap talk equilibrium, including receiver and sender welfare. Finally, we show that when Mylovanov's (2008) equilibrium that maximizes the receiver's payoff is in fact most informative and strictly Pareto dominant in the constant bias uniform quadratic setting.

The main contribution of our analysis is to facilitate the use of the veto model in applications, and to demonstrate the practical importance of the results we study the doctor-patient relationship. The institutional setting fits the veto environment quite well: doctors are more informed than patients, typically prefer more treatment, and have control since patients cannot self prescribe. In addition the predictions of the veto model are more in line with empirical evidence than those of previous models in the health literature. For example, under veto-based delegation patients are overtreated on average *and* are able to reject the doctor's recommendations. In contrast, the physician-induced demand framework predicts overtreatment without allowing non-compliance while pure cheap talk allows non-compliance but no excessive treatment.

By explicitly modeling communication the veto framework provides new insights compared to the workhorse physician-induced demand model. The doctor's financial incentive leads not only to excessive treatment but also to information loss as the doctor strategically misdiagnoses to avoid rejection, and patient welfare is potentially affected more by the information loss than by overtreatment. Thus by focusing only on average treatment levels and ignoring information loss, the estimated harm to patients from increasing doctors' financial incentives (e.g. higher reimbursement rates, allowing ownership of diagnostic labs, increasing cost of malpractice insurance, etc.) is substantially understated. Furthermore, the patient's preference for treatment affects only the informativeness of communication but not the average treatment level. Consequently a patient with a lower co-insurance payment receives the same amount of treatment but on average the treatment is better suited for the illness as communication improves due to a closer alignment of incentives. Thus even risk neutral patients find insurance valuable as a means to reduce the doctor's incentive to misdiagnose.

Appendix

This appendix is organized as follows. First we prove Lemmas 3 and 2 which establish the minimal pooling regions for equilibria at different values of the status quo \hat{a} . Then we identify the most informative equilibrium for intermediate (Proposition 3), low (Proposition 4) and highest (Proposition 6) values of the status quo, the proofs of which share a common approach. Then at the end we describe the most informative equilibrium for statuses quo between the intermediate and highest ranges (Proposition 5), which requires a different technique.

Proof of Lemma 3

We first recall several facts from Proposition 2. We refer to $a_s(\theta) = \theta - b_s$ as the sender's diagonal and observe that every interior pooling interval must intersect it. Further, if the right endpoint of a pooling interval is below the sender's diagonal then the next interval is also pooling and at its left endpoint is above the diagonal by the same amount. Since $a(\hat{\theta}) = \hat{a}$ is on the diagonal and the initial interval pools to the right, it must end below the diagonal and the second interval (θ_1, θ_2) , if it exists, must also be pooling.

It is also helpful to compute several expressions for interval endpoints and actions. The pooling action on interval (θ_1 , θ_2) must satisfy the sender's indifference condition at θ_1 , thus

$$\theta_1 + b_s = \frac{1}{2}(\hat{a} + a_1) \quad \Rightarrow \quad a_1 = 2\theta_1 + 2b_s - \hat{a}.$$
 (4)

Similarly, if (θ_2, θ_3) is also pooling then the sender's indifference at θ_2 implies

$$\theta_2 + b_s = \frac{1}{2}(a_1 + a_2) \quad \Rightarrow \quad a_2 = 2(\theta_2 - \theta_1) + \hat{a}.$$
 (5)

In addition, while the right endpoint θ_1 of the first pooling interval is unconstrained since the receiver must accept \hat{a} when it is prescribed, in the second pooling interval her posterior must be sufficiently high to accept, thus

$$\frac{1}{2}(\theta_1 + \theta_2) - b_r \ge \frac{1}{2}(a_1 + \hat{a}) \quad \Rightarrow \quad \theta_2 \ge \theta_1 + 2b_s + 2b_r, \tag{6}$$

with the second inequality following from plugging in the expression for a_1 from (4). We now proceed with the proof starting with the largest values for the status quo and working down.

(I)
$$\hat{a} \in (1 - b_s - 2b_r, 1 + b_s] \implies$$
 one pooling interval on $(\hat{\theta}, 1]$

Toward a contradiction suppose that $\theta_1 < 1$. Then $\theta_2 \ge \theta_1 + 2b_s + 2b_r > \hat{a} + b_s + 2b_r > 1$, where the first inequality is from (6), the second from $\theta_1 > \hat{\theta} = \hat{a} - b_s$, and the third by assumption on \hat{a} . The receiver's posterior constraint requires $\theta_2 > 1$ and therefore cannot be met.

(II)
$$\hat{a} \in (1 - 2b_s - 3b_r, 1 - b_s - 2b_r] \Rightarrow$$
 at most two pooling intervals on $(\hat{\theta}, 1]$

First suppose toward a contradiction that there exists a third interval (θ_2, θ_3) that is separating, which by Proposition 2(ii) implies that $\theta_2 = a_1 - b_s$. Then using (6), the receiver's constraint on (θ_1, θ_2) is satisfied only if

$$\frac{1}{2}(\theta_1 + (a_1 - b_s)) - b_r \ge \frac{1}{2}(a_1 + \hat{a}) \quad \Rightarrow \quad \theta_1 \ge \hat{a} + b_s + 2b_r,$$

which again by (6) requires that $\theta_2 \ge \theta_1 + 2b_s + 2b_r \ge \hat{a} + 3b_s + 4b_r$. But since by assumption $\hat{a} > 1 - 2b_s - 3b_r$, this implies $\theta_2 > 1$ which is a contradiction.

Alternatively, toward a contradiction suppose that (θ_2, θ_3) is pooling. Satisfying the receiver's posterior constraint requires $\frac{1}{2}(\theta_2 + \theta_3) \ge \frac{1}{2}(a_2 + \hat{a})$, a condition similar to that in (6). Thus

$$0 \le \frac{1}{2}(\theta_2 + \theta_3) - b_r - \frac{1}{2}(a_2 + \hat{a})$$

$$= \frac{1}{2}(\theta_3 - (\theta_2 - \theta_1) + (\theta_1 - 2\hat{a} - 2b_r))$$

$$\le \frac{1}{2}(1 - (2b_s + 2b_r) + ((1 - 2b_s - 2b_r) - 2\hat{a} - 2b_r))$$

$$= 1 - 2b_s - 3b_r - \hat{a},$$

where the second line plugs in for a_2 from (5) and the third line uses the facts that $\theta_3 \le 1$, $\theta_2 - \theta_1 \ge 2b_s + 2b_r$ (from (6)), and $\theta_1 \le 1 - 2b_s - 2b_r$ (so that $\theta_2 \le 1$). Since by assumption $\hat{a} > 1 - 2b_s - 3b_r$ this leads to a contradiction.

(III)
$$\hat{a} \in [b_s, 1 - 2b_s - 3b_r] \implies$$
 at most three pooling intervals on $(\hat{\theta}, 1]$

We have already argued that the first two intervals of any equilibrium must be pooling. If there is no third interval (θ_2 , θ_3) then the result is immediate. If the third interval is separating then $\theta_2 = a_1 - b_s$, and plugging this into the receiver's posterior condition for (θ_1 , θ_2) obtains

$$\frac{1}{2}(\theta_1 + (a_1 - b_s)) - b_r \ge \frac{1}{2}(a_1 + \hat{a}) \implies \theta_1 \ge \hat{a} + b_s + 2b_r \implies \theta_2 \ge \hat{a} + 3b_s + 4b_r,$$

with the last inequality following from (6). But then $(\hat{a} - b_s, \hat{a} + 3b_s + 4b_r) = (\hat{\theta}, \hat{\theta} + 4b_s + 4b_r)$ is covered by two pooling intervals. If instead (θ_2, θ_3) is pooling then by (5) and (6)

$$a_2 = \hat{a} + 2(\theta_2 - \theta_1) \ge \hat{a} + 4b_s + 4b_r$$

and in order to hit the sender's diagonal the third pooling interval must include the state $\theta = a_2 - b_s = \hat{a} + 3b_s + 4b_r$. Here $(\hat{a} - b_s, \hat{a} + 3b_s + 4b_r)$ is covered by three pooling intervals.

Proof of Lemma 2

Define $a_0 \equiv a(0)$. For part (i) of the lemma focusing on equilibria in which $a_0 = \hat{a}$, observe that the result was already demonstrated in part (III) the proof above. In particular the only difference here is that $\hat{a} - b_s < 0$, but none of the arguments presented above assume otherwise.

However part (ii) of Lemma 2 is different since $a_0 > \hat{a}$ and in these equilibria the status quo is never utilized. Instead observe that if there is a second pooling interval then to satisfy the sender's indifference at θ_1 it must be that

$$\theta_1 + b_s = \frac{1}{2}(a_0 + a_1) \quad \Rightarrow \quad a_1 = 2\theta_1 + 2b_s - a_0.$$
 (7)

Also, on the initial interval the receiver now has the ability to reject and to meet her constraint it must be that

$$\frac{1}{2}(0+\theta_1) - b_r \ge \frac{1}{2}(a_0 + \hat{a}) \quad \Rightarrow \quad \theta_1 \ge \hat{a} + 2b_r + a_0. \tag{8}$$

Then, meeting the receiver's posterior constraint on (θ_1, θ_2) requires $\frac{1}{2}(\theta_1 + \theta_2) - b_r \ge \frac{1}{2}(a_1 + \hat{a})$, which by plugging in (7) and (8) implies

$$\theta_2 \geq 2b_s + 4b_r + 2\hat{a}_r$$

which concludes the proof.

Proofs of Informativeness Propositions 3, 4, and 6

The approach for the proof of each proposition is to start with a candidate profile $z(\theta)$, transform it to a more informative profile $\tilde{z}(\theta)$, and then show that $\tilde{z}(\theta)$ performs worse than the conjectured best profile. Claim 1 establishes a convenient way to compute

the average value of z for any equilibrium profile, while Claims 2 and 3 describe two transformations of $z(\theta)$ that increase informativeness and are used repeatedly in the proofs.

Claim 1 *If equilibrium profile* $a(\theta)$ *covers interval* (θ_l, θ_h) *with at least two intervals then*

$$E[z \mid \theta \in (\theta_l, \theta_h)] = b_s + \frac{1}{2} \left(a(\theta_l) - (\theta_l + b_s) \right)^2 - \frac{1}{2} \left(a(\theta_h) - (\theta_h + b_s) \right)^2.$$

Proof of Claim: First, we argue that if an equilibrium profile intersects the sender's diagonal at states θ_l and θ_h (i.e., $a(\theta_l) = \theta_l + b_s$ and $a(\theta_h) = \theta_h + b_s$), then $E[z(\theta) | \theta \in (\theta_l, \theta_h)] = b_s$. To see this, observe that (θ_l, θ_h) is partitioned into separating and pooling intervals. Over all separating intervals the expected bias is b_s since $z(\theta) = b_s$ at every state. Next consider the leftmost pooling interval $[\theta_i, \theta_{i+1})$, if it exists, which is either preceded by a separating interval or starts with θ_l . In either case the profile begins at θ_l on the sender's diagonal, pools to the right until θ_{l+1} , then jumps symmetrically above the sender's diagonal and pools to the right at least until reaching the diagonal again. From θ_l until this next diagonal intersection there are two pooling intervals symmetric around the diagonal, and on this region the conditional expected bias is b_s . Then we again begin on the sender's diagonal and proceed the same way until we reach θ_h , which by construction must be on the sender's diagonal, maintaining a conditional mean of b_s all along the way.

Next, we find the value of $E[z \mid \theta \in (\theta_l, \theta_h)]$ by integrating $z(\theta)$ between the first and last intersection of the sender's diagonal, and then accounting for the states outside of this subset. For a given equilibrium profile $a(\theta)$, define $\theta \equiv \theta_l + z(\theta_l) - b_s$ and $\bar{\theta} \equiv \theta_h + z(\theta_h) - b_s$. Observe that if $z(\theta_l) > b_s$ then θ identifies the first diagonal intersection to the right of θ_l , while if $z(\theta_l) < b_s$ then θ identifies the first diagonal intersection to the left of θ_l if the pooling interval were extended. The state $\bar{\theta}$ is similarly defined. Then,

$$\begin{split} E[z \mid \theta \in (\theta_{l}, \theta_{h})] &= \int_{\underline{\theta}}^{\bar{\theta}} z(\theta) \, d\theta \, + \int_{\theta_{l}}^{\underline{\theta}} z(\theta) \, d\theta \, + \int_{\bar{\theta}}^{\theta_{h}} z(\theta) \, d\theta \\ &= (\bar{\theta} - \underline{\theta}) b_{s} \, + \, Sign[\underline{\theta} - \theta_{l}] \cdot \frac{1}{2} (\underline{\theta} - \theta_{l}) (\underline{\theta} - \theta_{l} + 2b_{s}) \, + \, Sign[\underline{\theta}_{h} - \bar{\theta}] \cdot \frac{1}{2} (\theta_{h} - \bar{\theta}) (\bar{\theta} - \theta_{h} + 2b_{s}) \\ &= b_{s} \, + \, \frac{1}{2} (\underline{\theta} - \theta_{l})^{2} \, - \, \frac{1}{2} (\bar{\theta} - \theta_{h})^{2}. \end{split}$$

In the second line, the first term follows from the argument above that the conditional average $z(\theta)$ on $(\underline{\theta}, \overline{\theta})$ equals b_s . The second and third terms in the first line integrate

 $z(\theta)$ over pooling intervals between θ_l and $\underline{\theta}$ and $\overline{\theta}$ and θ_h , and account for the fact that, for instance, it is possible that $\underline{\theta} < \theta_l$. The third line follows from the observation that $Sign[x] \cdot x = |x|$ and a regrouping of terms. By the definitions of $\underline{\theta}$ and $\overline{\theta}$, this line is equivalent to the statement of the claim.

Claim 2 If Θ is partitioned into Θ_A and Θ_B so that $E_{\Theta_A}[z(\theta)] \equiv z_A < z_B \equiv E_{\Theta_B}[z(\theta)]$, and

$$\tilde{z}(\theta) \equiv \begin{cases} z(\theta) + \delta_1 & \text{if } \theta \in \Theta_A \\ z(\theta) - \delta_2 & \text{if } \theta \in \Theta_B \end{cases},$$

with $z_A + \delta_1 \le z_B - \delta_2$ and $\delta_1, \delta_2 \ge 0$, then

$$\int_{\Theta} (z(\theta) - E_{\Theta}[z(\theta)])^2 d\theta \geq \int_{\Theta} (\tilde{z}(\theta) - E_{\Theta}[\tilde{z}(\theta)])^2 d\theta.$$

Proof of Claim: Figure 6 illustrates an example of the situation described in the claim. In the figure, note that even after the shift it remains that $\tilde{z}_B > \tilde{z}_A$. Evaluating this shift

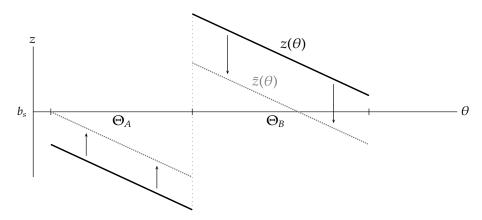


Figure 6: The variance is reduced when $z(\theta)$ is uniformly shifted up on Θ_A and down on Θ_B .

below,

$$\int_{\Theta} (z(\theta) - E_{\Theta}[z(\theta)])^{2} d\theta = \int_{\Theta_{A}} (z(\theta) - z_{A})^{2} d\theta + |\Theta_{A}|(E_{\Theta}[z(\theta)] - z_{A})^{2}$$

$$+ \int_{\Theta_{B}} (z(\theta) - z_{B})^{2} d\theta + |\Theta_{B}|(E_{\Theta}[z(\theta)] - z_{B})^{2}$$

$$\geq \int_{\Theta_{A}} (\tilde{z}(\theta) - \tilde{z}_{A})^{2} d\theta + |\Theta_{A}|(E_{\Theta}[\tilde{z}(\theta)] - \tilde{z}_{A})^{2}$$

$$+ \int_{\Theta_{B}} (\tilde{z}(\theta) - \tilde{z}_{B})^{2} d\theta + |\Theta_{B}|(E_{\Theta}[\tilde{z}(\theta)] - \tilde{z}_{B})^{2}$$

$$= \int_{\Theta} (\tilde{z}(\theta) - E_{\Theta}[\tilde{z}(\theta)])^{2} d\theta.$$

In the first line, the sum of square distances to $E_{\Theta}[z(\theta)]$ is separately taken over regions Θ_A and Θ_B , and within each region the sum is further decomposed into a sum of square distances to the conditional means z_A and z_B and a term to account for the difference between the conditional and unconditional means. In the second line, we switch to profile $\tilde{z}(\theta)$, which leaves the first and third terms in the first line unchanged. Furthermore, by construction $z_A \leq \tilde{z}_B \leq z_B$, and thus both the second and fourth terms in the first line are weakly smaller in the second line.

The next claim establishes a minimum amount of variance that arises in pooling regions when part of the pooling occurs below the diagonal and part above, as for example between θ_1 and θ_2 in Figure 3 parts V and VI.

Claim 3 For an equilibrium profile $a(\theta)$, if there are two disjoint pooling regions Θ_A and Θ_B with non-overlapping biases (i.e. $\sup_{\Theta_A} z(\theta) \leq \inf_{\Theta_B} z(\theta)$) then

$$\int_{\Theta_A \cup \Theta_B} (z(\theta) - E[z(\theta)])^2 d\theta \ge \frac{1}{12} \left| \Theta_A \cup \Theta_B \right|^3.$$

Proof of Claim: Since both regions are pooling, over region Θ_A the bias z is uniformly distributed on $\left(\sup_{\Theta_A} z(\theta) - |\Theta_A|, \sup_{\Theta_A} z(\theta)\right)$, and over region Θ_B the bias z is uniformly distributed on $\left(\inf_{\Theta_B} z(\theta), \inf_{\Theta_B} z(\theta) + |\Theta_B|\right)$ over region Θ_B . Define

$$\tilde{z}(\theta) = \begin{cases} z(\theta) + \inf_{\Theta_B} z(\theta) - \sup_{\Theta_A} z(\theta) & \text{if } \theta \in \Theta_A \\ z(\theta) & \text{if } \theta \in \Theta_B \end{cases}$$

and note that by Claim 2

$$\int_{\Theta_A \cup \Theta_B} (z(\theta) - E[z(\theta)])^2 d\theta \ge \int_{\Theta_A \cup \Theta_B} (\tilde{z}(\theta) - E[\tilde{z}(\theta)])^2 d\theta.$$

Note also that \tilde{z} is distributed uniformly on $\left(\inf_{\Theta_A} z(\theta), \inf_{\Theta_A} z(\theta) + |\Theta_A \cup \Theta_B|\right)$, thus

$$\int_{\Theta_A \cup \Theta_B} (\tilde{z}(\theta) - E[\tilde{z}(\theta)])^2 d\theta \geq \int_{\inf_{\Theta_A} z(\theta)}^{\inf_{\Theta_A} z(\theta) + |\Theta_A| + |\Theta_B|} (\tilde{z} - E[\tilde{z}(\theta)])^2 d\tilde{z} \geq \frac{1}{12} |\Theta_A \cup \Theta_B|^3,$$

where the final inequality is derived from the fact that the preceding integral is minimized if $E[\tilde{z}(\theta)] = \inf_{\Theta_A} z(\theta) + \frac{1}{2} (|\Theta_A| + |\Theta_B|)$.

We are now ready to prove the main propositions. We begin with Proposition 3, generalizing slightly to aid the proof of Proposition 4 which follows. Then similar logic is employed to prove Proposition 6 for the very highest statuses quo.

Proof of Proposition 3

Proposition 3 applies to statuses quo $\hat{a} \in [b_s, 1 - 3b_s - 4b_r]$, and we have shown that in this case $a_0 = \hat{a}$ in every equilibrium. With an eye towards the proof of Proposition 4 in which $\hat{a} \in [-b_r, b_s)$, here we find the most informative equilibrium in which $a_0 = \hat{a}$ for any $\hat{a} \in [-b_r, 1 - 3b_s - 4b_r]$. The generalized statement of Proposition 3 is below:

Generalized Proposition 3 If $\hat{a} < 1 - 3b_s - 4b_r$, then of the equilibria with $a_0 = \hat{a}$ the strictly most informative equilibrium has boundaries $(\theta_0, \theta_1, \theta_2, \theta_3) = (\max(\hat{\theta}, 0), \hat{a} + b_r, \hat{a} + 2b_s + 3b_r, \hat{a} + 3b_s + 4b_r)$ and action profile

$$a_1^*(\theta) = \begin{cases} \theta + b_s & \text{if } \theta \in (0, \theta_0) \\ \hat{a} & \text{if } \theta \in (\theta_0, \theta_1) \\ \hat{a} + 2(b_s + b_r) & \text{if } \theta \in (\theta_1, \theta_2) \\ \hat{a} + 4(b_s + b_r) & \text{if } \theta \in (\theta_2, \theta_3) \\ \theta + b_s & \text{if } \theta \in (\theta_3, 1) \end{cases}$$

with

$$E[z_1^*] = b_s + \frac{1}{2}(\min(\hat{\theta}, 0))^2 \quad and \quad Var(z_1^*) = \frac{4}{3}(b_s + b_r)^3 - \frac{1}{3}\left|\min(\hat{\theta}, 0)\right|^3 - \frac{1}{4}\left|\min(\hat{\theta}, 0)\right|^4.$$

Proof That $E[z_1^*] = b_s + \frac{1}{2}(\min(\hat{\theta}, 0))^2$ is implied by Claim 1, and the expression for variance can be computed explicitly. Recall that $\hat{\theta} \equiv \hat{a} - b_s$, $\bar{\theta} \equiv \hat{a} + 3b_s + 4b_r$, and that Lemmas 2(i) and 3(i) establish that $(\max(\hat{\theta}, 0), \bar{\theta})$ is covered by no separating and at most three pooling intervals. Consequently, on $(\hat{\theta}, \bar{\theta})$ there are either zero or precisely one diagonal intersections.

First consider a candidate profile $a(\theta)$ with zero intersections. This implies that $(\hat{\theta}, \bar{\theta})$ is covered either by a single pooling interval or two pooling intervals $(\max(\hat{\theta}, 0), \theta_1)$ and $(\theta_1, \bar{\theta})$, in which the first interval is entirely below the sender's diagonal and the second interval is entirely above it. Then,

$$Var(z) \ge \frac{1}{12}(\bar{\theta} - \max(\hat{\theta}, 0))^3 > \frac{1}{12}(4b_s + 4b_r)^3 \ge Var(z_1^*).$$

The first inequality follows from Claim 3, the second inequality comes from the parameter constraint $\hat{a} + 3b_s + 4b_r < 1$, and the final inequality follows from the expressions for variance in Propositions 3 and 4(i). Thus every candidate profile that does not intersect the sender's diagonal to the right of $\hat{\theta}$ is less informative than a_1^* .

Next consider profiles for which the second pooling interval intersects the sender's diagonal to the right of $\hat{\theta}$ precisely once, at state t. This includes profiles that to the right of $\hat{\theta}$ consist of two pooling intervals, or of three pooling intervals with the third interval ending above the sender's diagonal. Define

$$\tilde{z}(\theta) \equiv \begin{cases} \hat{a} + \theta & \text{if } \theta \in (\min(\hat{\theta}, 0), 0) \\ z(\theta) & \text{if } \theta \in (0, t) \\ z(\theta) + b_s - E[z \mid \theta \in (t, 1)] & \text{if } \theta \in (t, 1) \end{cases}.$$

The profile \tilde{z} alters the candidate z in two ways, as demonstrated in Figure 7. First, if $\hat{\theta} < 0$ then the initial pooling interval is extended to the left from state $\theta = 0$ to state $\theta = \hat{\theta}$, and in doing so ensures that $E[\tilde{z} \mid \theta \in (\hat{\theta}, t)] = b_s$. Also, for states (t, 1) the profile is uniformly shifted upward so that $E[\tilde{z} \mid \theta \in (t, 1)] = b_s$. Observe also that the region $(t, \bar{\theta})$ is covered by at most two pooling intervals (t, θ_2) and $(\theta_2, \bar{\theta})$, and that \tilde{z} is smaller

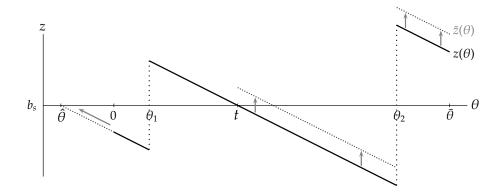


Figure 7: Alternate profile \tilde{z} extends the initial pooling interval to negative states if $\hat{\theta} < 0$ and uniformly increases $z(\theta)$ on $(t, \bar{\theta})$.

at every point in the former region than at any point in the latter region. Then,

$$Var(z) \geq Var(\tilde{z})$$

$$\geq \int_{\hat{\theta}}^{t} (\tilde{z}(\theta) - b_{s})^{2} d\theta + \int_{t}^{\theta} (\tilde{z}(\theta) - b_{s})^{2} d\theta$$

$$- \int_{\min(\hat{\theta},0)}^{0} (\hat{a} - \theta - b_{s})^{2} d\theta - (E[\tilde{z}] - b_{s})^{2}$$

$$\geq \frac{1}{12} (t - \hat{\theta})^{3} + \frac{1}{12} (\bar{\theta} - t)^{3} - \int_{\min(\hat{\theta},0)}^{0} (\hat{a} - \theta - b_{s})^{2} d\theta - (E[\tilde{z}] - b_{s})^{2}$$

$$\geq \frac{1}{6} \left(\frac{1}{2} (\bar{\theta} - \hat{\theta})\right)^{3} - \int_{\min(\hat{\theta},0)}^{0} (\hat{a} - \theta - b_{s})^{2} d\theta - (E[\tilde{z}] - b_{s})^{2}$$

$$= \frac{4}{3} (b_{s} + b_{r})^{3} - \int_{\min(\hat{\theta},0)}^{0} (\hat{a} - \theta - b_{s})^{2} d\theta - (E[\tilde{z}] - b_{s})^{2}$$

$$= Var(z_{1}^{*}).$$

In the first line, by moving from profile z to \tilde{z} we leave the profile unchanged on (0,t) but uniformly shift it up on (t,1). Since by Claim 1, $E[z \mid \theta \in (t,\bar{\theta})] < b_s \le E[z \mid \theta \in (0,t)]$, the shift brings these conditional means closer together which by Claim 2 reduces variance, thus obtaining the inequality. In the second line, we decompose variance by summing square distances from $\tilde{z}(\theta)$ to b_s for states $(\hat{\theta},\bar{\theta})$ in the first two terms, subtracting square distances over states $(\hat{\theta},0)$ if $\hat{\theta}<0$ in the third term, and accounting for the fact the true mean is $E[\tilde{z}]$ rather than b_s in the fourth term. If $\bar{\theta}=1$ then the second line holds with equality, however if $\bar{\theta}<1$ then the inequality ensues since square distances on

 $(\bar{\theta},1)$ are not included. The third inequality uses the fact that within each interval, both $(\hat{\theta},t)$ and $(t,\bar{\theta})$ are each spanned by two pooling intervals with non-overlapping values of $z(\theta)$, which by Claim 3 implies the first two terms on the right hand side. The fourth inequality uses the fact that $t=\frac{1}{2}(\bar{\theta}+\hat{\theta})$ minimizes the sum of the first two terms in the previous line, and the ensuing penultimate equality follows from plugging in $\bar{\theta}-\hat{\theta}=4b_s+4b_r$. Finally by Claim 1, $E[\tilde{z}]=E[z_1^*]$ and thus the last equality obtains.

Proof of Proposition 4

The proof of Proposition 3 above also characterizes the most informative equilibrium with $a_0 = \hat{a}$ when $\hat{a} \in [-b_r, b_s)$. Here we consider the remaining case $a_0 > \hat{a}$ and establish the existence of a threshold \underline{a} below which this case gives the most informative equilibrium.

Let $\bar{\theta} \equiv 2b_s + 4b_r + 2\hat{a}$, and since $\hat{a} < b_s$ Lemma 2(ii) states that every profile covers the region $(0, \bar{\theta})$ by at most two pooling intervals $(0, \theta_1)$ and $(\theta_1, \bar{\theta})$, and no separating intervals, as in the example in Figure 8. We first show that that $z(\theta)$ is non-overlapping

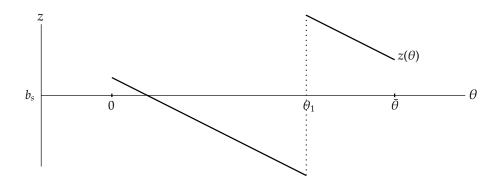


Figure 8: An example demonstrating no overlap in the distribution of z on the intervals $(0, \theta_1)$ and $(\theta_1, \bar{\theta})$, resulting from the fact that $\theta_1 \ge \frac{1}{2}\bar{\theta}$.

on these two intervals. Observe that

$$z^{-}(\bar{\theta}) = z^{+}(\theta_{1}) - (\bar{\theta} - \theta_{1})$$

$$= 2b_{s} - z^{-}(\theta_{1}) - (\bar{\theta} - \theta_{1})$$

$$= 2b_{s} - (a_{0} - \theta_{1}) - (\bar{\theta} - \theta_{1})$$

$$= 2b_{s} + 2\theta_{1} - a_{0} - \bar{\theta}$$

$$\geq 2b_{s} + 2(\hat{a} + 2b_{r} + a_{0}) - a_{0} - \bar{\theta}$$

$$= a_{0}$$

$$= z^{+}(0).$$

In the first line, the equality comes from the fact that over the pooling interval $(\theta_1, \bar{\theta})$ the value of $z(\theta)$ drops by the length of the interval. The second line then plugs in for the value of $z^+(\theta_1)$, using the fact that the profile is reflected above the line $z = b_s$ at the interval boundary. The third equality again comes from the fact that over the pooling interval $(0, \theta_1)$ the value of $z(\theta)$ drops by the length of the interval and that the starting value is $z(0) = a_0$. The ensuing inequality follows from the fact that $\theta_1 \ge \hat{a} + 2b_r + a_0$ in order to meet the receiver's posterior constraint, and the final two lines follow from the definitions of $\bar{\theta}$ and a_0 . Thus the values of $z(\theta)$ are non-overlapping on intervals $(0, \theta_1)$ and $(\theta_1, \bar{\theta})$, as depicted in Figure 8. Then,

$$Var(z) = \int_{0}^{\theta} (z(\theta) - E[z])^{2} d\theta + \int_{\bar{\theta}}^{1} (z(\theta) - E[z])^{2} d\theta$$

$$\geq \frac{1}{12} \bar{\theta}^{3} + \int_{\bar{\theta}}^{1} (z(\theta) - E[z])^{2} d\theta$$

$$\geq \frac{2}{3} (b_{s} + 2b_{r} + \hat{a})^{3}$$

$$= Var(z_{2}^{*}).$$

The first inequality follows by Claim 3 since we have shown that values of $z(\theta)$ are non-overlapping on $(0, \theta_1)$ and $(\theta_1, \bar{\theta})$, and is strict if $\theta_1 > \hat{a} + 2b_r + a_0$. The second inequality comes from the definition of $\bar{\theta}$ and is strict unless the region $(\bar{\theta}, 1)$ is separating, which occurs only if $z(\bar{\theta}) = b_s$. It can then be seen that $\theta_1 = \hat{a} + 2b_r + a_0$ and $z(\bar{\theta}) = b_s$ are satisfied in this class of profiles *only* by z_2^* , thus it is strictly the most informative.

Now, we show that for every b_s , b_r there is a threshold $\underline{a} < b_s$ such that the three pooling interval profile a_1^* of part (i) is most informative when $\hat{a} \ge \underline{a}$ and the two pooling interval profile a_2^* of part (ii) is most informative when $\hat{a} \le \underline{a}$. To show this we focus on

 $\Delta Var(\hat{a}) \equiv Var(z_1^*, \hat{a}) - Var(z_1^*, \hat{a})$ and establish that $\Delta Var(\hat{a})$ is monotone increasing:

$$\frac{d}{d\hat{a}} \left(\Delta Var(\hat{a}) \right) \equiv \frac{d}{d\hat{a}} \left(Var(z_2^*, \hat{a}) - Var(z_1^*, \hat{a}) \right) = 2(b_s + 2b_r + \hat{a})^2 - (b_s - \hat{a})^2 - (b_s - \hat{a})^3 \tag{9}$$

Recalling that $\hat{a} \in [-b_r, b_s)$ in Proposition 4, observe $\frac{d}{d\hat{a}} \Big(\Delta Var(-b_r) \Big) = 2(b_s + b_r)^2 - (b_s + b_r)^2 - (b_s + b_r)^3 > 0$ since $b_s + b_r < 1$ and $\frac{d}{d\hat{a}} \Big(\Delta Var(b_s) \Big) = 2(b_s + 2b_r)^2 - b_s^2 - b_s^3 \ge 2b_s^2 - b_s^2 - b_s^3 > 0$, where the last inequality follows from $b_s < 1$. Also, observe that $\frac{d^2}{d\hat{a}^2} \Big(\Delta Var(\hat{a}) \Big) = 4(b_s + b_r + \hat{a}) + 2(b_s - \hat{a}) + 3(b_s - \hat{a})^2 \ge 0$, and thus $\frac{d}{d\hat{a}} \Big(\Delta Var(\hat{a}) \Big) \ge 0$ for all $\hat{a} \in [-b_r, b_s)$.

Now note that $\Delta Var(b_s) > 0$, that is at the highest status quo in this region the three pooling interval equilibrium a_1^* is strictly more informative than the two pooling interval equilibrium a_2^* . Since $\Delta Var(\hat{a})$ is continuous there must be a nonempty set of statuses quo (\underline{a}, b_s) over which a_1^* is optimal. Then, since we showed $\Delta Var(\hat{a})$ is monotone increasing if there exists an $\underline{a} \in [-b_r, b_s)$ so that $\Delta Var(\underline{a}) = 0$, then for all $\hat{a} \in [-b_r, \underline{a})$ the two pooling interval profile a_2^* is optimal.

Finally, note that $\Delta Var(-b_r) < 0$ if and only if b_r is sufficiently small relative to b_s , thus $\underline{a} \ge -b_r$ and the two pooling interval equilibrium z_2^* is optimal only if b_r is sufficiently small.

Proof of Proposition 6

Since $\hat{a} \in [1 - b_s - 2b_r, 1 + b_s]$, Lemma 3(ii) shows that states $(\hat{\theta}, 1]$ must all pool on status quo action \hat{a} . For a fixed profile z, let $\delta \equiv E[z|\theta \in (0, \hat{\theta})] - b_s$ and define auxiliary profile

$$\tilde{z}(\theta) \equiv \begin{cases} z(\theta) - \delta & \text{if } \theta \in [0, \hat{\theta}] \\ z(\theta) & \text{if } \theta \in (\hat{\theta}, 1] \end{cases}$$

observing that $\delta \geq 0$ (Claim 1), that \tilde{z} is thus more informative than z (Claim 2), and that $E[\tilde{z}] = E[z^*]$. Then,

$$Var(z) \ge Var(\tilde{z}) \ge \int_{\hat{\theta}}^{1} (z(\theta) - b_s)^2 d\theta - (b_s - E[\tilde{z}])^2 = Var(z^*),$$

where the second inequality follows since square distances on states $[0, \hat{\theta})$ are omitted. The expressions for $E[z^*]$ and $Var(z^*)$ can be computed explicitly.

Proof of Informativeness Proposition 5

Here we consider statuses quo in the range $\hat{a} \in [1 - 3b_s - 4b_r, 1 - b_s - 2b_r]$ and our proof is more computational than those preceding. To get intuition for why this is necessary, observe that the sender's indifference condition makes all profiles symmetric around the bias b_s except at the edges of the parameter space. Previously the proposed equilibria had an expected bias that was as close as feasible to b_s , and then essentially minimized squared distances to b_s thereafter. Now at the upper edge of the status quo parameter space, pushing a profile toward an expected bias of b_s is only achieved with longer pooling intervals, which increases square distances to b_s . This tradeoff must be computed directly.

By the arguments just made in the proof of Proposition 6 it is optimal to separate on $[0,\hat{\theta}]$, and by Lemma 3(ii) no equilibrium has more than three pooling intervals covering $[\hat{\theta},1]$. Thus we focus on equilibrium profiles that separate up to $\hat{\theta}$ and have three of fewer pooling intervals thereafter. Also, recall from the proof of Lemma 3 that equations (4), (5), and (6) regarding sender and receiver constraints all apply. We first characterize the optimal equilibrium with three pooling intervals, then the optimal equilibrium with two profiles, and then compare the two. We also demonstrate an equilibrium with one pooling interval is never optimal.

Three pooling intervals

Claim 4 For any initial pooling interval $(\hat{\theta}, \theta_1)$, the optimal right endpoint of the second pooling interval (θ_1, θ_2) is either $\theta_2 = 1$ (largest feasible) or $\theta_2 = \theta_1 + 2b_s + 2b_r$ (smallest feasible).

Proof of Claim: To ease notation define $t \equiv 2\theta_1 - \hat{\theta}$, which as previously is the first diagonal intersection to the right of $\hat{\theta}$ for a profile with $z(\hat{\theta}) = b_s$. Also define $\varepsilon \equiv z(1) - b_s = 2\left(\theta_2 - \frac{1}{2}(t+1)\right)$ as the amount by which the third and final pooling interval ends above the diagonal (see Figure 9). From (6) we know that $\theta_2 \in [\theta_1 + 2b_s + 2b_r, 1]$, and therefore $\varepsilon \in [\hat{a} + 3b_s + 4b_r - 1, 1 - t]$. The variance of a profile with three pooling intervals and a fixed θ_1 (and thus a fixed t) can be expressed in terms ε (rather than θ_2) as follows:

$$Var_3(\varepsilon) = \int_{\hat{\theta}}^t (z(\theta) - b_s)^2 d\theta + \int_t^{1+\varepsilon} (a_2 - \theta - b_s)^2 d\theta - \int_1^{1+\varepsilon} (a_2 - \theta - b_s)^2 - \frac{1}{2} (E[z] - b_s)^2$$

$$= \frac{1}{12} (t - \hat{\theta})^3 + \frac{1}{12} (1 + \varepsilon - t)^3 - \frac{1}{3} \varepsilon^3 - \frac{1}{4} \varepsilon^4.$$
(10)

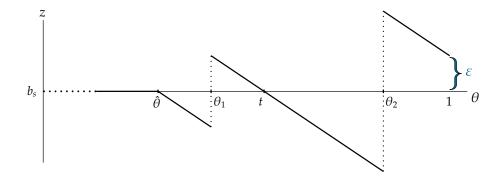


Figure 9: An example profile with three pooling intervals ending ε above b_s

In the first line, we extend the third pooling interval to $\theta = 1 + \varepsilon$ and integrate square distances to b_s on $(\hat{\theta}, 1 + \varepsilon)$ in the first two terms, subtract the square distances on $(1, 1 + \varepsilon)$ in the third term, and in the fourth term account for the fact that the true expected bias does not necessarily equal b_s . The second line then uses the fact that the profile z is symmetric around b_s on $(\hat{\theta}, t)$ and again on $(t, 1 + \varepsilon)$ for the first two terms, and obtains the fourth term from the fact that $E[z] = b_s - \frac{1}{2}b_s^2$ by Claim 1. Next,

$$\frac{dVar_3}{d\varepsilon} = \frac{1}{4}(1+\varepsilon-t)^2 - \varepsilon^2 - \varepsilon^3$$
$$\frac{d^2Var_3}{d\varepsilon^2} = \frac{1}{2}(1-t) - \frac{3}{2}\varepsilon - 3\varepsilon^2$$

The second derivative is strictly negative when evaluated at $\varepsilon = \frac{1}{3}(1-t)$ and is monotone decreasing in ε . Therefore

$$\frac{d^2Var_3}{d\varepsilon^2} > 0 \implies 1 - t > 3\varepsilon \implies \frac{dVar_3}{d\varepsilon} > 4\varepsilon^2 - \varepsilon^2 - \varepsilon^3 > 0,$$

so that for any ε where the second derivative is positive Var_3 is increasing. There are thus no local minima (zero first derivative, positive second derivative) and the global minimum occurs at one of the two corners.

The previous claim implies that if the most informative profile has three pooling intervals then θ_2 takes the minimal feasible value, since if the maximal $\theta_2 = 1$ is optimal then the result is an equilibrium with two pooling intervals. We now find the optimal value of θ_1 for this three pooling profile.

Claim 5 If $\theta_2 = \theta_1 + 2b_r + 2b_s$ (smallest feasible) then $\theta_1 = \hat{a} + b_r$ is optimal.

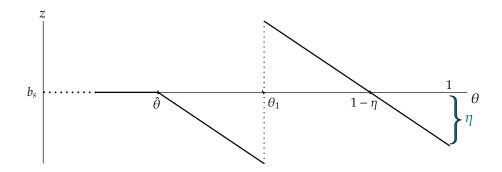


Figure 10: An example profile with two pooling intervals ending η below b_s

Proof of Claim: First note that $\theta_2 = \theta_1 + 2b_r + 2b_s \implies \varepsilon = \hat{a} + 3b_s + 4b_r - 1$, and thus ε is independent of the choice of θ_1 (and consequently t). Therefore, choosing t to minimize Var_3 requires minimizing only the first two terms in (10), which is achieved by $t = \frac{1}{2}(\hat{\theta} + 1 + \varepsilon)$. Recalling that $t = 2\theta_1 - \hat{\theta}$ and plugging in for ε then yields $\theta_1 = \hat{a} + b_r$.

Thus if the most informative equilibrium has three pooling intervals then it is characterized by $(\theta_1, \theta_2) = (\hat{a} + b_r, \hat{a} + 2b_s + 3b_r)$.

Two pooling intervals

We look for the optimal right endpoint θ_1 of the first pooling interval, and will show that for low values of $\hat{a} \in [1-3b_s, 4b_r, 1-b_s-2b_r]$ an interior θ_1 is optimal while for larger values of \hat{a} the largest feasible θ_1 that admits a second pooling interval is optimal.

Again to ease notation we now define $\eta \equiv b_s - z(1) = 1 + \hat{\theta} - 2\theta_1$ and observe $\eta = -\varepsilon$ from the three pooling case (see Figure 10). To meet the posterior constraint in the second pooling interval $\theta_2 = 1 \ge \theta_1 + 2b_s + 2b_r \implies \theta_1 \le 1 - 2b_s - 2b_r$, which in turn implies that $\eta \in [\hat{a} + 3b_s + 4b_r - 1, 1 - \hat{\theta}] \equiv [\underline{\eta}, \overline{\eta}]$. Every two pooling interval profile thus ends below the sender's diagonal. Variance in terms of η is

$$Var_2(\eta) = \int_{\hat{\theta}}^{1-\eta} (z(\theta) - b_s)^2 d\theta + \int_{1-\eta}^1 (z(\theta) - b_s)^2 d\theta - (E[z] - b_s)^2$$
$$= \frac{1}{12} (1 - \eta - \hat{\theta})^3 + \frac{1}{3} \eta^3 - \frac{1}{4} \eta^4.$$
(11)

The decomposition of variance into three terms is similar to that in the three pooling interval case. The first two terms sum the squared differences to b_s for states $(\hat{\theta}, 1)$ while

the third term accounts for the fact the true mean is $E[z] = b_s - \frac{1}{2}b_s^2$ (Claim 1) rather than b_s . We now look for the optimal η , showing the solution is interior for small \hat{a} and the smallest feasible η for larger \hat{a} .

Claim 6 There is a unique minimizer $\eta^* \in (\frac{1}{3}(1-\hat{\theta}), 1-\hat{\theta})$ that is interior and which solves $0 = -\frac{1}{4}(1-\hat{\theta}-\eta)^2 + \eta^2 - \eta^3$.

Proof of Claim: We demonstrate that $Var_2(\eta)$ is u-shaped over $\eta \in \left(\frac{1}{3}(1-\hat{\theta}), 1-\hat{\theta}\right)$ and for this compute several derivatives:

$$\frac{dVar_2}{d\eta} = -\frac{1}{4}(1 - \hat{\theta} - \eta)^2 + \eta^2 - \eta^3
\frac{d^2Var_2}{d\eta^2} = \frac{1}{2}(1 - \hat{\theta}) + \frac{3}{2}\eta - 3\eta^2
\frac{d^3Var_2}{d\eta^3} = \frac{1}{4} - 6\eta.$$
(12)

Observe that $\frac{dVar_2}{d\eta}\left(\frac{1}{3}(1-\hat{\theta})\right) < 0 < \frac{dVar_2}{d\eta}\left(1-\hat{\theta}\right)$ thus Var_2 starts downward sloping and ends upward sloping. Furthermore $\frac{d^2Var_2}{d\eta^2}\left(\frac{1}{3}(1-\hat{\theta})\right) > 0$ so variance is convex at the left endpoint. Thus we are guaranteed a unique interior solution as long as variance does not become concave and then again convex. But since variance starts convex at the left endpoint, if it becomes concave at some point then $\frac{d^3Var_2}{d\eta^3}$ must be negative, and by (??) the third derivative must remain negative thereafter, leading to a contradiction since variance thus remains concave.

The solution η^* to the first order condition from (12) is not always feasible. That is, η^* is guaranteed to be inside $\left(\frac{1}{3}(1-\hat{\theta}),1-\hat{\theta}\right)$ but the feasible set (from above) is $\left[\underline{\eta},\overline{\eta}\right]=(\hat{a}+3b_s+4b_r-1,1-\hat{\theta}]$, and the left boundary of the feasible set may potentially be larger than the interior solution. In this case the left boundary is itself the solution, as demonstrated in the following claim.

Claim 7 There exists a status quo $\hat{a} \in (1 - 2b_s - 3b_r, 1 - b_s - 2b_r)$ so that if $\hat{a} \in (1 - 3b_s - 4b_r, \hat{a}]$ then the optimal η^* solves the FOC given by (12), else if $\hat{a} \in (\hat{a}, 1 - b_s - 2b_r]$ then the optimal $\eta^* = \hat{a} + 3b_s + 4b_r - 1$, which is the smallest feasible value.

Proof of Claim: Recalling $\hat{\theta} \equiv \hat{a} - b_s$ and $[\underline{\eta}, \bar{\eta}] = [\hat{a} + 3b_s + 4b_r - 1, 1 - \hat{\theta}]$, for any given status quo \hat{a} the feasible set for η is

$$[\underline{\eta}, \bar{\eta}] = \left(\left(\frac{\hat{a} + 3b_s + 4b_r - 1}{1 + b_s - \hat{a}} \right) (1 - \hat{\theta}), 1 - \hat{\theta} \right) \equiv \left(\alpha(\hat{a})(1 - \hat{\theta}), 1 - \hat{\theta} \right).$$

Given $\hat{a} \in (1 - 3b_s - 4b_r, 1 - b_s - 2b_r)$, observe that $\alpha(1 - 3b_s - 4b_r) = 0$, $\alpha(1 - b_s - 2b_r) = 1$, and $\alpha(\hat{a})$ is strictly increasing. Now, using (12) observe that at the left boundary $\underline{\eta}$ the slope of variance

$$\frac{dVar_2}{d\eta}(\underline{\eta}) = -\frac{1}{4}\Big((1-\alpha)(1-\hat{\theta})\Big)^2 + \Big(\alpha(1-\hat{\theta})\Big)^2 - \Big(\alpha(1-\hat{\theta})\Big)^3$$

is negative at $\alpha = 0$, increasing in α , and positive at $\alpha = 1$. Thus there is a unique α at which $\frac{dVar_2}{d\eta}$ flips from negative to positive, and therefore a unique \hat{a} beyond which the solution to the FOC in (12) is no longer feasible and the optimal η becomes the left boundary.

Note that if $\hat{a} = 1 - 2b_s - 3b_r$ then $\alpha(\hat{a}) = \frac{1}{3}(1 - \hat{\theta})$ and thus $\frac{dVar_2}{d\eta}(\underline{\eta}) < 0$ and the optimal η is interior.

Finally, since the maximal $\bar{\eta} = 1 - \hat{\theta}$ corresponds to a profile with a single pooling interval on $(\hat{\theta}, 1]$ and since we have shown Var_2 is strictly increasing at $\bar{\eta}$, an equilibrium with a single pooling interval is always less informative than the optimal equilibrium with two pooling intervals.

Three pooling intervals versus two pooling intervals

We have found the optimal two and three pooling interval profiles, and that a profile with a single pooling interval is not optimal whenever a profile with two pooling intervals is feasible. Now we show that for lower values of \hat{a} a profile with three pooling intervals is optimal and for higher values a profile with two pooling intervals is optimal.

Claim 8 There exists a status quo $\tilde{a} \in (1 - 3b_s - 4b_r, 1 - 2b_s - 3b_r)$ so that the most informative equilibrium has three pooling intervals whenever $\hat{a} \in [1 - 3b_s - 4b_r, \tilde{a}]$ and two pooling intervals whenever $\hat{a} \in [\tilde{a}, 1 - b_s - 2b_r]$.

Proof of Claim: We cannot directly compare the variance of the best three interval and two interval profiles because we do not have a closed form expression for the latter. Instead we make use of the FOC characterization in Equation (12) and apply the envelope theorem.

Several preliminary steps are necessary to make the main argument. First note that since $\hat{a} \in (1 - 3b_s - 4b_r, 1 - 2b_s - 3b_r)$ then for profiles with two pooling intervals the optimal η^* is interior by Claim 7. Also, it can be seen that $\eta^* < \frac{2}{3}(1 - \hat{\theta})$ since by Equation (12)

$$\frac{dVar_2}{d\eta}\left(\frac{2}{3}(1-\hat{\theta})\right) = -\frac{1}{4}\left(\frac{1}{2}\eta\right)^2 + \eta^2 - \eta^3 = \eta^2\left(\frac{15}{16} - \eta\right) = \eta^2\left(\frac{15}{16} - \frac{2}{3} + \frac{2}{3}\hat{\theta}\right) > 0.$$

Next, totally differentiating the first order condition from (12) yields

$$0 = \left(\frac{1}{2}(1 - \hat{\theta} - \eta^*) + 2\eta^* - 3\eta^{*^2}\right)d\eta^* + \frac{1}{2}(1 - \hat{\theta} - \eta^*)d\hat{\theta} \implies \frac{d\eta^*}{d\hat{\theta}} = -\frac{\frac{1}{2}(1 - \hat{\theta} - \eta^*)}{\frac{1}{2}(1 - \hat{\theta} - \eta^*) + \eta^*(2 - 3\eta^*)}.$$

Since $\eta^* < \frac{2}{3}(1-\hat{\theta})$ implies $2-3\eta^* > 2\hat{\theta} > 0$, both terms in the denominator are positive, which in turn implies that $\frac{d\eta^*}{d\hat{\theta}} = \frac{d\eta^*}{d\hat{\theta}} > -1$.

Now to establish the statement of the Claim, the difference between the minimized variance with two and three pooling intervals is

$$Var_3^* - Var_2^* = \left(\frac{4}{3}(b_s + b_r)^3 - \frac{1}{3}\varepsilon^{*^3} - \frac{1}{4}\varepsilon^{*^4}\right) - \left(\frac{1}{12}(1 - \hat{\theta} - \eta^*)^3 + \frac{1}{3}\eta^{*^3} - \frac{1}{4}\eta^{*^4}\right),$$

in which by Claim 4, $\varepsilon^* = \hat{a} + 3b_s + 4b_r - 1$ and η^* solves the FOC in (12). Taking a derivative with respect to \hat{a} and applying the envelope theorem (i.e., $\frac{\partial Var_2}{\partial \eta^*} = 0$) obtains

$$\frac{d(Var_3^* - Var_2^*)}{d\hat{a}} = -\varepsilon^{*^2} - \varepsilon^{*^3} + \frac{1}{4}(1 - \hat{\theta} - \eta^*)^2,$$

and taking the derivative a second time yields

$$\frac{d^2(Var_3^* - Var_2^*)}{d\hat{a}^2} = -2\varepsilon^* - 3\varepsilon^{*^2} - \frac{1}{2}(1 - \hat{\theta} - \eta^*)\left(1 + \frac{d\eta^*}{d\hat{a}}\right) < 0,$$

with the inequality following from the previous observation that $\frac{d\eta^*}{d\hat{a}} > -1$.

We now use that $Var_3^* - Var_2^*$ is concave to prove the claim. Recall this proposition pertains to $\hat{a} \in [1 - 3b_s - 4b_r, 1 - b_s - 2b_r]$, and at the left boundary $Var_3^* - Var_2^* < 0$ since here the three pooling equilibrium is precisely the equilibrium at the *right* boundary of Proposition 3, which was shown to be uniquely optimal. On the other hand, at the right boundary $\hat{a} = 1 - 2b_s - 3b_r$ the minimal permissible θ_2 is 1 so that the three pooling interval equilibrium collapses to two pooling intervals and so is dominated by the best two pooling interval equilibrium. That is, $Var_3^* - Var_2^* > 0$ at $\hat{a} = 1 - 2b_s - 3b_r$. Then since $Var_3^* - Var_2^*$ is continuous and concave there exists a unique threshold \tilde{a} below which $Var_3^* - Var_2^*$ is negative and above which it is positive. This completes the proof of the claim and the proposition.

In summary, when $\hat{a} \in [1 - 3b_s - 4b_r, \tilde{a}]$ three pooling intervals minimize variance; when $\hat{a} \in [\tilde{a}, 1 - 2b_s - 3b_r]$ three pooling intervals can be supported but two pooling intervals with an interior solution is optimal; when $\hat{a} \in [1 - 2b_s - 3b_r, \tilde{a}]$ only two pooling intervals can be supported and the best has an interior solution; and finally, when $\hat{a} \in [\tilde{a}, 1 - b_s - 2b_r]$ only two pooling intervals are possible and the best has a boundary solution.

References

Alonso, R. and N. Matouschek (2008). "Optimal delegation." *The Review of Economic Studies*, 75(1), 259–293.

Battaglini, M., E.K. Lai, W. Lim, and J.T-y. Wang (2016). "The informational theory of legislative committees: an experimental analysis." *Working paper*.

Berwick, D.M. and A.D. Hackbarth (2012). "Eliminating waste in U.S. healthcare." *Journal of the American Medical Association*, 307(14): 1513–1516.

Brownlie, J., A. Greene, and A. Howson (2008). "Chapter 10: The health care outcomes of trust: a review of empirical evidence," in *Researching Trust and Health*. New York: Routledge.

Chandra, A., D. Cutler and Z. Song (2012). "Who ordered that? The economics of treatment choices in medical care" in *Handbook of Health Economics*, Vol 2 (M.V. Pauly, T.G. McGuire, and P.P. Barros, eds.). Oxford: Elsevier. 396–432.

Chen, J. and A. Vargas-Bustamante (2013). "Treatment compliance under physicianindustry relationship: a framework of health-care coordination in the USA." *International Journal for Quality in Health Care*, 1–8.

Chen, Y., Kartik, N. and Sobel, J. (2008). "Selecting Cheap-Talk Equilibria." *Econometrica*, 76: 117136.

Cho, I-K. and D.M. Kreps (1987). "Signaling games and stable equilibria." *Quarterly Journal of Economics*, 102(2), 179–222.

Christianson, J.B. and D. Conrad (2011). "Provider payment and incentives." In S.A. Glied and P.C. Smith (Eds.), *The Oxford handbook of health economics*. Oxford: Oxford University Press. 624–648.

Clemens, J. and J.D. Gottlieb (2014). "Do physicians' financial incentives affect medical treatment and patient health?" *American Economic Review*, 104(4), 1320–1349.

Crawford, V. P. and J. Sobel (1982). "Strategic information transmission." *Econometrica*, 50, 1431–1451.

Cutler, D.M. and R. Zeckhauser (2000). "The anatomy of health insurance." In A.J. Culyer and J.P. Newhouse (Eds.), *Handbook of health economics*. Amsterdam: North Holland, Elsevier. 563–643.

De Jaegher, K. and M. Jegers (2001). "The physician-patient relationship as a game of strategic information transmission." *Health Economics*, 10, 651–668.

Dessein, W. (2002). "Authority and communication in organizations." *Review of Economic Studies*, 69, 811–838.

Emanuel, E. J. and V. R. Fuchs (2008). "The perfect storm of overutilization." *Journal of the American Medical Association*, 299(23), 2789-91.

Evans, R. (1974). "Supplier-induced demand: some empirical evidence and implications," in M. Perlman, ed., *The Economics of Health and Medical Care*. London: Macmillan, 162–173.

Feldstein, M.S. (1973). "The welfare loss of excess health insurance." *Journal of Political Economy*, 81(2), 251–280.

Gibson, T.B., R.J. Ozminkowsk and R.Z. Goetzel (2005). "The effects of prescription drug cost sharing: a review of the evidence." *American Journal of Management Care*, 11(11), 730–740.

Gilligan, T. W. and K. Krehbiel (1987). "Collective decision-making and standing committees: an informational rationale for restrictive amendment procedures." *Journal of Law, Economics, and Organization*, 3(2), 287–335.

Goldman, D.P., G.F. Joyce and Y. Zheng (2007). "Prescription drug cost sharing: associations with medication and medical utilization and spending and health." *Journal of the American Medical Association*, 298(1), 61–69.

Gruber, J. and M. Owings (1996). "Physician financial incentives and cesarean section delivery." *The RAND Journal of Economics*, 27(1), 99–123.

Ivanov, M. (2010). "Informational control and organizational design." *Journal of Economic Theory*, 145, 721–751.

Johnson, E.M. and M.M. Rehavi (2013). "Physicians treating physicians: information and incentives in childbirth." *NBER Working Paper* 19242.

Kolotilin, A., H. Li and W. Li (2013). "Optimal limited authority for principal." *Journal of Economic Theory*, 148, 2344–2382.

Krehbiel, K. (2001). "Plausibility of signals by a heterogeneous committee." *The American Political Science Review*, 95(2), 453–457.

Krishna, V. and J. Morgan (2001). "Asymmetric information and legislative rules: some amendments." *The American Political Science Review*, 95(2), 435–452.

Manning, W.G., J.P. Newhouse, N. Duan, E.B. Keeler, A. Leibowitz (1987). "Health insurance and the demand for medical care: evidence from a randomized experiment." *American Economic Review*, 77(3), 251–277.

Marino, A.M. (2007). "Delegation versus veto in organizational games of strategic communication." *Journal of Public Economic Theory*, 9(6), 979–992.

McGuire, T. G. (2000). "Physician agency," in *Handbook of Health Economics*, Vol 1 (A.J. Culyer and J.P. Newhouse, eds.). Oxford: Elsevier. 461–536.

McGuire, T. G. (2012). "Demand for health insurance" in *Handbook of Health Economics*, Vol 2 (M.V. Pauly, T.G. McGuire, and P.P. Barros, eds.). Oxford: Elsevier. 317–396.

McGuire, T.G. and M.V. Pauly (1991). "Physician response to fee changes with multiple payers." *Journal of Health Economics*, 10, 385–410.

Melumad, N.D. and T. Shibano (1991). "Communication in settings with no transfers." *The RAND Journal of Economics*, 22(2), 173–198.

Melumad, N.D. and T. Shibano (1994). "The Securities and Exchange Commission and the Financial Accounting Standards Board: regulation through veto-based delegation." *Journal of Accounting Research*, 32, 1–37.

Mylovanov, T. (2008). "Veto-based delegation." Journal of Economic Theory, 138, 297–307.

Newhouse, J.P. and the Insurance Experiment Group (1993). *Free for all? Lessons from the RAND Health Insurance Experiment*. Cambridge, MA: Harvard University Press.

Pitchik, C. and A. Schotter (1987). "Honesty in a model of strategic information transmission." *American Economic Review*, 77(5), 1032–36.