Targeted advertising and costly consumer search*

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September, 2020

Abstract

We study a model of advertising targeting based on information about the consumer's likely ranking of products. With horizontally differentiated goods and costly search, ads convey to consumers a noisy, positive signal of their unknown willingness to pay for the firms' products. That implies a higher expected willingness to pay for a yet not sampled firm, which increases the incentives to search. However, when firms target their ads based on likely fit with consumer preferences, expected differentiation between products advertised to each consumer decreases. This reduces the incentives to search. The first effect is more important for lower search costs and pushes prices down. The second is more important for larger number of products and pushes prices up. Advertising intensity affects the precision of consumers' information. Thus, higher marginal cost of advertising results in higher endogenous segmentation and higher prices.

Keywords: random advertising, targeted advertising, horizontal differentiation, sequential search

JEL classification: L13, D83

*We are grateful to Simon Anderson, Dan Bernhard, José Luis Moraga-González, Régis Renault, Chris Wilson, Raluca Ursu, the participants of the IX$^{\text{th}}$ Workshop on the Economics of Advertising and Marketing, the 15th IIOS Conference, EARIE-2017 and the paricipants of seminars at IAE (CSIC) for their invaluable comments and suggestions. The authors acknowledge the financial support of the Spanish Ministry of Economy and Competitiveness through the Severo Ochoa Programme for Centres of Excellence in R&D (SEV-2015-0563), grants No. ECO2014-59959-P and No. ECO2015-74328-JIN (AEI/FEDER/UE), and 2016 FBBVA grant "Innovación e Información en la Economía Digital".

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1 Introduction

For decades now, search-theoretic models have been a standard tool to study, among other things, the incentives and consequences of advertising (see, for instance, Baye, Morgan, and Scholten (2006), Stahl II (1994)). More recently, and to a large extent as a response to the availability of enhanced information technologies, search models have also been standard tools to analyze firms' ability and incentives to target their advertising. Targeting has been modeled as the ability to make the (costly) decision of informing one consumer contingent on (a perhaps imperfect[1] signal of) her willingness to pay for the firm's product (e.g., Renault (2016), Bergemann and Bonatti (2015), Mayzlin and Shin (2011)).

The starting point of this paper is the observation that much of the information that firms use to target their ads is not directly related to the consumer's *absolute* willingness to pay for *a* product. Instead, in a world of horizontal product differentiation and data on consumer behavior available to firms, more and more of this information refers to the relative fit of a firm's product characteristics to the consumer's tastes, and so to the *relative* (with respect to competitors' products) willingness to pay for *the* firm's product. Online search habits, lists of favorite YouTube videos, or likes on Facebook may tell more about the fit of a product's particular characteristics to the consumer's inclinations (sport versus formal design shoes, sci-fi versus romantic fiction, etc.) than about the consumer's purchasing power, for instance.[2] Obviously, learning that a consumer's preferences favor one firm's product increases the consumer's expected willingness to pay for the firm's product. What is more important, though, is that it increases the expected gap between the willingness to pay for the firm's product and an arbitrary, other firm's product.

The purpose of this paper is to reevaluate the effects of targeting on consumer behavior and on firms' pricing and advertising strategies when this so far neglected type of information is used by firms to target their ads. Needless to say, firms use several sources of information, some directly related to consumers' (absolute) willingness to pay. The consequences of targeting based on such information have been extensively discussed in the literature. Thus, in order to highlight what our analysis contributes to this literature, in this paper we assume away any such information, and concentrate on how targeting based on ranking information affects firms' advertising and pricing behavior.

To do so, we compare market outcomes under targeted advertising with outcomes when advertising is not targeted (random). In the latter case, firms send their ads to random consumers. When firms target their ads, they send their ads to consumers who rank their products high with high probability.[3]

[1]During the experiment of Mediasmith with the data of different consumer data vendors, an increase in advertising accuracy through targeting varied from 5% to 183% (Marshall, 2015). Forbes (2015) notes that 54% of North American companies cite the identification of a target audience as a primary challenge. According to Loechner (2014), "*92% of Americans ignore at least one type of ad seen every day across six different types of media*", and the survey of Reuters shows that 39% and 47% of consumers in the UK and the US respectively use ad-blocking software (Austin and Newman, 2015) .

[2]Sometimes consumers do not know themselves how much they want to pay for one or another product. Franke, Keinz, and Steger (2009) find that consumer segmentation is more efficient when consumers understand their preferences better.

[3]In contrast, Athey and Ellison (2011), Chen and He (2011), Anderson and Renault (2015) and Chen and Zhang (2017) assume that consumers know the ranking of products that they search.

In both advertising regimes, there is room for consumer search.

The possibility of targeting based on relative willingness to pay (rankings) has several effects on firms' pricing and advertising incentives. For a given intensity of advertising, targeting increases the information content of ads. Consumers learn about the –probabilistically–most relevant sellers, given their idiosyncratic preferences, which raises the value of search.[4] More search increases the competitive pressure on firms, and so acts as a downward pressure on prices. We call this effect a *competition effect*. In addition, the products that consumers learn about are positively selected, and so closer substitutes on average. When this selection effect of advertising is strong, the value of search may in fact be lower in expected terms, and so demand less elastic,[5] which encourages sellers to raise their prices. We call this effect the *demand composition effect*.

This second, novel effect, then brings about the ingredients of the well-known Diamond (1971) paradox, and its strength increases with the number of firms (varieties). Indeed, a larger number of varieties implies (stronger selection and so) lower expected differentiation of the products that best fit the consumer's preferences. Thus, in expectation, the gains from search also shrink. As a consequence, the equilibrium price may be higher under targeted advertising when the (search cost and) the number of firms is large.

Targeting also affects the incentives of firms to advertise, which in turn affects the information that consumers may infer from receiving ads, and so their incentives to search. Indeed, firms optimally target those consumers for whom their product is likely to be among the best matches to the consumer's preferences. Therefore, other things equal, firms advertise first to those consumers for whom their product is more likely to be their best match. When firms' information is highly correlated, these consumers are the ones for whom other firms' products are less likely to rank high in their preferences. That is, consumers are less likely to receive ads from rivals. Thus, ex post the market is (partially, endogenously) segmented by the targeting of ads. The higher the cost of ads, the sharper this segmentation. This, which we term *endogenous monopolization* effect, results in an additional incentive for higher prices under targeting.[6]

When the demand composition (and monopolization) effect(s) dominate the competition effect, the ability to target may boost firms profits. This may be so even when the information used to that end is not proprietary. Thus, firms would have a positive willingness to pay for information provided by the same advertising intermediary that their rivals use. This is more likely, in particular, when the cost of advertising, the search cost, and/or the number of firms –varieties– is large.

The literature has considered targeting directly based on match values (de Cornière (2016), Renault

[4]In Kireyev et al. (2016), the display of some product information (not prices), which is qualitatively equivalent to targeted ads as consumers pick better products, raises search conversion and encourages consumers to click on ads more frequently.

[5]In his field experiment with wine buyers, Fong (2017) found that consumers who obtained ads that were more tailored to their preferences searched products less on average.

[6]Galeotti and Moraga-González (2008) and Manduchi (2004) study interesting models with homogeneous products and utility-irrelevant consumers' observable characteristics. Firms may target their ads to different-looking –identical, in all relevant characteristics– consumers, so as to segment the market and reduce competition.

(2016)).[7] Firms target their ads to consumers with a match value above certain threshold. That reduces the expected differentiation of products that each consumer learns about, which also reduces the incentives to search. However, match values are still (conditionally) independent, and so the incentives to search are (stationary and) independent of the number of firms/varieties in the market. On the contrary, when targeting is based on rankings, match values are endogenously correlated, and so the first observed match value conveys information about another not-yet-observed match value. This induces non stationary consumers' beliefs and search patterns. This new learning element that our model captures is the source of the demand composition effect on the equilibrium prices, an effect that is missing in Eliaz and Spiegler (2011)and Renault (2016), and is also different from the one identified by de Cornière (2016).[8],[9]

Information about rankings is central in models that study online search mediated by search engines, like Athey and Ellison (2011), Katona and Sarvary (2010), etc. Our model focuses on firms' pricing incentives and the intensity of advertising, as opposed to the relationship between the engine and the firms accessed through the engine. Also, we assume that ads themselves do not inform the consumer directly of the ranking of valuations. This is in contrast to Chen and He (2011), who study a clever model where consumers' beliefs do not change as they proceed with search.[10] Chen and He (2011), also discuss equilibria in their model where sellers appear randomly in the list of sponsored links. However, the particular distribution of willingness to pay assumed –with some probability, a commonly known number v, otherwise 0 – renders the price virtually exogenous (equal to v) in both equilibria.

We specify our model in Section 2. There, we present perhaps the simplest possible –imperfect– targeting technology based on rankings that still allows for meaningful competition. That technology allows a firm to identify two groups of consumers: those for whom its product is among the best two matches and those for whom it is not. As we show later (Section 6), this is only for simplicity and does not drive the insights or main results. Then we characterize consumer search with random and targeted advertising in Section 3. In that section, we also discuss the main effects that targeting has on this search. This is perhaps the core of the paper. The effects of targeting, particularly on prices, that we uncover all have to do with how targeting affects search behavior. In Section 4 we obtain the demand of firms

[7]Equivalently, it results firms targeting consumers with a larger probability to draw a positive match value (Eliaz and Spiegler (2011)).

[8]Specifically in his setting, a search engine charges more for advertising if targeting is finer, and these costs are passed through to consumers. In our setting, a price increase happens due to better consumer sorting according to their horizontal preferences, and we do not relate unit advertising costs to the precision of targeting.

[9]Ben Elhadj-Ben Brahim et al. (2011) study targeting in a Hotelling model with two firms, which imposes negative correlation of match values, and show that the negative correlation may lead to higher prices with finer targeting. Iyer et al. (2005) and can be viewed as a extreme version of the model of Ben Elhadj-Ben Brahim et al. (2011) with consumers only at the extremes of the line or at the middle point.

Esteves and Resende (2016) study firms' incentives to advertise on a Hotelling line like Ben Elhadj-Ben Brahim et al. (2011) but do not allow consumer search.

In Zhang and Katona (2012), a firm sends ads to consumers based on their preferences for media content that correlates with the binary distributed willingness to pay for a product and higher correlation helps to target ads better. Better targeting leads to monopolization when there are more price shoppers, and the result is reverse if the share of price shoppers is high.

[10]See also Chen and Zhang (2017), Anderson and Renault (2015) and the references therein.

and characterize symmetric equilibrium in prices and advertising for both regimes. Then, in Section 5, we specialize the model by defining parametric functional forms and obtaining explicit solutions. These illustrate the effects discussed in Section 3. As mentioned, in Section 6, we extend the model to allow for more precise and general ranking information. We show that the effects that we discuss in the simplified, baseline model are the effects that targeting has in general when it is based on rankings. We further discuss modeling assumptions and suggest some lines of future research in Section 7.

2 Model

Each of $N \geq 2$ symmetric firms offers one horizontally differentiated product to a continuum of consumers of measure one. The firms' marginal (production) costs are constant, identical, and normalized to zero. Firms also incur advertising costs.

A consumer $j \in [0,1]$ who buys a product of firm i at price P_i gets ex-post utility

$$u_i^j = z_i^j - P_i,$$

where z_i^j is a random variable measuring the value of the match between consumer preferences and product characteristics. We assume that all z_i^j are identically and independently distributed across consumers and products on the interval $[0,1]$ with a log-concave distribution function $F(z)$ and a log-concave density function $f(z)$. We normalize the utility of not buying to zero.

As in Wolinsky (1986), we assume that consumer j observes the match value z_i^j and the price P_i only upon visiting firm i. Moreover, z_i^j is then privately observed by the consumer. In fact, we assume away price discrimination. The consumer may visit firms sequentially by incurring a search cost s per visit, identical across consumers and firms. Additionally, we simplify the discussion by assuming that the first visit is free, and there are no recall costs.[11]

Firms simultaneously decide their prices and the number of ads they send to consumers. If a firm sends a measure $\mu \in [0,1]$ of ads, then it incurs advertising costs $C(\mu)$, where C is a differentiable, increasing, strictly-convex function with $C(0) = 0$. A consumer may only receive one ad from each particular firm. That is, firms have the possibility to address ads and not duplicate. Ads carry no information about z or P. Thus, as is standard in this literature, the only purpose of an ad is to allow the consumer to "locate" a firm. In particular, we assume that a consumer knows that (or believes that) there are N firms in the market, but it is prohibitively costly for her to locate any one of them.[12,13]

[11] Janssen and Parakhonyak (2014) show that recall costs do not affect qualitative results in sequential search models. Free first visits, a usual assumption, simplify the arguments without altering any of them.

[12] Promotional e-mails and ads provide with direct links and brand names that makes search easier without supplying any information about products or influencing search order. For instance,Mehta, Rajiv, and Srinivasan (2003) observe that "in-store display activities and feature ads do not influence quality perceptions, they do reduce consumer search costs for a brand, thereby significantly increasing the probability of the brand being considered."

[13] We are assuming that consumers cannot "locate" any firm without having received an ad. In the targeting case analyzed below, allowing a consumer to learn about other firm by other means is of no consequence: consumers would

Targeting technology. We consider two advertising cases: random and targeted. In the first setting, firms do not have any information about consumers. Thus, all consumers look alike and a firm randomly chooses to which consumers it sends its ads. In the second case, each firm has some information about how a consumer ranks its product with respect to the competitors'.[14] That is, firm i knows that for consumer j its product has some probability of being the most desirable (i.e., z_i^j is highest among all z_k^j), some probability of being the second most desirable, etc. In Section 6, we will discuss a more general model, but as anticipated, we will begin by make the simplest possible assumption about the information that firms have. We assume that each firm knows the subset of consumers to whom its product is one of the two most desirable.[15] We call those consumers *target* consumers.

When targeting is based on rankings, search behavior is not stationary even when match values are independent draws across firms. Thus, in order to more clearly identify the effects of targeting, we will focus on cases where firms' advertising behavior results in each consumer receiving (in equilibrium) at most two ads. Whether firms themselves distinguish who is most likely to value their product the most from who is most likely to have it as a second best is of little consequence. We choose here to make this additional simplifying assumption to facilitate grasping the insights which, as we show in Section 6, are quite immune to the assumption. As we shall discuss in Section 6 too, settings where firms' advertising results in consumers possibly receiving more than two ads would be more cumbersome, but nothing of substance would change.

Note that we will be assuming that firms have information about consumers' relative willingness to pay but not (directly) about consumers' absolute willingness to pay for their products. Product ranking is what search engines (or ad intermediaries in their platforms) provide to advertisers (e.g.Yang and Ghose (2010) and Yao and Mela (2011)). Also, strict regulations, like the General Data Protection Regulation (GDPR) in the EU, may prevent the collection of personal data, but not behavioral data that may shed (more) light on relative rankings.[16]

We follow the modeling convention in the consumer sequential search literature and assume that consumers hold passive equilibrium beliefs about prices and advertising levels. Because we focus on symmetric equilibrium, consumers expect that all firms charge the same price and send the same measure of ads.

give priority to received ads, as in Haan and Moraga-González (2011), and would infer that other visits would be of no value. In the random advertising case, allowing so could be homomorphic to assuming the advertising cost to be equal to zero for the first fraction of ads, which we can accommodate in our cost function C.

[14] Of course, the firm does not need to have that information, as long as the advertising intermediary in charge of targeting its ads does have it.

[15] That is, formally, given a ranking of z_i^j's for consumer j, $(z_{i(1)}^j, z_{i(2)}^j, ...z_{i(N)}^j)$, where $z_{i(k)}^j \geq z_{i(k+1)}^j$, for $k = 1, 2, ..., N-1$, firm $i(1)$ learns (observes a signal that says) that its product is the most valuable for consumer j with probality $1/2$ and the second most valuable with probaiblty $1/2$. Firm $i(2)$ lerans the same (observes the same signal). Firms $i(k)$ for $k > 2$ learn that their product is in the position $3, 4, ..., N$, each with probability $1/(N-2)$, in the ranking of match values for consumer j.

[16] As an example, the supermarket chain Target knows that their pregnant shoppers prefer unscented lotion vs scented lotion, and so have indirect information on the potential pregnancy of customers. However, they may not necessarily be able to observe customer's marital status, age, or income levels (see Hill (2012)).

The timing of the game that firms and consumers play is as follows. First, firms simultaneously and independently choose how many ads to send, to whom, and what prices to charge. After receiving ads, consumers decide which firm to visit first, if any. Upon visiting firm i and learning z_i^j and P_i, a consumer decides whether to buy, to search further, or to return to and buy from a previously visited firm. If she decides to search, further search may also be a choice.

3 Search behavior

From the consumers' point of view, all sellers from whom they receive ads are symmetric in both treatments. Thus, a consumer visits the firms she learns about in a random order. Upon visiting each firm, the consumer compares the expected gains from searching one more product with the search cost s. If the expected gains are higher than the cost, the consumer continues searching. Otherwise, the customer stops searching and buys the product providing the highest (positive) observed utility (if any). Thus, the consumer continues searching if the utility from the best alternative so far encountered is below a threshold that depends on whether advertising is targeted or not.

Random advertising. When ads are random, the observed match value of a particular product does not inform a consumer about not-yet-searched products. As a result, consumers apply a myopic stopping rule of Weitzman (1979) that we summarize here. Suppose that a consumer expects all firms to charge P^*. Also, suppose that after some visits, the best option the consumer has observed so far gives her positive utility u ($= z_i - P_i \geq 0$ for some visited seller i). A new visit to a firm l costs s and will give the consumer a benefit of $z_l - P^* - u$ if $z_l - P^* > u$, but no benefit otherwise. That is, the net expected value of that extra visit is

$$\int_{u+P^*}^{1} (z - P^* - u)\, dF(z) - s. \tag{1}$$

Thus, when all firms do charge the same price, consumers will keep searching after observing a match value above the price as long as (they have a new firm to search and) the highest match value they have found is below the threshold w that makes that net expected value of a new visit equal to zero. That is, w solves

$$\int_{w}^{1} (z - w)\, dF(z) = s. \tag{2}$$

When $P^* > w$, consumers will never search: they will visit –for free– one firm (if they receive an ad) and buy from it if and only if the match value observed is above P^*. Therefore, $P^* > w$ could be an equilibrium only if P^* is the monopoly price $P^M = \arg\max_P P(1 - F(P))$.

Targeted advertising. Contrary to the random advertising case, with targeting the observed match value of one product affects the expected match value of another good. Thus, we first need to figure how a consumer who receives two ads updates her beliefs about the match value in a new firm after sampling the match value in the other one. Suppose this consumer has observed z_l in the first sampled firm, firm

l. This z_l may be either the highest or the second-highest match value for this consumer. That is, z_l may be the first- or the second-order statistic of N realizations of F. Before observing z_l, the probability of each of these two events is 1/2. However, after observing z_l, the consumer updates the probability of these two events, and so the probability distribution of the match value in the other firm –say firm i– that this consumer knows about. In the Appendix we show how to compute the (conditional) density of that new match value for this case, which we denote by $g^c\left(z_i|\, z_l\right)$. Similarly, we denote by $G^c\left(z_i|\, z_l\right)$ the corresponding cdf. The probability of the conditioning event is the probability of the first- or second-order statistic of N independent realizations of F, each of them taken with probability 1/2. The density function of this random variable (i.e., of a target consumer's match value) is

$$g(z) = \frac{1}{2} N f(z) \left(F\left(z\right)^{N-1} + \left(N-1\right)\left(1 - F\left(z\right)\right) F\left(z\right)^{N-2} \right),$$

and we denote the corresponding cdf by $G(\cdot)$.

Similarly to the random advertising case, suppose that the consumer finds a match value higher than the price when visiting the first firm. She will visit a second firm if the expected gain from search is larger than s. Conjecturing a price of P_t^* in the new firm and having observed a positive utility u $(= z_l - P_l)$ in the first visit, this net gain is as in (1) with only substituting $g^c\left(z|\, z_l\right) dz$ for $dF(z)$ and P_t^* for P^*. In general, this gain is not monotone in z_l. However, a sufficient condition for this to be the case is that $f(z)$ is log-concave, as we are assuming. In such case, and as with random advertising, in a symmetric equilibrium with $P_i = P_t^*$ the optimal search rule is characterized by a cutoff w_t that solves:

$$\int_{w_t}^{1} \left(z - w_t\right) g^c\left(z|\, w_t\right) dz = s, \tag{3}$$

However, if $P_t^* \geq w_t$, then the consumer would never search, and so will buy from the first firm visited if and only if the match value there is larger than the price. Once more, for that to be part of an equilibrium, P_t^* must be the monopoly price for this case, $P_t^M = \arg\max_P P\left(1 - G(P)\right)$.

Maximum search costs. We are interested in cases other than monopoly. Thus to ensure that some consumers search in equilibrium, we impose an upper bound on the search cost. (As we have mentioned above, when there is no search, the only equilibrium price is the monopoly price.) Thus, let us assume that $s \leq s_r$, where $s_r = \int_{P^M}^1 \left(z - P^M\right) f(z) dz$. Likewise, in the target advertising case, let us assume that $s \leq s_t$, where $s_t = \int_{P_t^M}^1 \left(z - P_t^M\right) g^c\left(z|\, P_t^M\right) dz$.

3.1 The effects of targeting on search

We can discuss the main effects of ranking-based targeting by considering its effect on consumer search and the implications for pricing. In the next section, we will derive demand expressions in both regimes and characterize symmetric equilibria. However, the key to understand how targeting, and how the parameters of the problem, affect prices and other outcomes is to understand how they affect search.

Thus, suppose that firms send a measure $2/N$ of ads in both advertising regimes, and $N = 2$. In this case, $g^c(z_i|\, z_l) = f(z_i)$ for any z_l, and so (2) and (3) are the same. Indeed in both advertising regimes, receiving an ad from a firm conveys the same information: the firm exists and is one of the two firms that carry one of the two products that best match the consumer's preferences. Thus, consumers' search behavior would be the same in both advertising regimes provided that consumers conjecture the same price in both. From the point of view of firms, the situation is also the same in both advertising regimes: a product is one of the best two matches to each consumer's preferences. Therefore, we expect equilibrium prices to be the same in both advertising regimes, whatever the search cost is. Let this be our starting point.

Now consider $N > 2$ but assume that $s = 0$. Then consumers' search behavior is still the same in both advertising regimes, since $w = w_t = 1$. That is, before buying, consumers –costlessly– visit all firms they learn about, and buy from the firm providing the highest utility, if positive. However, from the point of view of firms the situation is now different. A consumer who visits a firm in the targeted advertising case will learn about the closest substitute as well. To all effects, that is all the consumer needs to know about the market: –in equilibrium– every firm will compete in prices with the closest competitor for perfectly informed consumers. On the contrary, since $2/N < 1$ now, a consumer in the random advertising case may not receive ads from all firms –possibly not even from another one. Thus, she may not learn about a firm's closest competitor. Thus, competition is fiercer in the targeting case, and this results in lower prices in the targeted advertising case even though firms compete for consumers with higher expected willingness to pay. We call this *competition effect*.

Next, assume that $s > 0$ (and still $N > 2$ and firms send $2/N$ ads each in both advertising regimes). Compared to the first case with $N = 2$, an increase in N does not affect (2). However, there are changes in (3): as N increases, g^c puts more and more weight on high values of z, and so w_t increases in N too. That is, an increase in N does not affect consumer search behavior in the random advertising case but it does affect that behavior in the targeted advertising case. With targeted ads, consumers are more choosy and require higher match values to terminate search after the first visit, so that, for *a given* observed match value during the first visit, the probability of search is higher as N gets higher. However as N increases, the probability of observing high match values during the first search is also higher –that is, g also puts more and more weight on high values of z–, which for *a given* threshold w_t implies a lower probability of search. That is, an increase in N triggers two opposite effects on the probability of search in the case of targeted advertising. Is it possible to determine which of the two is stronger? The answer is in the affirmative, at least for sufficiently large N. Indeed, as N grows, $g^c\left(z_i|\, z_l\right)$ approaches $\frac{f(z_i)}{1-F(z_l)}$ for $z_i > z_l$ and so w_t approaches the solution to

$$\int_{w_t}^{1} (z - w_t) \frac{f(z)}{1 - F(w_t)} dz = s \tag{4}$$

A solution to (4) is bounded away from 1.[17] Meanwhile, the probability that the match value observed in the first visit is above w_t, which equals $1 - G(w_t)$, approaches 1 for any value of $w_t < 1$. As a result, the probability of the second search approaches zero, and so the incentives to raise prices are higher than in the random advertising case. This we call the *demand composition effect.*

This demand composition effect is closely related to the Diamond paradox. Specifically when match values are perfectly correlated (products are homogeneous), the case where the Diamond's paradox was originally discussed, search vanishes no matter how small s is. Note that it is the correlation of values (plus the conjecture of equal prices in a symmetric equilibrium) that explains this run to monopoly pricing: consumers do not expect gains from an additional (costly) visit. In our targeted advertising case, the correlation between the first- and second-order statistics of N independent random draws of a random variable is increasing in N.[18] That is, the expected gain from an additional visit is also decreasing with N, and as N grows large, firms' equilibrium prices also converge to P_t^M ($> P^M$). (Note that in the random advertising case, the correlation between a consumer's expected match values in two firms from which she receives ads does not change with N.) Needless to say, the monopoly price is increasing in N too, as a larger N implies a larger expected value of both the first- and the second-order statistics. In fact, $P_t^M \to 1$ as N grows large.

Finally, let us consider the effect of the volume of advertising on search behavior. The value of μ does not affect consumers' search behavior in either regime. However, it does affect the number of firms a consumer visits on average. This number is increasing in μ. In particular, the lower μ, the higher the chances that a firm faces a consumer as a monopolist, which pushes the symmetric equilibrium price to the monopoly price. We call this effect of μ (and so of the cost of advertising) *endogenous monopolization effect.* This monopolization effect is common to both advertising regimes. However, remember that $P_t^M > P^M$. That is, the effect of monopolization on prices is stronger in the targeted advertising case. Thus, when the advertising levels are low (the advertising costs are high), the equilibrium price may be higher under targeted advertising than under random ads.

4 Equilibrium derivations

4.1 Random advertising

The derivations for this case are very similar to those in Wolinsky (1986). In a symmetric equilibrium, all firms charge the same P^* and send the same measure of ads μ^*. To characterize such price and advertising intensity, we analyze a small, unilateral deviation by a firm, say firm i, to $P \neq P^*$ and $\mu \neq \mu^*$. Consider a consumer who has received k ads, one of them from firm i. The probability of this event is $\mu B_{\mu^*}(k-1|\, N-1)$, where we use $B_q(x|\, X)$ to represent the probability function of a binomial

[17] For $w_t < 1$, the expression in the left-hand-side is positive, continuous and equals $E\left[z|\, z \geq w_t\right] - w_t$, which converges to 0 as $w_t \to 1$.

[18] It ranges from 0 correlation when $N = 2$ to one when N approaches infinity.

random variable with X draws and "probability of success q". That is,

$$B_{\mu^*}(k-1|\,N-1) = \binom{N-1}{k-1} (\mu^*)^{k-1} (1-\mu^*)^{N-k}.$$

Suppose that the consumer arrives at firm i after visiting $l < k$ other firms. The probability of this sampling order (i.e., that i would be visited in the $l+1^{th}$ position) is simply $1/k$. According to our discussion in the previous section, the consumer must have observed match values below w in all previous visits. Also, if the consumer observes at firm i a value z so that $z - P \geq w - P^*$, then she terminates search and buys from firm i. Thus, the probability that a consumer who has received an ad from firm i visits that firm and terminates search there is

$$\sum_{l=0}^{k-1} \frac{1}{k} F(w)^l \left(1 - F\left(w - P^* + P\right)\right), \tag{5}$$

which in Armstrong et al. (2009) is called *fresh demand* for product i.

Even if $z - P < w - P^*$, and so the consumer continues searching after a visit to firm i, she may eventually return to firm i and buy, once she has sampled all k firms she knows about. This happens if the utility $z - P$ is (positive and) the highest of all k observed utilities. The probability of this event, so called *returning demand* for product i, is:

$$\int_{P}^{w-P^*+P} F(z - P + P^*)^{k-1} f(z) dz. \tag{6}$$

Let $D^k(P; P^*)$ be the sum of (5) and (6) for k. Then the payoff of firm i is

$$P \sum_{k=1}^{N} \mu B_{\mu^*}(k-1|\,N-1) D^k(P; P^*) - C(\mu).$$

An interior, symmetric equilibrium is characterized by the first-order conditions for the problem of maximizing this payoff in P and μ when these values coincide with P^* and μ^*. The following lemma simply states these conditions.

Lemma 1. *In a symmetric, pure-strategy equilibrium with random advertising, all firms send μ^* ads and charge P^* determined by the system of equations*

$$\begin{aligned} \sum_{k=1}^{N} \mu^* B_{\mu^*}(k-1|\,N-1) \left(D^k(P^*; P^*) + P^* \left. \frac{\partial D^k(P; P^*)}{\partial P} \right|_{P=P^*} \right) &= 0, \\ P^* \sum_{k=1}^{N} B_{\mu^*}(k-1|\,N-1) D^k(P^*; P^*) - C'(\mu^*) &= 0. \end{aligned} \tag{7}$$

4.2 Targeted advertising

We now characterize a symmetric equilibrium for the targeted advertising case where all firms send $\mu_t^* \le 2/N$ ads and charge P_t^*.[19] That is, every firm sends ads to a share $\alpha^* \equiv \mu_t^* N/2$ of its target consumers. Given N, the choice of μ is equivalent to the choice of α, so we use the latter measure for deriving the payoff function.

Consider a firm i that contemplates a unilateral deviation to $P \neq P_t^*$ and $\alpha \neq \alpha^*$. There are two groups of target consumers who receive its ads. The first group do not obtain any other ad. The number of these consumers is $2\alpha(1-\alpha^*)/N$. The second group, $2\alpha\alpha^*/N$ of consumers, receive ads from both firm i and the other firms for which they are target consumers. The individual demand function of the first group is the monopoly demand $1 - G(P)$. (Recall that $G(z)$ is the distribution function of a target consumer's match value.) Thus, the demand from this group is

$$\alpha(1-\alpha^*)\frac{2}{N}(1-G(P)). \tag{8}$$

To compute the demand of the second group, assume that P is (slightly) above P_t^*.[20] Half of these consumers will first visit another firm. Then, if they observe a match value below w_t they will visit firm i and buy if the match value is above P and also above the alternative plus $(P - P_t^*)$. That is, a total of

$$\begin{aligned} & \alpha\alpha^*\frac{1}{N}\int_0^{w_t} g(z)\left[1 - G^c(\max\{z + P - P_t^*, P\}|\, z)\right] dz \\ = \;& \alpha\alpha^*\frac{1}{N}\left(\int_0^{P_t^*} g(z)\left[1 - G^c(P|\, z)\right] dz + \int_{P_t^*}^{w_t} g(z)\left[1 - G^c(z + P - P_t^*|\, z)\right] dz\right), \end{aligned} \tag{9}$$

where z above represents the value of the match in the other firm.[21]

The rest of these consumers visit firm i first. The consumer is surprised by the price observed here. Indeed, there is an (unanticipated) extra cost $P - P_t^*$ of purchasing at firm i. Yet, the (expected) gains from searching, although different from the gains in equilibrium, are very similar, and require simply adding this extra cost. Thus, the consumer's optimal search response is given by (3) with only substituting $w_t + (P - P_t^*)$ for w_t in the lower-bound of the integral and in the difference $z - w_t$ (but not in the conditional density). That is, there is a new cut-off point $w_t(P)$ defined as the solution to the modified equation. Then the consumer buys from the firm if the match value is above $w_t(P)$ or if (it is below but above P and) the value in the other firm is below the value in firm i minus $(P - P_t^*)$.

[19] We will assume $C(\mu)$ to be such that indeed μ_t^* is indeed not larger than $2/N$. If the marginal cost of advertising is (sufficiently) lower, firms may send ads to consumers other than their target consumers. This would affect the density g^c and g, but not in fundamental ways.

[20] Differently from the random advertising case, the demand function of a deviating firm under upward and downward deviation is different. We provide the detailed derivations of other deviations the Supplementary Appendix.

[21] Note that in all cases this value would have to be below the value in firm i, and so we use the corresponding expression for g^c.

That is, a total of

$$\alpha\alpha^* \frac{1}{N}\left\{(1 - G(w_t(P)) + \int_P^{w_t(P)} g(z)G^c(z - P + P_t^* |\, z)dz\right\}, \tag{10}$$

where this time z above represents the value in firm i.

Thus, if we let $D_t(P, \alpha; P_t^*, \alpha^*)$ represent the sum of (8), (9), and (10), the firm's profit is

$$D_t(P, \alpha; P_t^*, \alpha^*)P - C\left(\frac{2\alpha}{N}\right).$$

In the appendix we present the expression for $D_t(P, \alpha; P_t^*, \alpha^*)$ in terms of the distribution F and its density. The following lemma simply states the necessary conditions for a symmetric equilibrium in terms of the first order conditions for the firm's profit maximization.

Lemma 2. *In a symmetric, pure-strategy equilibrium with targeted advertising, all firms send* $\mu_t^* = \frac{2\alpha^*}{N}$ *ads to their target consumers and charge price* P_t^* *characterized by*

$$\begin{aligned} D_t(P_t^*, \alpha; P_t^*, \alpha^*) + P_t^* \left.\frac{\partial D_t(P, \alpha; P_t^*, \alpha^*)}{\partial P}\right|_{P=P_t^*} &= 0, \\ \left.\frac{\partial D_t(P_t^*, \alpha; P_t^*, \alpha^*)}{\partial \alpha}\right|_{\alpha=\alpha^*} P_t^* - \frac{2}{N}C'\left(\frac{2\alpha^*}{N}\right) &= 0. \end{aligned} \tag{11}$$

4.3 Advertising and profitability

Note that in both regimes we have that equilibrium is characterized by

$$\frac{P \times Q}{\mu} = C'(\mu),$$

where Q is a firm's demand, and we recall that in the targeted advertising case $\mu = \frac{2}{N}\alpha$. That is, $Q = D_t(P_t^*, \alpha^*; P_t^*, \alpha^*)$ in the targeted advertising case and $Q = \sum_{k=1}^{N} \mu B_{\mu^*}(k-1|\, N-1)D^k(P; P^*)$ in the random advertising case. Indeed, in our model, an ad's marginal return equals an ad's average return, which in equilibrium equals marginal advertising cost.

Thus, since a firm's profit is $P \times Q - C(\mu)$, we have that in both regimes a firm's profit is $\mu C'(\mu) - C(\mu)$ in equilibrium. The derivative of this expression with respect to μ is simply $\mu C''(\mu) > 0$. Thus, comparing profits in both regimes amounts to comparing advertising efforts. If firms send more ads in one regime it means that profits are larger in that regime. Likewise, if the effort in advertising increases with a change in one parameter, then profits also increase with that change.

5 Prices, profits, and advertising

In this section, we present the solutions to (7) and (11) specializing the functional forms in the model to illustrate the combined effects discussed in Section 3.1. Thus, we consider the family of match-value distributions, $F(z) = z^r$ for $r \geq 1$ and $z \in [0, 1]$, and cost functions, $C(\mu) = c\mu^b$, for $c > 0$ and $b > 1$. The solutions can be computed for values of the parameters (N, s) for different functions, i.e., parameters (c, b, r).[22] The main purpose of the exercise is to illustrate how the –equilibrium– relative strength of the effects of targeting based on ranking information, discussed in Section 3.1, are affected by the cost of search, the number of firms (degree of variety competition), and the cost of advertising. We summarize the behavior of equilibrium market outcome in a few figures where we plot these outcomes against the values of the parameters. (We include c in the variables of interest, as it directly measures how costly advertising is.)

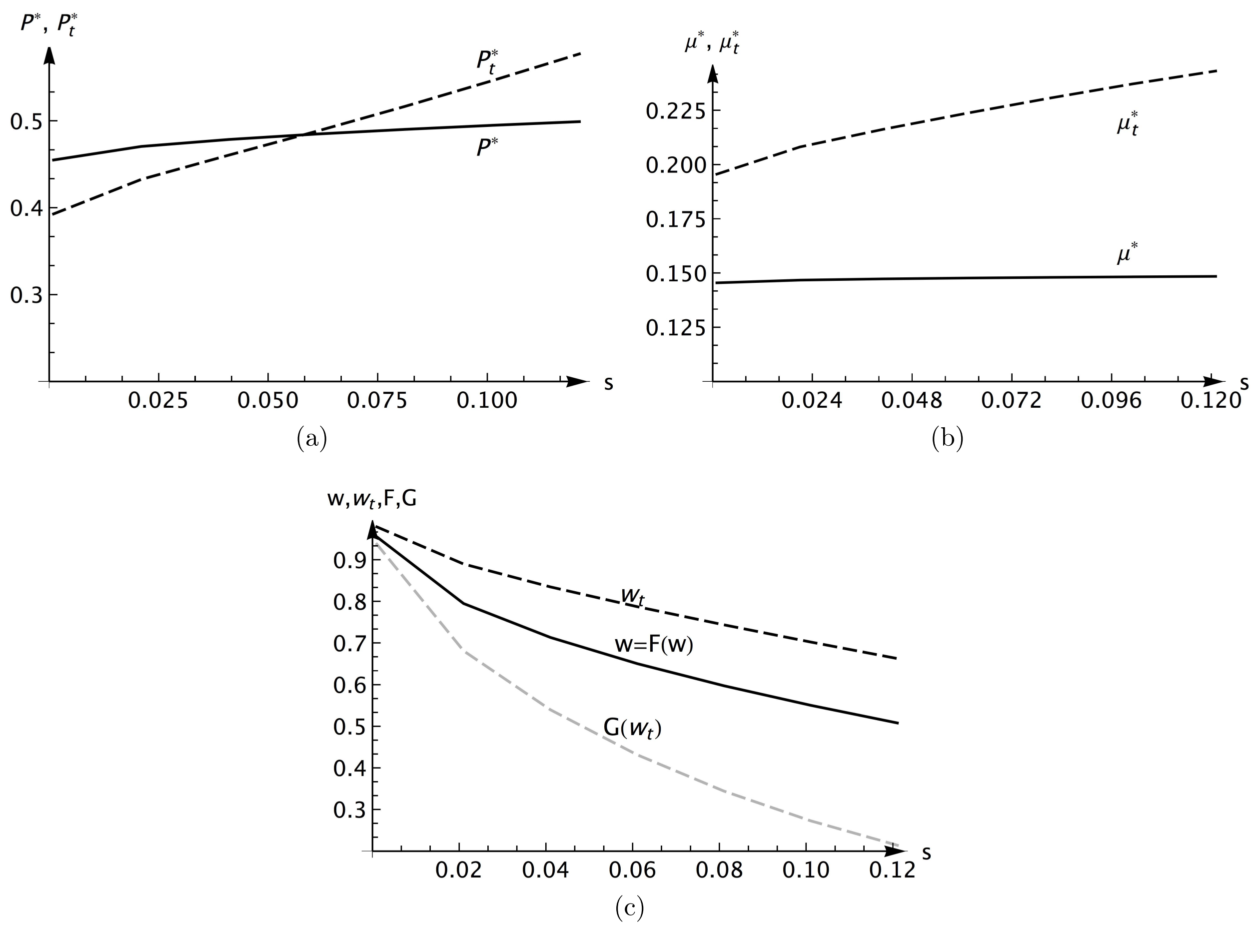


Figure 1: Equilibrium prices, advertising and search intensity with respect to s.

In Figure 1, we plot prices, search intensity (i.e., both w and w_t, and $F(w)$ and $G(w_t)$), and advertising intensity (and so profits), as a function of the cost of search, s. (In this figure, we have set $r = 1$, $b = 2$,

[22]The, existence of symmetric equilibrium is not guaranteed in general like in Christou and Vettas (2008). The cases studied here do have a symmetric equilibrium.

$N = 6$, and $c = .7$.) Note that for very low values of s, so that search is intense in both regimes, prices are lower under targeting. Indeed, that is where the competition effect is stronger, and so firms compete more fiercely under targeting. As the cost of search increases, consumers search less and so demand is more inelastic and consequently prices increase. Although this is so in both regimes, the relative weight of the competition effect decreases as compared with the demand composition effect, which eventually results in higher prices under targeted advertising. Indeed, even though firms may intensify their advertising more when ads are targeted, (and so the endogenous monopolization effect is less acute in the targeting advertising case) for larger values of s the demand composition effect dominates the competition effect –consumers search less–, and as a result prices are higher under targeted advertising.

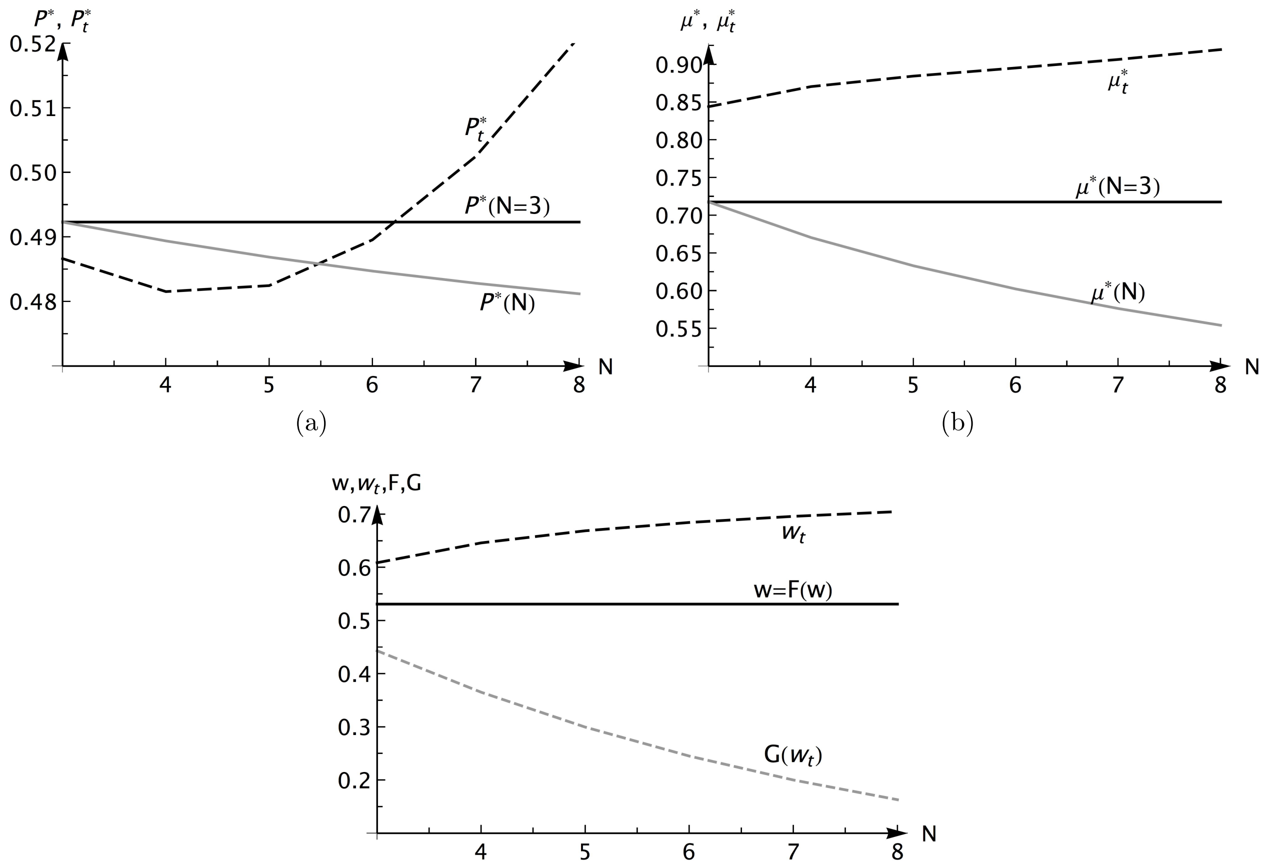


Figure 2: Equilibrium prices, advertising and search intensity with respect to N.

This is more clearly observed in Figure 2, where we have plotted the same outcomes as a function of N. (In this figure, $r = 1$, $s = c = .11$, and $b = 3$.) We need to explain this comparative statics exercise with respect to N. As the number of firms grows, the size of the market for each of them shrinks. In particular, the number of target consumers for each firm shrinks as well, so to all effects the (marginal) cost of advertising (for any given proportion of relevant consumers) decreases. A way of separating the effects of variety and of advertising cost is to let the size of the total consumer population grow with the number of firms. This is what we do in Figure 2 for the targeted advertising case. (We took $N = 3$

as the initial number of firms, that is, when the number of consumers is 1.) However, in the random advertising case, this same adjustment induces a change in the distribution of the number of other ads for a consumer receiving one ad from the firm.[23] Keeping both the marginal cost of advertising and the ad distribution unchanged as N increases is (from a firm's perspective) equivalent to keeping the number of firms that interact with each firm–i.e., that may advertise to the same consumer the firm advertises– fixed. That is, guaranteeing that each consumer may possibly receive ads from the same number of firms, even though there are more firms in the market. (This is equivalent to assuming a type of market segmentation which does not affect the potential market in the random advertising case.)

With this caveat, we see that it is possible that the equilibrium price under targeting decreases as N grows, as we have already noted. However, eventually the increased correlation between the match values of competing firms (firms that target the same consumers) drives this price up, and eventually above the price under random advertising. Advertising intensity is higher with targeted advertising, and increases with N, but search intensity is still lower and decreasing, as we anticipated in our discussion in Section 3.1.

Figure 3 plots the same outcomes against the cost of advertising, c. (In this figure, we have set $r = 1$, $b = 2$, $N = 6$, and $s = .04$.) Of course, search intensity is independent of the cost of advertising, so we do not report this outcome. As should expect, advertising intensity is lower the higher the cost of advertising, but the reduction is higher in the targeted advertising case. Consequently, prices increase in both regimes with the cost of advertising, but the endogenous monopolization effect is stronger in the targeted advertising regime. Thus, for high enough cost of advertising, the price in the latter regime may be higher than the price under random advertising.

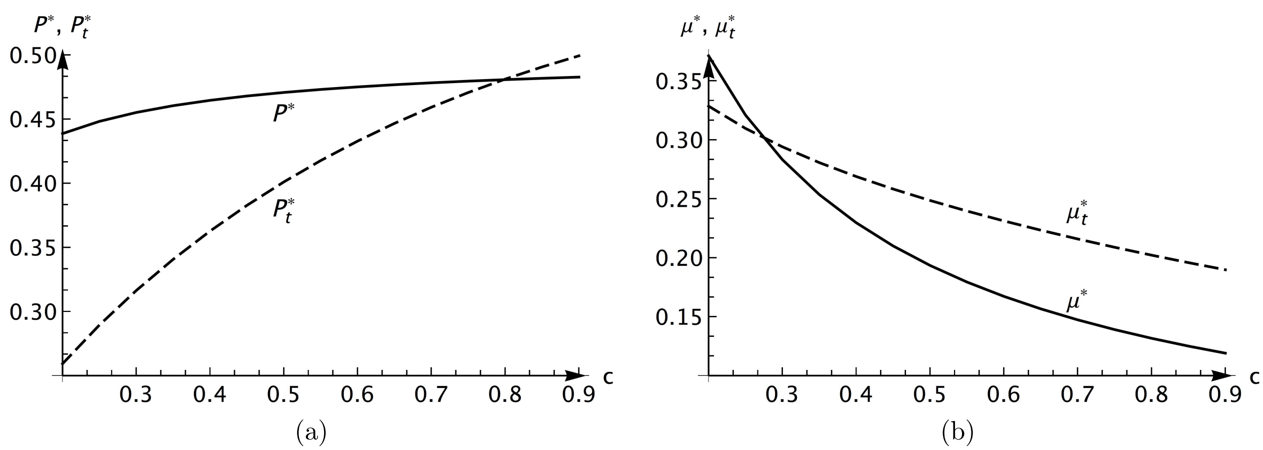


Figure 3: Equilibrium prices, advertising and search intensity with respect to c.

Recall that profits and advertising intensity measure the same phenomenon. Thus, whenever $\mu_t^* > \mu^*$, firms' profits under targeted advertising are higher than under random advertising, and vice versa.

With respect to welfare, and as usual in this type of models, the effect of targeting is complex.

[23]The average number of ads that each consumer would get is not affected but the variance would be increasing in N, for instance.

Targeting typically improves the match between products and consumers, which is good both for total welfare and consumer surplus. Also, and usually, targeting reduces the necessary search, and so search costs, to identify the best options for the consumer. Higher prices may reduce both welfare and consumer surplus, so when targeted advertising results in higher prices this is a possibility. In Figure 4, we plot (for $r = 2$, $b = 2$, $N = 6$, and $c = 2$) the consumer surplus under both regimes for different values of s. As we know, high search costs typically result in prices being higher under targeted advertising. That, in its turn, may outweight the positive effect of targeting on consumer surplus. In this example, this is the case for search costs above 0.085 approximately. At that point, the price under targeted advertising is around 0.71 whereas the price with random advertising is around 0.56, and this difference in price compensates for the reduced search costs and higher probability of better fits.

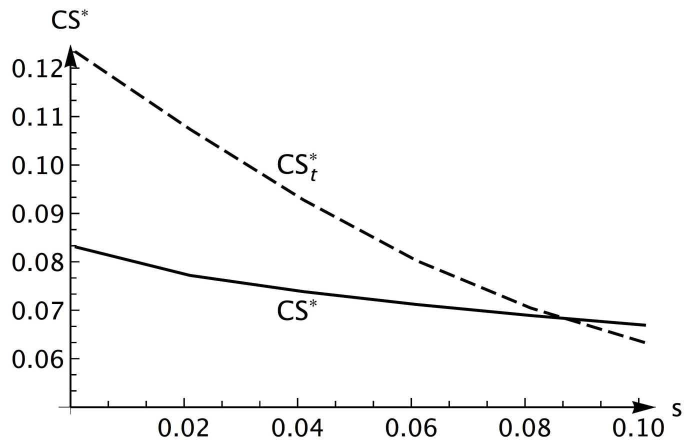


Figure 4: Consumer surplus with respect to s

In fact, as targeting becomes more precise (and advertising not too inexpensive), targeting necessarily increases total welfare, but only at the expense of consumers. Indeed, consider an increase in N, which, from the point of view of the firms, increases the information content of their signals on consumer (and, from the consumer's point of view, the information content of ads). As we have argued in Subsection 3.1, for large enough values of N, search happens with vanishing probability under targeted advertising (and is unaffected in the random advertising case). As a consequence, under targeted advertising the equilibrium price approaches the monopoly price, which approaches itself 1. Dead-weight loss approaches 0 (all consumers find and buy products they value close to the highest possible with probability approaching 1), but all the surplus is appropriated by the sellers (as the price approaches the willingness to pay with probability 1). That is, when the demand composition effect dominates, and even though total surplus is higher under targeted advertising, consumers will eventually be worse off than under random advertising.

6 More discriminating targeting

The previous baseline model is perhaps the simplest possible model of targeting based on ranking that still allows for some competition among firms.[24] However, the main insights gained with this simple model are far from an artifact of its simplifying assumptions. Let us analyze this point by briefly looking at a generalization of the model. Suppose that for each consumer, all firms observe a signal that tells whose product is the best match for that consumer and whose product is the second best match. This ranking is correct with probability $\beta \geq 1/2$ and wrong (in that the ranking of the two identified firms is reversed) with probability $1-\beta$.

Firms would optimally send ads to those consumers identified as having their product as the best match for them. We can call them *type 1* target consumers. If the cost of advertising is sufficiently low, they may also send ads to a proportion $\widehat{\alpha}$ of those for whom their product is most likely the second best match. We can call them *type 2* target consumers. (Again, to keep the analysis simple, suppose that the cost of advertising is not so low as for irms to be interested in sending ads to other, less promising consumers.) This second case is the one of interest: if firms only advertise to type 1 target consumers, then firms to all effects are monopolies.

Consumers' search behavior would be unchanged by this generalization. Indeed, a consumer who gets two ads knows nothing but what our consumer in the baseline model knew. From the point of view of firms, however, consumers' demand is now different. Type 1 target consumers for firm i may receive one or two ads. If they receive two ads, they still visit the other firm first with probability 1/2, and then, if they observe a match value below w_t, defined as in Section 3, visit firm i. With probability 1/2 they visit firm i first, and then they may either buy immediately, or visit the other firm they know about. If this happens, they may still return. The probabilities of selling to these consumers in each of the cases are different though.

Type 2 target consumers who receive an ad from firm i will receive another ad from the firm for whom they are type 1 target consumers. They too will visit firm i first with probability 1/2 and the other firm first with probability 1/2. They too may buy immediately or visit both firms, and so may return to firm i after visiting both firms. Once again, the probabilities of these events will differ from the ones computed in Section 3. Let $g_k(\cdot)$ be the density function of a type k target consumer's match value, for $k=1,2$, and $G_k(\cdot)$ the corresponding cdf. Also, let $g^c_k(\cdot\,|\,z)$ be the density of such random variable conditional on z being the match value in the other firm for which the consumer is a target consumer, and $G^c_k(\cdot\,|\,z)$ be the corresponding (conditional) cdf. In the appendix we detail these densities.

As in Section 3, suppose firm i charges $P > P^*_t$ and sends ads to (all its type 1 target consumers and) a proportion $\widehat{\alpha} \neq \widehat{\alpha}^*$ of its type 2 target consumers, when consumers conjecture equilibrium values P^*_t and $\widehat{\alpha}^*$. Then, of the $\frac{1}{N}$ type 1 target consumers of firm i, a total of $(1-\widehat{\alpha}^*)\frac{1}{N}$ will receive one ad

[24] A simpler model would allow firms to identify the consumers for whom their product ranks first in their preferences. That would easily lead to monopoly (for high enough cost of advertising), or a sort of monopoly with noise (otherwise).

only (from firm i) and then the total demand from these consumers will be

$$(1-\widehat{\alpha}^*)\frac{1}{N}(1-G_1(P)). \tag{12}$$

The rest of type 1 target consumers of firm i will receive two ads. Half will visit first the other firm and then visit firm i if they find a value below w_t at the other firm. Thus, the total demand from these consumers will be

$$\widehat{\alpha}^*\frac{1}{2N}\left(\int_0^{P_t^*} g_2(z)\left[1-G_1^c(P|\,z)\right]dz+\int_{P_t^*}^{w_t} g_2(z)\left[1-G_1^c(z+P-P_t^*|\,z)\right]dz\right), \tag{13}$$

where z above represents the value of the match value in the other firm.

The other half of these consumers will visit firm i first. As in the baseline model, the consumer will buy from firm i if the match value is above $w_t(P)$ or if (it is below, but above P and) the value in the other firm is below the value in firm i minus $(P-P_t^*)$. That is, a total of

$$\widehat{\alpha}^*\frac{1}{2N}\left\{(1-G_1(w_t(P))+\int_P^{w_t(P)} g_1(z)G_2^c(z-P+P_t^*|\,z)dz\right\}, \tag{14}$$

where in this case z above represents the value in firm i (and so $g_2^c(z_i|\,z_l)$ in the computation of G_2^c takes the form of $z_l \leq z_i$).

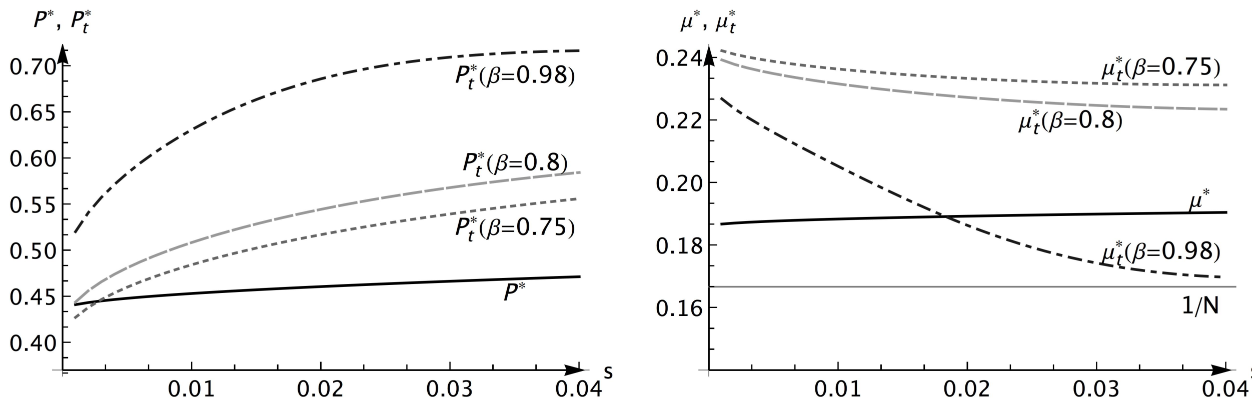
The demand from type 1 target consumers is the sum of (12), (13), and (14). Type 2 consumers who get an ad from firm i (there are $\alpha\frac{1}{N}$ of them) will all get another ad from the firm for which they are type 1. Half of them will visit that other firm and then will visit firm i if their match value there is below w_t. Then, the total demand from these consumers will be

$$\alpha\frac{1}{2N}\left(\int_0^{P_t^*} g_1(z)\left[1-G_2^c(P|\,z)\right]dz+\int_{P_t^*}^{w_t} g_1(z)\left[1-G_2^c(z+P-P_t^*|\,z)\right]dz\right). \tag{15}$$

Similarly as before, the demand of the other half of type 2 target consumers who receive an ad from firm i is

$$\alpha\frac{1}{2N}\left\{(1-G_2(w_t(P))+\int_P^{w_t(P)} g_2(z)G_1^c(z-P+P_t^*|\,z)dz\right\}. \tag{16}$$

The total demand for firm i, $D_t^{\beta}(P,\alpha;P_t^*,\widehat{\alpha}^*)$ is simply the sum of (12) through (16). As in the baseline model, we have computed equilibrium outcomes for different values of the parameters. Some are presented in Figure 5. Note that distinguishing more from less likely best-matched consumers, allows firms to better partition the market. As a result, prices in the target advertising case are higher the more precise the rankings are. Indeed, firms have less incentives to compete for consumers whose rivals will target when they have little hopes of being able to offer these consumers a higher value. In fact, as the curves for advertising intensity show in Figure 5, as information becomes more precise, firms reduce the spending in ads targeted to consumers other than the most promising ones. Thus, the endogenous monopolization effect is stronger when ranking information is more precise.



(a) (b)

Figure 5: Equilibrium prices, advertising and search intensity with more precise targeting. Parameter values: $r = 1$, $c = 0.51$, $N = 6$, $b = 2$.

Other than that, the effects that we have uncovered in our baseline model still drive the effects of targeting on prices, advertising efforts, and profits.

7 Discussion: assumptions, implications, and further research

We have investigated the effects of targeting based on rankings in the simplest possible framework. Targeting based on rankings introduces some technical complications that make it particularly important to keep complexities to a minimum. Indeed, as opposed to random advertising, targeting will necessarily result in search not being stationary, as consumers' beliefs necessarily change with their search experience.

We have assumed that consumers *know* the number of varieties. The alternative, without abandoning the fully rational paradigm, would be to assume uncertainty, and some probability distribution (prior beliefs) that consumers would then update as they observe the realizations of their match values. (Note that even in the random advertising case, consumers' beliefs would need to be updated, but only based on the number of ads received.) There are several reasons for assuming full information on N. The least important of them is that we want to depart the least possible from the standard in the literature. That allows for easier reference to other effects discussed elsewhere, in particular for targeting directly based on match values. But more importantly, it seems to us that it makes sense to begin with the model where the main effects of targeting are least obscured by the interaction with other effects. Of course, we should be confident that the simplifications do not drive the effects we are highlighting. We think the danger does not exist in this case.

For instance, if consumers update their beliefs on the number of firms and are aware of the way firms target their ads, then a good realization in one visit will make them more optimistic about the number of varieties. That would give them an additional incentive to search. Yet, with sufficient regularity on the beliefs (so that posteriors are still monotonic, for instance) search will still be defined by a cutoff on match values. Also, the larger the (realized, i.e., actual) number of firms the higher the probability of high realizations of target consumers' match values, and so the lower the probability of search. (In fact, the effect is reinforced: consumers would not update their beliefs based on the values they observe as much as if they *observed* the increase in N!) Thus, a larger number of firms and varieties should still result in higher prices.

We have discussed the symmetric firms case. Once more, we think that it makes sense to first consider the cleanest case, where it is the type of information used for targeting, rather than, for instance, how this information affects existing asymmetries, what drives the results. Asymmetries (in the cost function, in the distribution of match values, etc.) introduce other interesting questions worth investigating. For instance, would ex-ante more salient firms have incentives to shut down rivals' access to ranking information by signing exclusive contracts with intermediaries? Nevertheless, the three main affects that we have uncovered in this analysis will still be key forces behind the answers to those questions.

Another simplifying assumption –in the targeted advertising case– is that firms' information allows them to distinguish –only– consumers for which their products are most likely the best match –our type 1 target consumers– and those for which they are most likely the second best match –our type 2 target consumers–, and these from the rest. We could assume that firms can also distinguish those for whom their products are most likely the third best, and the fourth best, etc. Obviously, firms would still prefer to send ads to type 1 target consumers and, once they cover that segment, to type 2 target consumers. Only if the cost of advertising is sufficiently low, they would also send ads to what we could then call type 3 target consumers, etc. That is, unless the cost of advertising is low, Section 6 still describes equilibrium in this more general model. Perhaps more importantly, even if the cost of advertising is lower, the three effects of targeting that we have uncovered would still characterize the effects of targeting on search and so on market outcomes. Targeting would allow consumers to learn about their most preferred varieties (competition effect), it would introduce endogenous correlations of match values (demand composition effect), and firms would endogenously segment the market into competing with a smaller number of firms (the main aspect of what we called endogenous monopolization effect).

One implication of our analysis is that, unless advertising and search costs are low (and so the competition effect is dominant), using ranking information to target ads is profitable for firms offering differentiated products even if the rivals also target their ads. Moreover, this is so even if rivals use the same information. That is, even when information is not proprietary.[25] This is important in order to understand the appeal of, for instance, online advertising intermediaries. The firm offering the closest substitute will target a firm's most attractive potential customers, and this is a downside of using

[25] We have not consider the firms' choice of whether to target ads or not. To formally analyze this issue, we would need to consider the case where some firms target and some don't. This introduces again the necessity of deciding on how to model consumers' beliefs along, possibly, out of equilibrium paths. Although the possibility of existence of equilibria where firms do not target has some interest, we think that this interest is rather marginal for the goals of this paper, and would not help clarifying the basic effects of targeting based on rankings.

this common information on rankings. However, the induced correlation on match values (and so the reduced expected differentiation) reduces the consumers' search incentives. Moreover, it also induces an endogenous segmentation of the market, more acute the more refined the information. Both effects will push prices –and profitability– up.[26]

[26]As opposed to the baseline model, when we introduce the beta parameter, profitability is not directly measured by advertising intensity. Indeed, equilibrium conditions do not include that revenues by ad–i.e., average revenue per ad–equals marginal advertising cost.

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Appendix

A.1 The derivation of the search threshold w under random advertising

The expected utility from search $E\left[\max\left\{u_i, u_l\right\}\right] - s$ equals

$$-s + \int_{u_i}^{1-P_l} u_l dF\left(u_l\right) + \int_{0-P_l}^{u_i} u_i dF\left(u_l\right). \tag{A1}$$

We subtract u_i from (A1), replace u_i and u_l with respectively $z_i - P_i$ and $z_l - P^*$ and obtain the expected change in utility due to this search:

$$\int_{z_i - P_i + P^*}^{1} \left(z_l - P^* - \left(z_i - P_i\right)\right) dF\left(z_l\right) - s, \tag{A2}$$

where the integral is the net gain from searching firm l. Further, if we replace $z_i - P_i + P^*$ with w in (A2) and set it equal to zero, we obtain the point at which a consumer is indifferent between searching firm l and taking product i. This gives

$$\int_{w}^{1} \left(z - w\right) dF\left(z\right) = s, \tag{A3}$$

which gives a unique solution w. Then a consumer is indifferent between searching and taking product i if $z_i - P_i = w - P^*$.

A.2 The derivation of densities and the search threshold w_t of target consumers

Suppose that the consumer has received ads from firms i and l and she visits firm l first. The density functions of the first-order and second-order statistics are respectively $Nf\left(z_l\right)F\left(z_l\right)^{N-1}$ and $N\left(N-1\right)f(z_l)F\left(z_l\right)^{N-2}\left(1 - F\left(z_l\right)\right)$. By applying the Bayes rule, we obtain that the probability that the match value z_l is the highest existing for the consumer equals

$$\frac{Nf\left(z_l\right)F\left(z_l\right)^{N-1}}{Nf\left(z_l\right)F\left(z_l\right)^{N-1} + N\left(N-1\right)f(z_l)F\left(z_l\right)^{N-2}\left(1 - F\left(z_l\right)\right)} = \frac{F\left(z_l\right)}{F\left(z_l\right) + \left(N-1\right)\left(1 - F\left(z_l\right)\right)}. \tag{A4}$$

Likewise, z_l may be the second highest match value for the consumer, an event with conditional probability

$$\frac{N\left(N-1\right)f\left(z_l\right)F\left(z_l\right)^{N-2}\left(1 - F\left(z_l\right)\right)}{Nf\left(z_l\right)F\left(z_l\right)^{N-1} + N\left(N-1\right)f(z_l)F\left(z_l\right)^{N-2}\left(1 - F\left(z_l\right)\right)} = \frac{\left(N-1\right)\left(1 - F\left(z_l\right)\right)}{F\left(z_l\right) + \left(N-1\right)\left(1 - F\left(z_l\right)\right)}. \tag{A5}$$

The unconditional probability that all $N-1$ match values are less than z_l equals $F(z_l)^{N-1}$ and the unconditional probability that one match value is greater than z_l is $1-F(z_l)$. Thus, the density function of z_i conditional on z_l is

$$g^c\left(z_i|\, z_l\right) = \begin{cases} \frac{(N-1)f(z_i)F(z_i)^{N-2}}{F(z_l)^{N-2}[F(z_l)+(N-1)(1-F(z_l))]} & \text{if } z_i \leq z_l, \\ \frac{(N-1)f(z_i)}{F(z_l)+(N-1)(1-F(z_l))} & \text{if } z_i > z_l. \end{cases} \tag{A6}$$

Further, we use the same routine to compute the gains from search as in a random advertising case and obtain that the expected gains from searching product 2, having observed match value z_1 and price P_1, with $z_1 \geq P_1$, equal

$$\int_{z_1-P_1+P_t^*}^{1} \left(z_2 - P_t^* - z_1 + P_1\right) g^c\left(z_2|z_1\right) dz_2. \tag{A7}$$

Similarly to the random advertising case, the consumer will continue searching after the first visit if z_l is below the threshold w_t that equates (A7) to s. The expression (A7) is decreasing in z_l.[27] Thus, the threshold is well defined and is decreasing in s. In an equilibrium, where $P_l = P_t^*$, that value solves an equation similar to (A3) with only substituting $g\left(z_i|w_t\right)$ for $f\left(z\right)$ and w_t for w:

$$\int_{w_t}^{1} \left(z - w_t\right) \frac{\left(N-1\right) f(z)}{F(w_t) + \left(N-1\right)\left(1 - F(w_t)\right)} dz = s. \tag{A8}$$

A.3 The demand under targeted advertising

We present here the expression for $D_t(P, \alpha; P_t^*, \alpha^*)$ in terms of the distribution F and density f. A total of $\alpha(1-\alpha^*)\frac{2}{N}$ consumers will get only one ad from this firm, and buy if their match value is above P, so that $\alpha(1-\alpha^*)\frac{2}{N}(1-G(P))$. Substituting for G, this is

$$\alpha(1-\alpha^*)\frac{1}{N}\left[N(1-F(P)^{N-1}) - (N-2)(1-F(P)^N)\right].$$

Next, we substituting for G^c and g in the expression for the demand of consumers who obtain ads from two firms and visit first the other firm. There z, the value of the match in the other firm, is below the value at firm i, and so we use the corresponding expression for g^c. Thus, this demand is

$$\alpha\alpha^* \frac{1}{N}\left(\frac{1}{2}N\left(1-F\left(P\right)\right)F\left(P_t^*\right)^{N-1} + \int_{P_t^*}^{w_t} \frac{1}{2}N\left(N-1\right)f(z)F\left(z\right)^{N-2}\left[1 - F\left(z+P-P_t^*\right)\right]dz\right).$$

[27] See the Supplementary Appendix for the details.

The demand of consumers who get ads from two firms, and visit firm i first is

$$\alpha\alpha^*\frac{1}{N}\{\frac{1}{2}\left[N(1-F(w_t(P))^{N-1})-(N-2)(1-F(w_t(P))^N)\right]+$$
$$\int_P^{w_t(P)}\frac{1}{2}Nf(z)F(z-P+P_t^*)^{N-1}dz\},$$

since in this case z above represents the value in this firm, and so $g^c\left(z_i|\,z_l\right)$ in the computation of G^c takes the form of $z_i \leq z_l$.

Then, the total demand is

$$\alpha(1-\alpha^*)\frac{1}{N}\left[N(1-F(P)^{N-1})-(N-2)(1-F(P)^N)\right]+$$
$$\alpha\alpha^*\frac{1}{N}\left(\frac{1}{2}N\left(1-F\left(P\right)\right)F\left(P_t^*\right)^{N-1}+\int_{P_t^*}^{w_t}\frac{1}{2}N\left(N-1\right)f(z)F\left(z\right)^{N-2}\left[1-F\left(z+P-P_t^*\right)\right]dz\right.$$
$$\left.+\frac{1}{2}\left[N(1-F(w_t(P))^{N-1})-(N-2)(1-F(w_t(P))^N)\right]+\int_P^{w_t(P)}\frac{1}{2}Nf(z)F(z-P+P_t^*)^{N-1}dz\right).$$

A.4 Density functions for types 1 and 2 target consumers

The probability density of the match value of a type 1 target consumer is

$$g_1(z)=Nf(z)\left(\beta F\left(z\right)^{N-1}+\left(1-\beta\right)\left(N-1\right)\left(1-F\left(z\right)\right)F\left(z\right)^{N-2}\right),$$

whereas the probability density of a type 2 target consumer is

$$g_2(z)=Nf(z)\left((1-\beta)F\left(z\right)^{N-1}+\beta\left(N-1\right)\left(1-F\left(z\right)\right)F\left(z\right)^{N-2}\right).$$

Also, from a firm's point of view, the probability density of the match value z_i of a type 1 target consumer who has visited the firm for which she is type 2 target consumer and observed a match value of z_l there, is

$$g_1^c(\,z_i|\,z_l)=\begin{cases}\frac{(1-\beta)(N-1)f(z_i)F(z_i)^{N-2}}{F(z_l)^{N-2}((1-\beta)F(z_l)+\beta(N-1)(1-F(z_l)))} & \text{if } z_i\leq z_l,\\ \frac{\beta f(z_i)(N-1)}{(1-\beta)F(z_l)+\beta(N-1)(1-F(z_l))} & \text{if } z_i> z_l.\end{cases}$$

We may similarly compute $g_2^c(\,z_i|\,z_l)$, the probability density of a type 2 target consumer's match value in firm i conditional on having a match value z_l at the firm for which she is a type 1 target consumer.

$$g_2^c(\,z_i|\,z_l)=\begin{cases}\frac{\beta(N-1)f(z_i)F(z_i)^{N-2}}{F(z_l)^{N-2}(\beta F(z_l)+(1-\beta)(N-1)(1-F(z_l)))} & \text{if } z_i\leq z_l,\\ \frac{(1-\beta)f(z_i)(N-1)}{\beta F(z_l)+(1-\beta)(N-1)(1-F(z_l))} & \text{if } z_i> z_l.\end{cases}$$