



Capacity Investment and Product Line Decisions of a Multiproduct Leader and a Focus Strategy Entrant*

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ABSTRACT

In this article, I investigate the capacity investment cost conditions where a multiproduct market leader may respond to a focus strategy entrant by using different strategies such as changing the product mix, production volumes, quality levels, and/or by investing in more capacity. The products offered in the market are quality differentiated and customers are heterogeneous in their willingness to pay for quality. The capacity investment costs of the two firms (i.e., the leader and the entrant) may also be different. The classical Stackelberg model predicts that an incumbent does not change its position in response to entry. However, when heterogeneous customer base, product differentiation, and capacity costs are taken into consideration, I find that the leader with a low capacity cost may choose to expand its product line and increase its production. The leader with low capacity cost may introduce a product that it was holding back when the entrant has to bear the high-capacity cost and cannibalization threat is relatively small. Nevertheless, the extent of production volume strategies reduces as the capacity cost increases for the leader. I also find that when the leader has the power to set the industry standards by deciding the quality levels, as a response to a high-quality focused entrant, the leader increases both levels of quality and production of the low-quality product. Moreover, when the capacity investment cost is high for both the entrant and the leader, I find that market prices may increase with entry. [Submitted: July 19, 2012. Revised: October 3, 2012. Accepted: December 12, 2012.]

Subject Areas: *Capacity Investment, Competition, Focus Strategy, OM-Marketing Interface, and Product Line.*

INTRODUCTION

There are numerous empirical studies that show how market leaders attack new entrants in a variety of ways (Smiley, 1988; Geroski, 1991; Oster & Strong, 2001). Smiley (1988) shows that 26% of the established firms choose to respond by increasing their product variety. Geroski (1995) reports some case studies where the incumbents introduce new products which they had been holding back. Use of capacity is another common response to the potential threat of entry. For

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example, in the airline industry, the launch of low-cost subsidiaries like United Shuttle and Delta Express (new product introduction) is recorded to be a “defensive response carefully targeted to the *threat* of Southwest and other low-cost, low-fare airlines” in a DOT report (Oster & Strong, 2001, p. 16). This report also provides specific examples, such as Northwest’s actions toward Reno Air’s entry in Reno–Minneapolis market in 1993. Northwest not only launched a new nonstop service (increasing quality), it has also started offering additional nonstop flights from Reno to markets served by Reno Air (new product introduction). Another example involves competition in the Detroit–Philadelphia market. Spirit Airlines entered this market at the end of 1995 with high capacity at the low fare range. When Spirit Airlines announced its entry in other markets served by Northwest, Northwest responded by increasing its low fare seat capacity to over 35 times more than before (use of capacity) in these markets (Oster & Strong, 2001).

Major airlines have responded similarly to focus strategy competition at the high-end market. For example, in 2007–2008, American Airlines (AA) had increased its service frequency and award miles program (use of capacity and increasing quality) in the London–New York route with lower prices until the bankruptcy of the new all-business carrier, EOS. These actions toward EOS are recorded to be predatory in the press (Rowell, 2009).

Market leaders often have a first mover advantage (Lee & Ng, 2007). They have higher market shares and a better business performance. They have enough technological capability and support facilities to produce multiple products or withhold one if necessary. An entrant firm is motivated to steal business in such a market dominated by a leader. If the leader chooses to change its product mix in response to the entrant, then it has to watch out for cannibalization among its own products. Moreover, these changes may require additional resources. Hence, there are intricate relationships in this game which make it difficult to predict the optimal decision a priori. However, the existing analytical literature, by and large, overlook the impact of such policies involving capacity and product line policies. Yet in a vertically differentiated industry with multiple products (potentially consuming different amounts of resources per unit), a market leader may benefit from better utilization of costly resources under some operational conditions. In this article, I study such a competition between a market pioneer and a focus strategy entrant and identify such conditions analytically.

In particular, I address the following research questions: What are the operational conditions that trigger a strategy change for the firms? How are the product mix and supply in the market affected by an asymmetric competition? How are capacity investment decisions affected by the cost of such investments in a vertically differentiated market? What are the implications on the quality levels of the products?

In this stylized model, a sequential entry (Stackelberg) game between a multiproduct market leader and a focus strategy follower is studied. The leader has the capability to produce two vertically differentiated products with different qualities, unit production costs, and unit resource consumptions. It moves first and chooses which product(s) to offer, their quality levels, and how much to offer in the market. Sequentially, the entrant that has a focus (on either high or low quality) product observes the leader’s decisions and determines its own production

quantity and necessary capacity investments. Among others the major findings are as follows:

- It is shown that the leader would be better off by expanding the product line to include the entrant's focus product when the capacity cost is high for the entrant. The entrant generates an interest in the market for its focus product, but the increasing capacity cost puts operational pressure on the entrant to lower the production. This creates an incentive for the leader (with a lower capacity cost) to proliferate and introduce this particular product, which it would hold back as a monopolist.

- Common wisdom suggests that the capacity investments should decrease as the relevant costs increase. In contrast, I find that the entrant's capacity investment may be increasing as the cost of capacity increases. When the leader offers only the high-quality product and the high-quality product requires lots of resources for production, the low-quality focused entrant is better off by increasing the supply which requires increasing the capacity investment.

- I also find that as a response to a high-quality focused entrant, the leader chooses to increase both levels of quality and the production of the low-quality product. It may even increase the production of the high-quality product depending on its capacity investment cost. Moreover, contrary to intuition, I find that market prices may increase with entry despite the competition, when the capacity investment cost is high for both the entrant and the leader.

The rest of the article is organized as follows. The literature review is presented in the next section followed by model assumptions. Next, the problem is solved focusing on the product variety implications. Then, the implications on quality decisions are analyzed. Finally, I discuss managerial insights and conclude the article.

RELATED LITERATURE

Firms' Reactions to Entry

There is a vast literature in marketing and economics on firms' reactions to entry. Geroski (1995) provides a summary of the stylized facts and empirical generalizations on the subject. Empirical findings suggest that increasing product variety is a common strategy in response to entry for companies (Smiley, 1988; Neven, 1989; Geroski, 1995). In a recent paper, Dunn (2008) investigates the airline industry. He provides evidence that a firm is more likely to start a high-quality service in response to entry if the firm has an existing low-quality service in the market. The author finds that cannibalization effects are more likely to diminish when there is "the threat of business stealing" in the market. However, analytical investigation is nonexistent in this domain especially when the products are differentiated. In this article, I fill this gap and analytically study the product variety changes in the face of sequential entry. I explain when a firm should proliferate as a response to competition when there are quality and resource consumption differences among the products.

There are a number of analytical papers in the entry literature that look at the capacity investment as an entry-deterring strategy of an incumbent firm (Spence,

1977; Dixit, 1980). However, these papers have largely ignored the product variety and resource utilization aspects of the problem. The majority of these papers focus on homogeneous products and fixed costs of capacity. The common finding is that the incumbent firm may invest in capacity to deter entry; and this investment may result in idle capacity. However, there are conflicting empirical findings on this claim. There are a number of studies which reject this hypothesis (Geroski, 1995). On the other hand, Conlin and Kadiyali (2006) find evidence that firms with larger market shares (market leaders) have more (entry deterring) idle capacity than firms with smaller market shares (followers).

I contribute to this line of literature by considering a differentiated product line that leads to additional trade-offs such as cannibalization; and by introducing potential differences in resource consumptions of products. I show that quality differentiation and resource utilization issues provide a better explanation to how firms may or may not use capacity investments and production volume changes in response to entry. In particular, when the products consume different amounts of the costly resource, the interplay between the products for resources and the competitive interaction between the firms for better market share in a vertically differentiated industry leads to different outcomes than before.

Vertically Differentiated Product Lines

There is also a rich literature on the firm's choice of vertically differentiated product lines (Mussa & Rosen, 1978; Moorthy, 1984; Johnson & Myatt, 2003). In these studies, the major finding is that a monopolist needs to either increase the differentiation between the products or delete the low-quality product from the product line in order to mitigate the effects of cannibalization. Surprisingly, the number of papers that study product variety choices in the case of sequential entry is quite low. This article contributes to the literature by analyzing the impact of asymmetric competition on the decisions of the capacity-constrained multiproduct firms.

Tsikriktsis (2007) empirically studies the effects of focus strategy, capacity utilization, and conformance quality on profits of the airline industry. He shows that the quality defects have a greater impact on the profitability of focus strategy airlines than the full-service carriers. Intuitively, focus strategy firms are more sensitive to downward changes in their operational parameters in general. One possible explanation I offer to his findings is the fact that the diverse firms can adapt to these changes (such as the fuel cost increase) by modifying their product lines, maybe even shifting operations from one product to another. However, focus strategy firms have to survive with their focus product alone. Based on my study in this article, I believe that the parametric changes in the industry could leave a focus strategy firm to operate under suboptimal conditions and lose more money than their diversified rivals.

Boyaci and Ray (2003) and Chayet, Kouvelis, and Yu (2009) consider capacity management issues for offering differentiated product lines. They formulate the problem as queuing models and focus on the capacity investment decisions of monopolists in congested systems. Chayet et al. (2009) show that increasing congestion costs results in less variety in the product line. This article shows that

Table 1: Comparison of this paper with the literature.

	Method	Vertical Differentiation	Competition	Capacity Investment
Mussa and Rosen (1978)	Analytical	Yes	No	No
Dixit (1980)	Analytical	No	Yes	Yes
Smiley (1988)	Empirical	Yes	Yes	Yes
Johnson and Myatt (2003)	Analytical	Yes	Yes	No
Conlin and Kadiyali (2006)	Empirical	No	Yes	Yes
Dunn (2008)	Empirical	Yes	Yes	No
Chayet et al. (2009)	Analytical	Yes	No	Yes
Yayla-Küllü et al. (2011)	Analytical	Yes	No	Yes
Haruvy et al. (2013)	Analytical	Yes	Yes	No
This article	Analytical	Yes	Yes	Yes

this may not be true when the resource consumptions of the products differ. In addition, the impact of competition on these decisions is discussed here.

Notably, Johnson and Myatt (2003) provide an analysis of the multiproduct quality competition in duopoly and oligopoly markets, but they ignore the capacity constraints of the firms. They aim to explain why the firms adjust their product lines (by introducing fighting brands or reducing the variety) in response to competition. The authors study symmetric firms as well as the competition between a multiproduct firm and a low-quality firm. They show that in the asymmetric competition, the incumbent continues to act like a monopolist for quality levels that are greater than the entrant’s highest quality. In contrast, I show that all products’ supplies may be affected due to the costly resource decisions of the firms. Moreover, Johnson and Myatt (2003) do not consider a high-quality entrant while I do, in addition to the capacity investment decisions. Hence, this is the first article that considers both high- and low-quality entry in a comprehensive model including capacity constraints.

More recently, Haruvy, Miao, and Stecke (2013) numerically study a sequential entry game with two periods. The leader chooses a single product in the first period and has the potential to offer a second differentiated product in the second period in response to the entrant. They focus on identifying quality and innovation strategies for the incumbent while my focus is on the capacity investments and production quantity decisions. They ignore the capacity dimension altogether which is the key parameter in this article.

In short, I contribute by considering costly capacity choices and potential asymmetry in the product line strategy of competing firms. A tabulated summary comparison of this article with the literature is given in Table 1. I take the costs of different actions into account in order to understand the consequences of these actions better. For example, if the leader changes its product mix, then it has to watch out for cannibalization among its own products. Moreover, an increase in production may increase the resource consumptions of the products which means more capacity is necessary. At that point the firm needs to consider the unit

production costs and the capacity investment costs as much as the competitive forces. Hence, introducing all these operational costs and the competitive trade-offs is a necessary contribution to the literature and I aim to fill this gap with this article.

MODEL

In this stylized model, there are two products, a high (H)- and a low (L)-quality product. Each unit of product $i = H, L$ has quality q_i . As the first mover, the leader has the power to choose these quality levels and set the standards in the market. The leader chooses from a technologically feasible range of qualities: $q_i \in [q, \bar{q}]$. It is shown in the empirical literature that the new entrants have no power in shaping the industry structure and they often start as followers of the standards set by the market pioneers (Geroski, 1995) (unless the entrant is an innovator, which is not the focus of this article). In the airline example, this would correspond to an airline choosing from a number of available cabins such as economy, premium economy, business, and first class cabins. Although the type of service customers will receive within each cabin is pretty much set by the industry standards, minor changes might be made in terms of the service quality. For example, in a transatlantic flight, customers expect and accept to be uncomfortable in an economy class seat, whereas industry standards dictate that a first class customer gets gourmet food on board (Source: Skytrax Web site: airlinequality.com).

Following the literature, I assume that customers will only buy a positive quality ($q > 0$) or choose not to buy anything (Moorthy, 1984). Unit production cost is $c_i = c(q_i)$. It is an increasing function of quality ($c_H \geq c_L$), and it is linear in quantity (Mussa & Rosen, 1978; Moorthy, 1984). Products may also have different resource consumptions. Each unit of product i requires $s_i = s(q_i)$ units of capacity. Indeed, for the airline example, business class seats are perceived as better quality ($q_H > q_L$); they are bigger in size ($s_H > s_L$); and it costs more to serve them ($c_H > c_L$) due to the greater number of flight attendants, food and drinks, and so forth. Yayla-Küllü and Tansitpong (2011) also provide empirical support that the resource consumption differences between the products have a significant impact on airlines' capacity allocation decisions.

I adopt the classical vertical differentiation demand model (Mussa & Rosen, 1978; Moorthy, 1984; Tirole, 1988). Customers vary in their willingness to pay for quality. Following the literature, I assume that the customer types (θ) are uniformly distributed in the unit interval $[0,1]$ with unit total mass (cf. Tirole, 1988, p. 296). It is necessary to assume a uniform demand distribution to make the analysis tractable. This enables us to keep the focus on the effects of capacity costs on product line choice and to derive insights. Note also that this is a common assumption in the literature when analysis with more general distributions is intractable (cf. Johnson & Myatt, 2006, p. 594).

When a type θ customer buys product i at price p_i , her utility is equal to $U(q_i, p_i, \theta) = \theta q_i - p_i$. By checking the conditions for the marginal customers, demands for the two products can be found. One such marginal customer is indifferent between buying a high- or low-quality product; the other one is indifferent between buying the low-quality product or nothing at all. Then, demands for the

two product types are as follows:

$$D_H(p_L, p_H) = 1 - \frac{p_H - p_L}{q_H - q_L}, \quad D_L(p_L, p_H) = \frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L}. \quad (1)$$

Segmentation, product proliferation, and cannibalization are well-known conflicting forces for a firm in such a quality-differentiated demand model (Mussa & Rosen, 1978; Moorthy, 1984; Tirole, 1988). In a model that does not take the capacity constraint and competitive interactions into account, if a firm offers a single product of high quality (product *h*), the total sales would be $(1 - \frac{p_h}{q_h})$. Introducing a second product of lower quality (product *l*) may increase the total sales by $(\frac{p_h}{q_h} - \frac{p_l}{q_l})$, which would be the benefit of *proliferation*. Moreover, through price discrimination, the firm further benefits from offering multiple products in the market. However, under the same conditions, some customers switch from buying the high-quality product to buying the low-quality product, reducing the high-quality sales by $(\frac{p_h - p_l}{q_h - q_l} - \frac{p_h}{q_h})$, which is known as *cannibalization*. In the existing literature, product line choice centers on deciding the quality differentiation in the product line, where firms analyze the trade-offs associated with the benefits of product proliferation and the effects of cannibalization that arise from the introduction of a new product. Analysis of such models in terms of competition, cost, and market conditions provides insights into whether firms should offer a high- or a low-quality product (Yayla-Küllü, Parlakturk, & Swaminathan, 2011, 2013).

Suppose a full-service carrier (e.g., British Airways) has to make choices for its high-end segment in a market (e.g., Las Vegas–London route). It has the option of offering both first and business class cabins, or it can choose to offer only one or the other on board. If both classes are available (proliferation), a greater customer body could be served. However, some first class cabin customers might switch to business class due to lower prices (cannibalization). The problem gets more interesting when an all-business class carrier (e.g., MaxJet) enters such a market, especially at a time when the fuel prices are rising and rapidly constraining the capacity in the air.

In this stylized model there are two firms, *X* and *Y*. Firm *X* has the ability to move first and assume a leader position. It can serve a subset of the two products $\{H, L\}$. It needs to decide for the quality levels of the products ($q_H, q_L \in [q, \bar{q}]$), how much of each product ($x_H, x_L \geq 0$) to offer in the market, and the required capacity (K_X) necessary to produce this quantity. On the other hand, entrant *Y* is a new, focus strategy firm. It has a fixed product line and produces one product. Entrant *Y* observes the decisions of the leader *X*, and sequentially determines how much of its product ($y_j \geq 0$, where *j* is either *H* or *L*) to offer in the market and the required capacity (K_Y) necessary to produce this quantity. Note that the customers are indifferent buying from either firm. In the end, prices are set to clear the market based on the total amount of production in the industry: $p_H(x_H, x_L, y_j)$ and $p_L(x_H, x_L, y_j)$.

It is assumed that the entrant has to pay γ_y to purchase a unit of capacity and capacity purchase is not lumpy. On the other hand, as the pioneer in the market, the leader has a distinct cost advantage (Geroski, 1995; Lee & Ng, 2007). It has to pay only γ_x where $0 \leq \gamma_x \leq \gamma_y$ to purchase a unit of capacity. This assumption also fits well with the airline industry, where the aircraft operational costs are

extensive. An airline should pay for aircraft leasing, fuel, and labor costs before allocating the aircraft space among different cabin and seat types. Aircrafts are expensive. Airlines typically use one, or a combination, of the following techniques to pay for their fleet: Cash, operating leasing and sale/leasebacks, bank loans/finance leases, export credit guaranteed loans, tax leases, and manufacturer support (source: <http://www.airfinancejournal.com/>). Note that, only very few well-established firms can afford to pay by cash. All of the other financing options are priced based on the risk of the buyer airline. As mentioned earlier, it is common knowledge that a new entrant firm is a much riskier option than an established market leader (Geroski, 1995; Lee & Ng, 2007). Hence, one would expect that an established airline should be able to receive better financing options compared to a new entrant airline.

In short, the decision time line is as follows: (i) The entrant announces entry in the market. Its focus product, either H or L , is common knowledge. (ii) The leader makes necessary capacity investments and announces its product line, quality levels, and production quantities. A leader can offer any combination of the two products. (iii) The entrant observes the leader's decisions, makes necessary capacity investments, and announces its production quantity. (iv) Customers self-select from the menu of offerings and market clearing prices are observed by all agents.

This time line is consistent with many industries including the airline industry. Starting scheduled flights from one airport to another necessitates a great deal of paperwork and entry is public information long before any quantity decisions are made. Note also that a focus strategy firm such as Southwest has a fixed focus product and does not change this decision before they enter a given market identified by routes in the airline industry. However, I note that a full-service carrier such as Delta may or may not have a business class on board in addition to economy class in every route. Moreover, there are clear distinctions between market leaders and followers in every market. Many markets (identified by origin-destination pairs) have a dominant airline, mostly due to well-known hub-and-spoke systems. Hence, Stackelberg-type competition is prevalent in such markets where the dominant airline is the market leader and others are followers.

IMPLICATIONS ON PRODUCT VARIETY

In this section, solutions of the game for various market settings are presented and the conditions that trigger changes in product mix, supply, and capacity investment decisions are discussed. I first study a monopolist's problem and establish the basis for comparisons. Then, two Stackelberg games that may have different outcomes due to the leader's capacity investment cost are analyzed.

The quality levels are fixed to two variants; this enables us to direct our attention to the research questions about the product variety choices. There are numerous studies in the literature that model the product line problem with n exogenous levels of quality (Bhargava & Choudhary, 2001; Johnson & Myatt, 2003; Jing, 2006). This is essentially equivalent to having a continuous range of qualities, as n could be an arbitrarily large number. The major finding is that the firm would offer a full product line if the cost to quality ratio is increasing ($c_H/q_H > c_L/q_L$),

and only the highest quality product otherwise ($c_H/q_H \leq c_L/q_L$). Having $n > 2$ products does not change this product variety choice (Bhargava & Choudhary, 2001; Johnson & Myatt, 2003). In another study, I have also shown that the same insights can be achieved with two exogenous quality levels as when there are $n \geq 3$ levels of quality (Yayla-Küllü et al., 2013). Hence, I will discuss how my findings with asymmetric competition and resource considerations are different than such a product line choice without loss of generality.

Note also that in the airline industry, product types (identified by different class cabins) are notably similar across different firms. Industry standards are set very clearly. For example, economy seats offered by American, Delta, and United in their Boeing 757–200 aircraft all have a pitch of 31 inches and a width of 17–17.2 inches. Similarly, first class seats on this aircraft have a pitch of 38–39 inches and a width of 20.5–21 inches. Likewise, economy seats of these three airlines on their Canadair CRJ-700 aircraft also have pitch of 31 inches and width of 17–17.5 inches (source: seatguru.com). Indeed, there are ranking firms such as Skytrax (airlinequality.com) that evaluate each airline based on these clearly set industry standards.

In this stylized model, as the market leader, firm X expects the entrant to behave optimally and makes decisions taking her response into account. Hence, the leader solves the following problem:

$$\begin{aligned} \text{Max}_{(K_X \geq 0, x_L \geq 0, x_H \geq 0)} & (p_H(x_H, x_L, y_j^*(x_H, x_L)) - c_H)x_H \\ & + (p_L(x_H, x_L, y_j^*(x_H, x_L)) - c_L)x_L - \gamma_X K_X, \end{aligned} \tag{2}$$

subject to $s_H x_H + s_L x_L \leq K_X$.

Observing the optimal decisions of the leader, entrant Y with a focus product j solves the following problem:

$$\text{Max}_{(K_Y \geq 0, y_j \geq 0)} (p_j(x_H^*, x_L^*, y_j) - c_j)y_j - \gamma_Y K_Y, \tag{3}$$

subject to $s_j y_j \leq K_Y$.

Threshold capacity cost levels $\hat{\gamma}$ that may be functions of product types are explicitly stated in Appendix A. These thresholds are useful for describing how firms' strategies change. All proofs appear in Appendix B.

Lemma 1: Since the objective functions given in problems (2) and (3) are jointly concave in their corresponding decision variables on convex solution sets defined by linear constraints, subgame perfect Nash equilibrium can be obtained by solving the first-order conditions and backward induction.

The lemma shows that the general form of the problem with asymmetric costs is easily solvable. In the following subsections, I characterize the subgame perfect Nash equilibrium in closed form when the capacity cost for the leader is in the two ends of the spectrum: either zero or equal to the entrant's cost. Then, I complement these results with numerical examples to show how they generalize to a wider range of parameters. Keeping the capacity cost parameters with two variants makes my analysis tractable allowing me to focus on the key research question, and characterize the impact of asymmetric competition on firms product line choices when resources are costly. Note again that following each result, I will

present numerical examples to show how these results continue to hold in a wider range of parameters, and how these are contradicting to the existing literature.

A Monopolist's Product Line and Capacity Investments

A monopolist that needs to make product line decisions ($x_L^M \geq 0, x_H^M \geq 0$) and capacity investments ($K_M \geq 0$) has to solve the following problem:

$$\text{Max}_{(K_M \geq 0, x_L^M \geq 0, x_H^M \geq 0)} (p_H(x_H^M, x_L^M) - c_H)x_H^M + (p_L(x_H^M, x_L^M) - c_L)x_L^M - \gamma_x K_M, \quad (4)$$

subject to $s_H x_H^M + s_L x_L^M \leq K_M$.

Following Lemma 1, the optimal solution can be obtained by solving the first-order conditions. Next proposition presents this solution for such a monopolist.

Proposition 1: The monopolist's optimal product line configuration based on the capacity investment costs is characterized as follows:

- (a) When $(c_H/c_L > q_H/q_L)$,
 - (i) For $q_L - c_L > q_H - c_H$, when $\gamma_x < \hat{\gamma}_1^L$, the firm offers only product L.
 - (ii) For $\frac{s_L}{s_H} < \frac{q_L - c_L}{q_H - c_H} < 1$,
 - When $\gamma_x < \hat{\gamma}_2$, the firm offers both products L and H;
 - When $\hat{\gamma}_2 \leq \gamma_x < \hat{\gamma}_1^L$, the firm offers only product L.
 - (iii) For $\frac{s_L}{s_H} > \frac{q_L - c_L}{q_H - c_H}$,
 - When $\gamma_x < \hat{\gamma}_3$, the firm offers both products L and H;
 - When $\hat{\gamma}_3 \leq \gamma_x < \hat{\gamma}_1^H$, the firm offers only product H.
- (b) When $(c_H/c_L \leq q_H/q_L)$,
 - (i) For $\frac{s_L}{s_H} < \frac{q_L - c_L}{q_H - c_H} < 1$,
 - When $\gamma_x < \hat{\gamma}_3$, the firm offers only product H;
 - When $\hat{\gamma}_3 \leq \gamma_x < \hat{\gamma}_2$, the firm offers both products L and H;
 - When $\hat{\gamma}_2 \leq \gamma_x < \hat{\gamma}_1^L$, the firm offers only product L.
 - (ii) For $\frac{s_L}{s_H} > \frac{q_L - c_L}{q_H - c_H}$, when $\gamma_x < \hat{\gamma}_1^H$, the firm offers only product H.

For capacity investment costs beyond $\hat{\gamma}_1^i$, it is not profitable for the firm to operate.

This proposition establishes the basis of a firm's product line strategies for different levels of quality, resource consumption, and unit production and capacity investment costs. Proposition 1a explains what happens when the cost to quality ratio is increasing. In all cases of Proposition 1a, when capacity costs are very low, the firm offers full line of products. As expected, this finding is in line with the literature that disregards capacity costs altogether (Mussa & Rosen, 1978; Johnson & Myatt, 2003). However, when the capacity costs starts increasing, resource consumption of each product starts to play a role in the product line decisions. When the capacity cost is high, only the product with a better profit margin per unit resource ($(q_i - c_i)/s_i$) is offered to the market. Note that this product can be either of the two products.

For example, in the airline industry, when the unit operating cost of a business class seat is very high compared to the service provided with respect to an economy

class seat, for high aircraft operating costs (leasing, fuel, labor, etc.), the airline is better off focusing on only one class in a given route. Indeed, many full-line carriers offer only economy class in most domestic flights.

Proposition 1b explains what happens when the cost to quality ratio favors the high-quality product (H) from all angles. Once again, when capacity costs are very low, the firm holds back the low-quality product and offers only H , as expected from existing results in the literature (Bhargava & Choudhary, 2001; Johnson & Myatt, 2003). However, the interesting results surface when product H consumes a lot of the costly resources. In part (i) of Proposition 1b, I find that the firm has to give up the production of seemingly advantageous product H and start offering the low-quality product L because it has a better profit margin per unit resource consumed. Note that for very high-capacity cost levels, the optimal product line is “only L ,” which is indeed the opposite of what is predicted (“only H ”) by earlier studies that ignore capacity (Bhargava & Choudhary, 2001; Johnson & Myatt, 2003).

This phenomenon is observed in the international markets where many airlines do not offer first class cabins. As a result of this analysis, I believe that part of the reason is the amount of space required to offer a first class seat. It is notably more than an economy class seat. For example, the amount of space required to offer a first class seat is 5.33 times that of an economy class seat in Turkish Airlines’ Boeing 777. Especially for high aircraft operating costs, an airline might be better off eliminating the first class seat from its offerings due to its high space consumption even when the unit operating cost of a first class seat is not very high as in Proposition 1b.

Next, I will study the competitive model and discuss how these results change when a focus strategy firm enters the market.

Stackelberg Competition with $\gamma_y \gg \gamma_x \geq 0$

In this section, I study the market setting where the capacity cost for the leader is very low compared to the entrant’s costs (i.e., $\gamma_y \gg \gamma_x \geq 0$). This might happen when an established firm is operating in multiple markets and it is relatively easy to move capacity from one market to the next. For example, for an airline, transferring aircrafts from one route to another may be relatively easy. Another possibility is that the established market leader may be operating with some idle capacity. Conlin and Kadiyali (2006) find empirical evidence that market leaders have more idle capacity than firms with smaller market shares. These leaders are predicted to use their idle capacity as a signal to increase production in case an entry materializes in their market (Spence, 1977; Dixit, 1980). Indeed, in such a situation, capacity investments may be unnecessary for the leader. On the other hand, the entrant has to make costly capacity investments before it can start operations in the market. The size of the operation (production or service) is limited by its invested capacity.

I analytically study this setting at the extreme case where $\gamma_x = 0$. After discussing analytical results, I present a numerical example that relaxes this assumption.

The classical Stackelberg model predicts that an incumbent should ignore an entrant and keep its monopoly position in terms of market supplies (Tirole,

1988). However, these classical models focus on homogeneous markets and ignore customer heterogeneity. On the other hand, the literature that study the vertically differentiated product line design problem ignore the sequential market entry and Stackelberg competition. In the following paragraphs, I show how the previous results change when all these issues are studied simultaneously.

The general conclusion of the literature that study product differentiation is that the production quantity of a firm decreases considerably when faced with competition (Johnson & Myatt, 2003). This strong conclusion is true under the assumption of strategic symmetry of the competing firms. In contrast, Proposition 2 shows that the leader's optimal production quantity may be greater than its monopoly quantity when firms are not symmetric.

There are many case studies that demonstrate how an entrant stimulates an incumbent to introduce new products and processes that were being held back (Geroski, 1995). Analytically, when the low-quality product has a greater unit profit margin ($q_L - c_L > q_H - c_H$), possibly due to high production costs of H , it is intuitive to suggest that a monopolist should hold the high-quality product back and offer only L as discussed in the previous section. In addition, previous literature suggests that the decreasing cost to quality ratio favors the high-quality product and a monopolist should hold back the low-quality product and offer only H in their product lines due to the cannibalization effect (Bhargava & Choudhary, 2001; Johnson & Myatt, 2003). However, when there is sequential entry in the market and the leader has a capacity cost advantage over the entrant, I find that the market leader may be better off by expanding its product line and offering both products. I also find that the leader's optimal production quantity may actually be greater than the monopolist's quantity in a capacity constrained, asymmetric, vertically differentiated setting as formally presented in the following proposition.

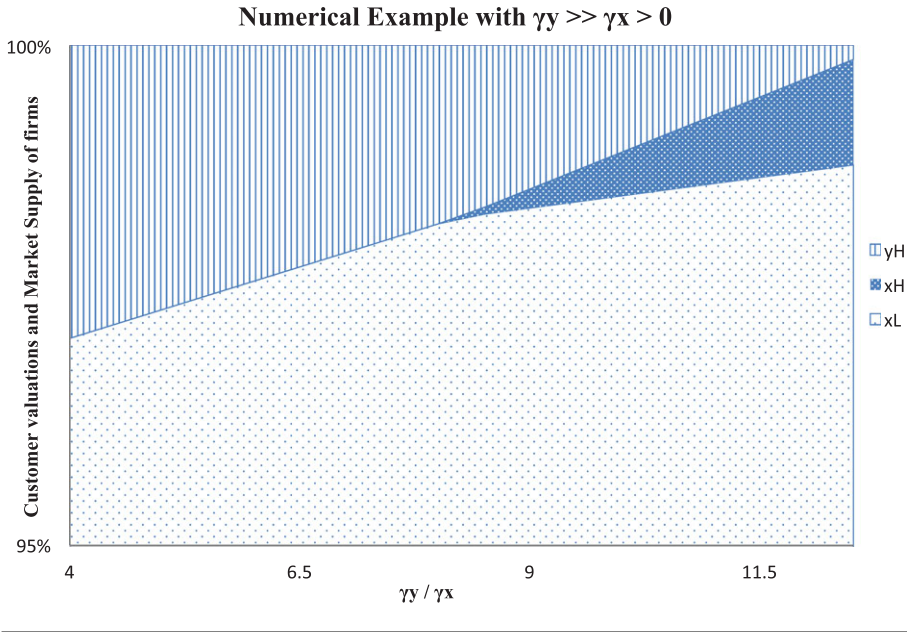
Proposition 2: Incumbent firm increases its product variety and market supply of all products in response to:

- (a) An H focused entrant when $c_H/q_H > c_L/q_L$, $\frac{4q_H - q_L}{3q_H} > \frac{q_L - c_L}{q_H - c_H} > 1$, and $\hat{\gamma}_4 < \gamma_y < \frac{1}{3}\hat{\gamma}_1^H$.
- (b) An L focused entrant when $c_H/q_H \leq c_L/q_L$, $\frac{4q_H - q_L}{3q_L} > \frac{q_H - c_H}{q_L - c_L} > 1$, and $\hat{\gamma}_5 < \gamma_y < \frac{1}{3}\hat{\gamma}_1^L$.

In Proposition 2a (2b), a greater unit profit margin (decreasing cost to quality ratio) makes L (H) the advantageous product and it would be best for a monopolist to offer only L (H) and avoid cannibalization between the two products. Holding back the less profitable product is the best strategy for the monopolist in this case. It is also true for the sequential entry game when the ratio of the potential profit margin of the advantageous product to the other one is greater than a certain threshold. The incumbent's losses due to cannibalization are greater than its losses due to competition and thus it avoids offering one more product.

On the other hand, when the profit margin ratio is below this certain threshold and the H (L) focused entrant is already stealing business at the high (low) end, the incumbent fights back by increasing the current production to make up for the decreasing prices. As the capacity cost increases, the entrant reduces H (L)

Figure 1: Increasing product variety in response to H focus entrant.



production. It helps the prices to recover and provides more incentive for the incumbent to increase L (H) production. However, when γ_y reaches the first threshold and the entrant continues to reduce production, it is not profitable for the incumbent to further increase L (H) production. Beyond this threshold, by offering the entrant’s product, the incumbent steals demand which would otherwise be satisfied by the entrant and improves its profits. As a result, the leader finds it more profitable to introduce a product that was being held back in response to a focus strategy entrant with a high capacity investment cost. When the capacity cost is much higher, beyond the second threshold, the entrant can no longer operate profitably and ceases operations.

For example, in the airline industry, imagine a market (such as New York–Raleigh route) where a full-service carrier dominates and optimally provides an all-economy service. My results suggest that if an all-business focus carrier enters this market and aircraft operational costs are at a high range as in Proposition 2a, it is in the best interest of the market leader airline to start offering business class seats in addition to its economy class seats.

Next, I will present an example for Proposition 2 in Figure 1. In this example, both the leader and the entrant have positive capacity costs and $q_H = 3.05$, $q_L = 1.5$, $c_H = 2.5$, $c_L = 0.9$, $s_H = 1.5$, $s_L = 1$. I would like to show that the analytical results shown in the proposition continue to hold when the leader is also capacity constrained. This example fits into Proposition 2a where the low-quality product would be the better product and a monopolist would only offer that. However, the

example shows that the leader optimally increases its product variety in response to an H focused entrant beyond a certain capacity cost threshold. For example, when the capacity cost of the entrant is 10 times that of the leader, only the top 1.5% of the market (that is ordered by the customers' valuations for quality) is willing to pay for the high-quality product. Out of this 1.5%, the entrant serves 1.07% and the leader serves 0.44%. The leader also offers product L and the next 21.44% of the market purchases it. Note that, the graph only shows the top 5% of the market for brevity. At this point, it becomes a counterexample and shows that offering only L may not be the optimal strategy when sequential entry, product differentiation, and capacity costs are taken into consideration.

Stackelberg Competition with $\gamma_y \geq \gamma_x \gg 0$

In this section, the second case when the leader's cost is equally high as the entrant's cost for capacity investment is analyzed. If an industry norm is to operate the business via leasing of equipment and/or rental facilities such as aircraft leasing and fuel costs, then this assumption would be acceptable. Then, even an established firm would have to pay the lease every month to continue operations in the market and increases in the financial rates would affect all players equally.

Similar to the previous section, I analytically study this setting at the extreme case where $\gamma_x = \gamma_y$. After discussing analytical results, I present numerical examples that relax this assumption. In this case, I focus more on the capacity investment decisions and how they are affected by the changes in capacity costs. Note that in this case optimal quantity decisions determine the necessary capacity investments for both firms.

The classical Stackelberg model predicts that an incumbent should ignore an entrant and keep its monopoly position in terms of market supplies. However, I find that for a range of capacity investment cost levels, an incumbent firm may be better off by moving away from its monopoly position and increasing its market supply which necessitates an increase in its capacity investment as formally presented in the following proposition.

Proposition 3: Suppose K_M and K_X are the optimal monopoly and Stackelberg leader capacity levels, respectively. Then, $K_X > K_M$ in response to:

- (a) An H focused entrant when $c_H/q_H > c_L/q_L$, $\frac{s_L}{s_H} < \frac{q_L - c_L}{q_H - c_H} < 1$, and $\hat{\gamma}_2 \leq \gamma_y < \hat{\gamma}_6$.
- (b) An L focused entrant when $c_H/q_H \leq c_L/q_L$, $\frac{q_L - c_L}{s_L} > \frac{q_H - c_H}{s_H}$, and $\hat{\gamma}_7 \leq \gamma_y < \hat{\gamma}_3$.

In Proposition 3a, both the increasing cost to quality ratio ($c_H/q_H > c_L/q_L$) and the profit margin per unit resource consumed ($\frac{q_H - c_H}{s_H} < \frac{q_L - c_L}{s_L}$) favor L . For lower levels of γ_y , the leader optimally offers both products. As γ_y increases, the leader reduces the production of both products with the pressure of costly resources. The reduction is more aggressive in the high end because H generates less potential profit when resources consumed are taken into account. Eventually, at the first threshold for γ_y , the leader ceases the high-quality production altogether.

It is better to offer only L beyond this threshold. Until this point, the leader behaves like a monopolist. It is in the best interest of the leader to ignore the entrant.

Beyond the first threshold (given in the Proposition 3a), the two firms engage in a pure asymmetric competition: the leader offers only L while the entrant offers H . At this high-capacity cost range, focusing on H is inefficient and generates low profits per unit resource consumed for the entrant. The incumbent has incentive to increase the production. Note that the leader's profit is much closer to a monopoly when there is an H focused entrant than in the case when there is a symmetric (or L focused) entrant in the market. Moreover, due to low profit margin per unit resource consumed, the H focused entrant is affected more than the leader with the increase in γ_y . As a result, the entrant reduces production more aggressively as γ_y increases. This creates additional incentive for the incumbent to increase production to satisfy the demand that is left over from the entrant. As a result, the relative difference in production between a Stackelberg leader and a monopolist increases with γ_y up to the second threshold. Beyond the second threshold, the entrant chooses to stay out of the market.

In Proposition 3b, while decreasing cost to quality favors H , profit margin per unit resource consumed ($\frac{q_H - c_H}{s_H} < \frac{q_L - c_L}{s_L}$) favors L . When γ_y is low enough, the incumbent firm offers only H and has incentive to increase production due to the pure asymmetric competition. Like the previous case, the incumbent increases production to make up for the low prices and get close to the monopoly profits. This incentive decreases as γ_y increases and the best utilization of the expensive resource becomes the leading economic force for the incumbent. When γ_y reaches the second threshold, the incumbent starts producing L in the market, which causes a head to head competition at the low-end market. At these high levels of γ_y , where the leader also offers L , it is better to ignore the entrant. The incumbent rather concentrates on its own resource utilization than the competition. High-capacity cost and closer competition become the legitimate economic reasons that cause the incumbent to ignore the entrant.

Common wisdom suggests that the optimal capacity investment decreases as the cost of capacity increases. In a stark contrast, I find that the L focused entrant's capacity investment level may be increasing with the cost of capacity. This result is formally stated in the following proposition.

Proposition 4: Suppose K_Y is the L focused entrant's optimal capacity level. Then, $\partial_{\gamma_y} K_Y > 0$ when $c_H/q_H \leq c_L/q_L$, $\frac{q_L - c_L}{s_L} > \frac{q_H - c_H}{s_H}$, $\frac{s_H}{s_L} > \frac{4q_H - q_L}{2q_L}$, and $\hat{\gamma}_7 \leq \gamma_y < \hat{\gamma}_3$.

When the cost to quality ratio is decreasing and γ_y is lower than the second threshold, the leader chooses to offer only H . Note that the unit resource consumption of H is unprofitably high; not only is the profit margin per unit resource consumed worse for H , but the resource consumption of H is also much higher than L . Hence, the increase in the cost of capacity affects the leader quite strongly and investment in the resources becomes prohibitively costly. As γ_y increases, the leader reduces the production more aggressively than the L focused entrant. At this range, the entrant has incentive to satisfy the demand that is left over from the leader. In other words, the entrant steals business from the market leader. Similar to intuition explained before, increasing the production becomes more profitable

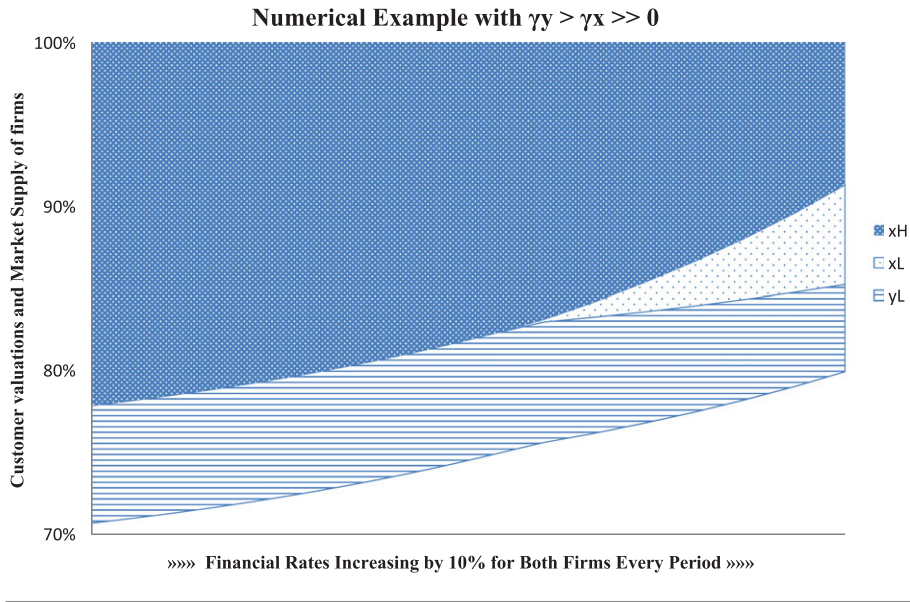
for the entrant even if the cost of investment increases. Note that this result does not arise when the firms are strategically symmetric as in Yayla-Küllü et al. (2013). It is an outcome of the strategic asymmetry between the competing firms.

In order to understand this result better, let's look at a hypothetical example. Suppose that there are two airlines operating in the market between cities A and B and there are 1,000 potential customers in a given time unit. Airline *X* is the market leader and its offerings are observable by Airline *Y* which is an all-economy class airline entrant. The market is heterogenous such that a customer with the highest quality valuation is willing to pay \$3,000 for a first class seat and \$1,000 for an economy class seat. Suppose that it costs \$1,500 to offer a first class seat whereas it costs \$600 to offer an economy class seat. These operating costs do not include the opportunity cost but only items such as fuel, flight attendants, service, and food. Suppose also that a first class seat occupies seven times the space of an economy class seat. The airline industry has a wide range of seat sizes. An economy class seat can be as small as 28" by 17.5". On the other hand, a first class suite can get as spacious as a private bedroom with a stand-alone bed, a personal full length wardrobe, and a 23" LCD TV. In such a market, the capacity investment costs can be equally low for the two firms. Note that rescheduling a plane to a new route within an already existing fleet has negligible costs. Then, increasing capacity by adding flights on a given route at a given time does not necessarily require capital investment. Let's say the capacity investment cost is $\gamma_y = \$0$ per unit space. Then, the entrant optimally offers only 70 economy class seats. If the capacity investment cost is greater, (let's say $\gamma_y = \$70$ per unit space), then the entrant offers 81 economy class seats. The entrant has increased its capacity investment even though the costs are more in the latter case. Note that in the low cost case, the leader offers 260 first class seats while in the high cost case, it offers 169 first class seats.

In this example, there are two forces at play for the leader. The first one is the cannibalization concern: the leader does not offer economy class seats because first class seats are very profitable. It is not good for the company to offer economy class seats and cannibalize its own sales. The second force is the utilization of costly resources: the leader has to reduce the supply in the market to keep its costs at a minimum and prices as high as possible. Note that one extra first class seat costs seven times more compared to an economy class seat due to costly resource investments. This causes the leader to reduce its supply more aggressively than the low-quality focused entrant. In this setting, the entrant is also affected by the increasing cost. However, the faster reduction of supply in the first class market generates opportunity (through increasing prices) for the economy class market. By increasing its supply, the entrant is now able to sell more seats for higher prices which pays off for the increase in the capacity cost.

Similar to Figure 1, Figure 2 presents an example relaxing the leader's capacity cost assumption for Propositions 3 and 4 when the entrant is *L* focused. In this example, the leader's capacity costs are smaller than those of the entrant's and $q_H = 3$, $q_L = 1$, $c_H = 1.5$, $c_L = 0.6$, $s_H = 7$, and $s_L = 1$. This example shows the case where the high-quality product would be the better product for low capacity costs and the leader would only offer that up to a capacity cost threshold. For example, when the capacity costs are relatively lower, the leader covers the

Figure 2: Entrant’s capacity investments increasing in costs.



top 21.8% of the market by offering product *H* and the entrant covers the next 7.2% by offering product *L*. When the capacity costs increase to an intermediate level, the leader lowers its production down to 17.7% while the entrant increases it up to 7.4%. At this point this example becomes a counterexample and shows that increasing capacity investments with increasing costs may indeed be the optimal strategy when sequential entry, product differentiation, and capacity costs are taken into consideration.

IMPLICATIONS ON QUALITY

In this section, I look at another strategy that can be put into action by the leader in response to a focus strategy entrant. A possible change in the quality levels of the products when there is a threat of entry in the market is investigated. Note that it may not always be feasible to change the quality levels of the products since there could be technological and/or time constraints that prevent firms utilizing this lever.

In order to extend to endogenous quality levels, I need to define functional forms for unit production costs and unit resource consumptions. For unit production costs, I follow the literature (cf. Moorthy, 1984, p. 292) and assume a quadratic function of quality ($c(q) = \alpha q^2$). For unit resource consumptions, I also assume a quadratic function ($s(q) = q^2$) for analytical tractability. However, note that a quadratic cost function results in increasing cost to quality ratio ($c_H/q_H > c_L/q_L$)

and a quadratic resource consumption function results in $(s_H/q_H > s_L/q_L)$ which limits the discussion only to these cases unlike the previous section.

I also limit the discussion to the cases when the market leader offers both products and capacity cost is in the two extremes. This makes it possible to direct our attention to the research questions about the quality choices in response to a focus strategy entrant.

A Monopolist's Quality Decision and Capacity Investments

A monopolist that needs to make quality choices ($q_i^M \in [q, \bar{q}]$) and capacity investments solves the following problem:

$$\text{Max } (p_H(x_H^M, x_L^M) - c(q_H^M))x_H^M + (p_L(x_H^M, x_L^M) - c(q_L^M))x_L^M - \gamma_x K_M, \quad (5)$$

$$\text{subject to } s(q_H^M)x_H^M + s(q_L^M)x_L^M \leq K_M,$$

$$K_M > 0, x_i^M > 0, q_i^M \in [q, \bar{q}], i = H, L.$$

I find that a monopolist reduces the quality levels to control the increasing capacity cost. The characterization of this result is provided in the following lemma. It will help to discuss findings for the competitive setting.

Lemma 2: When $\frac{2}{5\bar{q}} < \gamma_x + \alpha < \frac{1}{5\bar{q}}$, $q_H^M = \frac{2}{5(\alpha + \gamma_x)}$, $q_L^M = \frac{1}{5(\alpha + \gamma_x)}$, $x_H^M = \frac{1}{5}$, $x_L^M = \frac{1}{5}$.

For a large range of cost parameters, the firm optimally fixes the production quantity at a constant level for both products. When γ_x increases, the monopolist reduces capacity investment through reducing the quality levels offered in the market. A similar observation is true for the unit production cost parameter (α). An increase in α is offset by a reduction in the quality levels rather than a reduction in the production quantity.

For an airline, when the product line is fixed at two products, such as business and economy cabins, I find that when the capacity costs are increasing, the firm optimally reduces the overall service quality (therein customers' willingness to pay as measured by θq_i).

Stackelberg Competition with $\gamma_y \gg \gamma_x = 0$

As in the previous section, I start by analyzing the capacity decision in two cases. In case 1, I study the model where the leader's capacity investment cost is lower than the entrant's cost. For analytical tractability, I will study the extreme case where it is zero. On the other hand, the new entrant needs to make costly resource investments before it can start operating in the market. Then, leader firm X 's problem is revised as follows:

$$\begin{aligned} &\text{Max } (p_H(x_H, x_L, y_j(x_H, x_L)) - c(q_H))x_H + (p_L(x_H, x_L, y_j(x_H, x_L)) \\ &\quad - c(q_L))x_L, \quad (6) \\ &\text{subject to } x_i \geq 0, q_i \in [q, \bar{q}], \quad i = H, L \end{aligned}$$

The entrant, firm Y 's problem is given as follows:

$$\text{Max}_{(K_Y \geq 0, y_j \geq 0)} (p_j(x_H, x_L, y_j) - c(q_j))y_j - \gamma_y K_Y \text{ subject to } s(q_j)y_j \leq K_Y. \quad (7)$$

I find that the production quantity of the high-quality product may increase as a response to an H focused entrant when the leader has no capacity constraint and unit production costs are below a certain threshold. This finding is in contrast with the earlier papers (Ashiya, 2002; Barbot, 2008). With no consideration of either the unit production costs or the capacity costs, Ashiya (2002) and Barbot (2008) suggest that the leader should exit the high-quality market when a high-quality entry occurs. I show that optimal choices change when such operational factors are taken into consideration.

Proposition 5: Suppose x_i^* and q_i^* are the Stackelberg leader's optimal production quantity and quality level for $i = H, L$. Then, in response to an H focused entrant, $q_i^* > q_i^M$ for $i = H, L$ and $x_L^* > x_L^M$ when $\gamma_y \geq \alpha > \frac{2}{5q}$. In addition, $x_H^* > x_H^M$ when $\alpha < \frac{2\gamma_y}{3} + \frac{2}{5q}$.

The leader has multiple incentives to increase the quality levels in this case. First, it wants to increase the prices that will be driven down due to competition. Second, the new entrant is affected more by an increase in quality due to the capacity investment cost. Remember in this case, the leader has a negligible capacity cost, and both the unit production costs and unit resource consumptions increase with quality. When the entrant's cost of capacity is relatively high, the entrant chooses to reduce production due to costly capacity investments. Eventually, the leader sets the industry standard for quality to the highest level feasible and puts the new entrant in a tough position in terms of profitability. Moreover, when the unit production cost parameter (α) is below a certain threshold, I show that the leader also increases its H production. Because the leader has no capacity investment cost and this cost increases considerably for the entrant, the entrant reduces production considerably. This creates incentive for the leader to increase its H production and steal business from the entrant. The leader recovers its profits by this increase in production as long as the production costs remain below a certain threshold.

This result explains the behavior of AA toward EOS in the London–New York route in 2007–2008 (Rowell, 2009). The dominant airline (AA) with multiple products (both business class and economy class) faced a business class focus entrant (EOS) in the market. AA attacked the entrant by increasing its flight frequency (in other words, production quantity) and by offering double frequent flier miles and new services which increase the customers' valuations of the service (in other words, better overall quality of service). In the end, increasing financial burden due to costly resources pushed the new entrant (EOS) over the edge and it went bankrupt before the end of 2008.

Stackelberg Competition with $\gamma_y = \gamma_x \gg 0$

In order to allow for the capacity decision, the leader firm X 's problem is revised in the following and the entrant firm Y 's problem is kept as in Equation (7):

$$\begin{aligned} \text{Max } & (p_H(x_H, x_L, y_j(x_H, x_L)) - c(q_H))x_H \\ & + (p_L(x_H, x_L, y_j(x_H, x_L)) - c(q_L))x_L - \gamma_x K_X, \end{aligned} \tag{8}$$

$$\text{subject to } s(q_H)x_H + s(q_L)x_L \leq K_X,$$

$$K_X \geq 0, x_i \geq 0, q_i \in [\underline{q}, \bar{q}], i = H, L.$$

I find that the leader adjusts the quality levels of products, shift operations toward the other product, and moves away from the entrant’s market. This result is presented in the following proposition.

Proposition 6: Suppose x_i^* and q_i^* are the Stackelberg leader’s optimal production quantity and quality level for $i = H, L$. Then,

- (a) $q_i^* > q_i^M$ for $i = H, L$, $x_L^* > x_L^M$ and $x_H^* < x_H^M$ in response to an H focused entrant when $\gamma_y + \alpha > \frac{2}{5\bar{q}}$.
- (b) $q_i^* < q_i^M$ for $i = H, L$, $x_L^* < x_L^M$ and $x_H^* > x_H^M$ in response to an L focused entrant, when $\gamma_y + \alpha < \frac{1}{5\bar{q}}$.

This proposition shows that the optimal quality increases in response to an H focused entrant when capacity investment and production costs exceed a certain threshold. Upon entry, the best response for the leader is to reduce its operations at the high end and shift resources to the low end. The concern with the low end is that the price is much lower than its monopoly level because of the competition if the leader ignores the entrant and keeps the same position as a monopolist (in terms of both the quality and the quantity). Therefore, the leader increases the quality levels which increases the prices. The prices under competition may be greater than its monopoly level due to increased quality. Interestingly, the impact of competition may be not only the supply increase but also the price increase as formally presented in Corollary 1.

I also find that the strategy is reversed in response to an L focused entrant when capacity investment and production cost parameters are below a certain threshold. Like the previous case, best reaction for the leader is to reduce its operations at the low end and shift resources to the high end. However, increasing H production would require substantially more resources which are very costly due to relatively high-quality levels. For that reason, it is best for the leader to reduce the quality levels which will reduce the costs and resource consumption levels for H production. Although the prices are much lower than its monopoly levels, the production and capacity costs are also much lower. This helps the leader to offset the losses due to competition.

Corollary 1: Suppose that p_i^M and p_i^* are the market clearing prices under monopoly and Stackelberg game with an H focused entrant for $i = H, L$, respectively. Then, $p_H^* > p_H^M$ when $\frac{1}{5\bar{q}} > \gamma_y + \alpha > \frac{7}{15\bar{q}}$. Moreover, $p_L^* > p_L^M$ when $\frac{1}{5\bar{q}} > \gamma_y + \alpha > \frac{\sqrt{217}-5}{20\bar{q}}$.

There are few empirical papers that attempt to investigate this controversial claim of increasing prices with competition. Frank and Salkever (1997) show that the price increase after entry is indeed observed in practice. Their findings are confirmed by a recent study in Regan (2008). In the airline industry, Tretheway and Kincaid (2005) refute the widely accepted belief that the hub premiums of some airlines are due to near-monopoly powers. Lee and Luengo-Prado (2005) find that there exist some markets where the prices are higher even when there is greater competition. At the time of their study, Delta passengers had paid 12% less per mile in markets to and from Salt Lake City, which is a hub where the airline enjoys a near-monopoly power, compared to travel throughout the remainder of its network where the firm has to compete with numerous airlines.

In this article, I offer an alternative explanation to these findings. The leader raises the quality standards in the industry when the entrant is as capable as the leader in terms of high-quality production. This increase in quality eventually leads to an increase in prices.

I also investigate how asymmetric competition affects the total supply in the market. I find that when the capacity investment and production costs are fairly high, an H focused entrant may cause a decrease in the overall supply. This result is formally presented in the following corollary.

Corollary 2: Suppose x_i^M , x_i^* , and y_H^* are the monopolist's, Stackelberg leader's, and the H focused entrant's optimal production quantities for $i = H, L$, respectively. Then, $x_H^* + x_L^* + y_H^* < x_H^M + x_L^M$ when $\frac{1}{5q} > \gamma_y + \alpha > \frac{7}{10q}$.

Existing literature on symmetric competition suggests that the total supply in the market increases and prices decrease as the number of competing firms increases (Gal-Or, 1983; De Fraja, 1996; Johnson & Myatt, 2006). On the contrary, this corollary shows that the total amount of production may decrease and prices may increase with entry when strategic asymmetries, resource consumptions of different products, and capacity investments are taken into account. This decrease in supply is mainly due to the sharp decrease in high-quality production as explained in Proposition 6. Even the increase in low-quality production cannot offset such a decrease and the total supply is reduced compared to its monopoly levels when the resources are costly and the entrant has a high-quality focus.

MANAGERIAL INSIGHTS

In this section I will synthesize the findings of the previous sections, and summarize the managerial insights.

First of all, the proliferation lever is expected to be pulled in response to a focus strategy entrant when the capacity cost is in the low range for the leader. The leader with low capacity cost may introduce a product that it was holding back when the entrant has to bear the high capacity cost and cannibalization threat is relatively small (profit margin ratio is below a certain threshold). When the cost is high for the leader, it is no longer profitable to increase variety in response to a focus strategy entrant.

Second, the production volume lever is expected to get used both when the capacity cost is high and when it is not. However, the extent of this use

changes as the capacity cost increases for the leader. When it is low, the production volume of a market leader would be nondecreasing for all parametric settings. On the other hand, as the capacity cost increases, changing the production volume loses its powerful advantage. It is observed only when there is pure asymmetric competition with the entrant. If the entrant's focus product is the less advantageous product, then it is in the best interest of the market leader (that offers only the more advantageous product) to increase its production volume compared to its monopoly level.

Ignoring the new focus strategy entrant surfaces as a legitimate strategy when the capacity cost is in the high range for the leader. The leader chooses to ignore the entrant when it is already offering the same product as the entrant. If the leader faces a head to head competition with the entrant at either end of the market, then keeping the same position in the market is the best option for the leader.

When the leader has the opportunity to change the quality levels of the products, it would use this opportunity by increasing the quality levels in response to a high-quality focused entrant both when its capacity investment cost is low and when it is high. When its capacity cost is low enough, I even expect the leader to increase the production volume of both products in response to a high-quality focused entrant that has a high-capacity investment cost. The condition for the leader's volume increase occurs only when the unit production costs are below a certain threshold. In response to a low-quality focused entrant, the leader with high capacity cost decreases the quality levels in the market. The production volume lever moves in opposite directions for the two products. While the leader increases the high-quality production, it decreases the low-quality production.

CONCLUSIONS

In this article, I study a multiproduct firm competing with a focus strategy entrant in a vertically differentiated market. There are two products that have different quality, unit production, and unit resource consumption levels. Because the customers are heterogenous in their willingness to pay, cannibalization within the products of the same firm and the entrant's business stealing actions are major concerns for the leader. Moreover, as products may potentially consume different levels of the costly resource, optimal resource utilization may provide a leverage as well as a disadvantage for the leader in its fight with the new entrant. Hence, the leader should find the balance between cannibalization, business stealing, and resource utilization forces in this competitive setting.

In the main model, the multiproduct firm is the leader and has the capability to produce both products. First, the leader decides how much of each product to offer in the market. Note that the leader may decide to offer only one of the products in a positive quantity limiting the product variety in the market. Next, the focus strategy entrant determines its quantity and decides on its capacity. I show that the leader may have to increase the product variety in response to a focus strategy entrant. The parametric regions where it is optimal for the firm to offer the entrant's focus product in addition to the most profitable product are identified. Offering the same product is especially important when the capacity cost is high for the entrant firm.

Interestingly, I also find that the capacity investment level of an entrant may be increasing with the cost of capacity. When the cost to quality ratio is decreasing and the cost of capacity is below a certain threshold, the leader chooses to offer only the high-quality product. The incentive to reduce production is greater for the leader than for an entrant with a focus on low-quality production, as the high-quality product may consume a greater amount of the costly resource. When the low-quality product has greater profit margin per unit resource consumed, the entrant is at a great advantage. Eventually, increasing the production becomes more profitable for the low-quality focused entrant even if the cost of investment increases for a certain range of capacity costs.

I also find that when the leader has the power to set the industry standards by deciding the quality levels, as a response to a high-quality focused entrant, the leader increases both levels of quality and production of the low-quality product, and may decrease or increase the production of the high-quality product depending on its capacity investment cost. Moreover, when the capacity investment cost is high for both the entrant and the leader, the prices may increase with entry.

Like all models, this article has limitations. I consider the case when the demand is deterministic. This enables full characterization of the best response functions of both firms. It would be interesting to discuss these findings in the face of uncertainty. Future research can extend my work further and study the impact of focus strategy competition on product variety and capacity choices when the demand is stochastic in nature. Furthermore, following the literature, I assumed that consumer preferences are distributed uniformly. This makes the analysis tractable, making it possible to keep the focus on effects of costly resources on the product line and investment choices of firms. While I verified that these insights can carry over to nonuniform distributions through some numerical examples (which are not reported in this article), it would be worthwhile for future work to further study what happens under nonuniform distributions in general.

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APPENDIX A: CAPACITY COST THRESHOLDS

We define the following threshold capacity cost levels that may depend on the type of the product $i = H, L$. We refer to these threshold capacity cost levels for describing the firms' optimal policies.

$$\hat{\gamma}_1^i = \frac{q_i - c_i}{s_i}, \quad (\text{A1})$$

$$\hat{\gamma}_2 = \frac{q_H - c_H - q_L + c_L}{s_H - s_L}, \quad (\text{A2})$$

$$\hat{\gamma}_3 = \frac{c_L q_H - c_H q_L}{q_L s_H - q_H s_L}, \quad (\text{A3})$$

$$\hat{\gamma}_4 = \frac{q_H(q_L - c_L - (q_H - c_H))}{(q_H - q_L)s_H}, \quad (\text{A4})$$

$$\hat{\gamma}_5 = \frac{c_L q_H - c_H q_L}{(q_H - q_L)s_L}, \quad (\text{A5})$$

$$\hat{\gamma}_6 = \frac{q_H(2c_L + 4q_H - 3q_L) + c_H(-4q_H + q_L)}{4q_H s_H - q_L s_H - 2q_H s_L}, \quad (\text{A6})$$

$$\hat{\gamma}_7 = \frac{4c_L q_H - 2c_H q_L - c_L q_L - 2q_H q_L + q_L^2}{2q_L s_H - 4q_H s_L + q_L s_L}. \quad (\text{A7})$$

APPENDIX B: PROOFS

Proof of Lemma 1: Following the main model, demands for the two product types are given as $D_H(p_L, p_H) = 1 - \frac{p_H - p_L}{q_H - q_L}$, and $D_L(p_L, p_H) = \frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L}$. Then prices are set to clear the market based on the total amount of production in the industry: $p_H(x_H, x_L, y_j)$ and $p_L(x_H, x_L, y_j)$.

Following the fact that $x_i + y_i = D_i(p_H, p_L)$, the above price–quantity equations can be solved for prices as follows: $p_H = q_H(1 - x_H - y_H) - q_L x_L$ and $p_L = q_L(1 - x_L - x_H - y_H)$ for the high-quality entrant and $p_H = q_H(1 - x_H) - q_L(x_L + y_L)$ and $p_L = q_L(1 - x_L - y_L - x_H)$ for the low-quality entrant.

For the leader against a high-quality entrant, $\partial_{x_H, x_H}((q_H(1 - x_H - y_H) - q_L x_L - c_H)x_H + (q_L(1 - x_L - x_H - y_H) - c_L)x_L - \gamma_x K_X) = -2q_H$ and $\partial_{x_L, x_L}((q_H(1 - x_H - y_H) - q_L x_L - c_H)x_H + (q_L(1 - x_L - x_H - y_H) - c_L)x_L - \gamma_x K_X) = -2q_L$ and $\partial_{x_H, x_L}((q_H(1 - x_H - y_H) - q_L x_L - c_H)x_H + (q_L(1 - x_L - x_H - y_H) - c_L)x_L - \gamma_x K_X) = -2q_L$ and $\partial_{K_X, K_X}((q_H(1 - x_H - y_H) - q_L x_L - c_H)x_H + (q_L(1 - x_L - x_H - y_H) - c_L)x_L - \gamma_x K_X) = 0$. For the high-quality entrant, $\partial_{y_H, y_H}((q_H(1 - x_H - y_H) - q_L x_L - c_H)y_H - \gamma_y K_Y) = -2q_H$ and $\partial_{K_Y, K_Y}((q_H(1 - x_H - y_H) - q_L x_L - c_H)y_H - \gamma_y K_Y) = 0$.

For the leader against a low-quality entrant, $\partial_{x_H, x_H}((q_H(1 - x_H) - q_L(x_L + y_L) - c_H)x_H + (q_L(1 - x_L - y_L - x_H) - c_L)x_L - \gamma_x K_X) = -2q_H$ and $\partial_{x_L, x_L}((q_H(1 - x_H) - q_L(x_L + y_L) - c_H)x_H + (q_L(1 - x_L - y_L - x_H) - c_L)x_L - \gamma_x K_X) = -2q_L$ and $\partial_{x_H, x_L}((q_H(1 - x_H) - q_L(x_L + y_L) - c_H)x_H + (q_L(1 - x_L - y_L - x_H) - c_L)x_L - \gamma_x K_X) = -2q_L$ and $\partial_{K_X, K_X}((q_H(1 - x_H) - q_L(x_L + y_L) - c_H)x_H + (q_L(1 - x_L - y_L - x_H) - c_L)x_L - \gamma_x K_X) = 0$. For the low-quality entrant, $\partial_{y_L, y_L}((q_L(1 - x_L - y_L - x_H) - c_L)y_L - \gamma_y K_Y) = -2q_L$ and $\partial_{K_Y, K_Y}((q_L(1 - x_L - y_L - x_H) - c_L)y_L - \gamma_y K_Y) = 0$.

Hence, objective functions given in problems (2) and (3) are jointly concave in their corresponding decision variables. \square

Proof of Proposition 1: Following the price–demand equations (1) and the fact that $x_i = D_i(p_H, p_L)$, these equations can be solved for prices as follows: $p_H = q_H(1 - x_H) - q_L x_L$ and $p_L = q_L(1 - x_L - x_H)$. Note that the capacity constraint is always satisfied by equality at the optimal solution ($s_H x_H + s_L x_L = K_X$). We plug in this fact and price functions in the objective function. The resulting objective

function is jointly concave on the variables of the problem. Solving for the first-order conditions, we find three alternative solutions for the monopolist depending on the Lagrangian multipliers as follows:

Solution 1: Offer both products: $x_H^M = \frac{-c_H+c_L+q_H-q_L-\gamma_x s_H+\gamma_x s_L}{2(q_H-q_L)}$ and $x_L^M = -\frac{c_L q_H-c_H q_L-\gamma_x q_L s_H+\gamma_x q_H s_L}{2q_H q_L-2q_L^2}$. Solution 2: Offer only H when $\frac{c_L q_H-c_H q_L-\gamma_x q_L s_H+\gamma_x q_H s_L}{q_H} \geq 0$: $x_L^M = 0$ and $x_H^M = \frac{q_H-c_H-\gamma_x s_H}{2q_H}$. Solution 3: Offer only L when $c_H - c_L - q_H + q_L + \gamma_x s_H - \gamma_x s_L \geq 0$: $x_H^M = 0$ and $x_L^M = \frac{q_L-c_L-\gamma_x s_L}{2q_L}$.

By checking the feasibility and Lagrangian multiplier conditions, we find the corresponding feasible optimal alternative for each parametric set as follows:

- (a) When $(c_H/c_L > q_H/q_L)$,
 - (i) For $q_L - c_L > q_H - c_H$, when $\gamma_x < \frac{q_L-c_L}{s_L}$, $x_H^M = 0$ and $x_L^M = \frac{q_L-c_L-\gamma_x s_L}{2q_L}$.
 - (ii) For $\frac{s_L}{s_H} < \frac{q_L-c_L}{q_H-c_H} < 1$, when $\gamma_x < \frac{q_H-c_H-q_L+c_L}{s_H-s_L}$, $x_H^M = \frac{-c_H+c_L+q_H-q_L-\gamma_x s_H+\gamma_x s_L}{2(q_H-q_L)}$ and $x_L^M = -\frac{c_L q_H-c_H q_L-\gamma_x q_L s_H+\gamma_x q_H s_L}{2q_H q_L-2q_L^2}$; when $\frac{q_H-c_H-q_L+c_L}{s_H-s_L} \leq \gamma_x < \frac{q_L-c_L}{s_L}$, $x_H^M = 0$ and $x_L^M = \frac{q_L-c_L-\gamma_x s_L}{2q_L}$.
 - (iii) For $\frac{s_L}{s_H} > \frac{q_L-c_L}{q_H-c_H}$, when $\gamma_x < \frac{c_1 q_H - c_H q_L}{q_1 s_H - q_H s_L}$, $x_H^M = \frac{-c_H+c_L+q_H-q_L-\gamma_x s_H+\gamma_x s_L}{2(q_H-q_L)}$ and $x_L^M = -\frac{c_L q_H-c_H q_L-\gamma_x q_L s_H+\gamma_x q_H s_L}{2q_H q_L-2q_L^2}$; when $\frac{c_1 q_H - c_H q_L}{q_1 s_H - q_H s_L} \leq \gamma_x < \frac{q_H-c_H}{s_H}$, $x_L^M = 0$ and $x_H^M = \frac{q_H-c_H-\gamma_x s_H}{2q_H}$.
- (b) When $(c_H/c_L \leq q_H/q_L)$,
 - (i) For $\frac{s_L}{s_H} < \frac{q_L-c_L}{q_H-c_H} < 1$, when $\gamma_x < \frac{c_1 q_H - c_H q_L}{q_1 s_H - q_H s_L}$, $x_H^M = 0$ and $x_L^M = \frac{q_L-c_L-\gamma_x s_L}{2q_L}$; when $\frac{c_1 q_H - c_H q_L}{q_1 s_H - q_H s_L} \leq \gamma_x < \frac{q_H-c_H-q_L+c_L}{s_H-s_L}$, $x_H^M = \frac{-c_H+c_L+q_H-q_L-\gamma_x s_H+\gamma_x s_L}{2(q_H-q_L)}$ and $x_L^M = -\frac{c_L q_H-c_H q_L-\gamma_x q_L s_H+\gamma_x q_H s_L}{2q_H q_L-2q_L^2}$; when $\frac{q_H-c_H-q_L+c_L}{s_H-s_L} \leq \gamma_x < \frac{q_L-c_L}{s_L}$, $x_H^M = 0$ and $x_L^M = \frac{q_L-c_L-\gamma_x s_L}{2q_L}$.
 - (ii) For $\frac{s_L}{s_H} > \frac{q_L-c_L}{q_H-c_H}$, when $\gamma_x < \frac{q_H-c_H}{s_H}$, $x_L^M = 0$ and $x_H^M = \frac{q_H-c_H-\gamma_x s_H}{2q_H}$. □

Proof of Proposition 2: We first identify the solutions of the game presented in our model. We use backward induction and start by solving the entrant’s problem (3). Then, we show that among the optimal solution alternatives, the leader chooses to offer both products over others whenever feasible. The proof is completed when we show that for the range of parameters in the proposition, offering both products is a feasible optimal solution alternative.

Following the fact that $x_i + y_i = D_i(p_H, p_L)$, above price–quantity equations can be solved for prices as follows: $p_H = q_H(1 - x_H - y_H) - q_L x_L$ and $p_L = q_L(1 - x_L - x_H - y_H)$ for the high-quality entrant and $p_H = q_H(1 - x_H) - q_L(x_L + y_L)$ and $p_L = q_L(1 - x_L - y_L - x_H)$ for the low-quality entrant.

(a) We need to solve the game for the high-quality focused entrant. Note that the capacity constraint is always satisfied by equality at the optimal solution ($s_j y_j = K_Y$). Using this fact and plugging in the price functions in objective function (3), and solving for the first-order conditions, we find that $y_H(x_L, x_H) = -\frac{c_H+\gamma_x s_H+q_H(-1+x_H)+q_L x_L}{2q_H}$. Plugging this back into objective function (2) with

$\gamma_x = 0$, and solving for the first-order conditions, we find three alternative solutions for the leader depending on the Lagrangian multipliers as follows:

Solution 1: Offer both products: $x_H = \frac{-c_H q_H + c_L q_H + (q_H - q_L)(q_H + \gamma_y s_H)}{2q_H(q_H - q_L)}$, $x_L = -\frac{c_L q_H - c_H q_L}{2q_H q_L - 2q_L^2}$. Solution 2: Offer only H when $c_L - \frac{c_H q_L}{q_H} \geq 0$: $x_H = \frac{-c_H + q_H + \gamma_y s_H}{2q_H}$, $x_L = 0$. Solution 3: Offer only L when $\frac{c_H q_H - c_L q_H - (q_H - q_L)(q_H + \gamma_y s_H)}{2q_H - q_L} \geq 0$: $x_H = 0$, $x_L = -\frac{-2c_L q_H + q_L(c_H + q_H + \gamma_y s_H)}{2q_L(-2q_H + q_L)}$.

Then, we plug in these options one by one and calculate the profits for the leader as follows:

Profit with Solution 1: $\pi_1 = \frac{2c_H^2 q_H^2 - 4c_H c_L q_H q_L + q_L(c_H^2(q_H + q_L) - 2c_H(q_H - q_L)(q_H + \gamma_y s_H) + (q_H - q_L)(q_H + \gamma_y s_H)^2)}{8q_H(q_H - q_L)q_L}$.

Profit with Solution 2: $\pi_2 = \frac{(-c_H + q_H + \gamma_y s_H)^2}{8q_H}$.

Profit with Solution 3: $\pi_3 = \frac{(-2c_L q_H + q_L(c_H + q_H + \gamma_y s_H))^2}{8q_H(2q_H - q_L)q_L}$.

Since $q_H > q_L$, $\pi_1 - \pi_2 = \frac{(c_L q_H - c_H q_L)^2}{4q_H(q_H - q_L)q_L} > 0$ and $\pi_1 - \pi_3 = \frac{(-c_H q_H + c_L q_H + (q_H - q_L)(q_H + \gamma_y s_H))^2}{4q_H(2q_H^2 - 3q_H q_L + q_L^2)} > 0$.

For the conditions presented in Proposition 2a, offering both products (Solution 1) is a feasible optimal solution alternative ($x_H = \frac{-c_H q_H + c_L q_H + (q_H - q_L)(q_H + \gamma_y s_H)}{2q_H(q_H - q_L)} > 0$, $x_L = -\frac{c_L q_H - c_H q_L}{2q_H q_L - 2q_L^2} > 0$). As this is the most profitable option, Solution 1 is the optimal best response for the leader under the conditions presented in Proposition 2a. Following Proposition 1a part (i), the monopolist’s optimal product line would be offering only L for the same conditions which completes the proof for Proposition 2a.

(b) Following similar steps, we solve the game for the low-quality focused entrant. Plugging in the price functions in objective function (3), and solving for the first-order conditions, we find that $y_L(x_L, x_H) = -\frac{c_L + \gamma_y s_L + q_L(-1 + x_H + x_L)}{2q_L}$. Plugging this back into objective function (2), and solving for the first-order conditions, we find three alternative solutions for the leader depending on the Lagrangian multipliers as follows:

Solution 1: Offer both products: $x_H = \frac{-c_H + c_L + q_H - q_L}{2(q_H - q_L)}$, $x_L = \frac{-c_L q_H + c_H q_L + \gamma_y(q_H - q_L)s_L}{2(q_H - q_L)q_L}$. Solution 2: Offer only H when $\frac{c_L q_H - c_H q_L + \gamma_y(-q_H + q_L)s_L}{2q_H - q_L} \geq 0$: $x_H = \frac{-2c_H + c_L + 2q_H - q_L + \gamma_y s_L}{4q_H - 2q_L}$, $x_L = 0$. Solution 3: Offer only L when $c_H - c_L - q_H + q_L \geq 0$: $x_H = 0$, $x_L = \frac{-c_L + q_L + \gamma_y s_L}{2q_L}$.

Then, we plug in these options one by one and calculate the profits for the leader as follows: Profit with Solution 1:

$\pi_1 = \frac{(2c_H^2 q_L - 4c_H c_L q_L + c_L^2(q_H + q_L) + 2c_L(q_H - q_L)(q_L - \gamma_y s_L) + (q_H - q_L)(-q_L(4c_H - 2q_H + q_L) + 2\gamma_y q_L s_L + \gamma_y^2 s_L^2))}{8(q_H - q_L)q_L}$.

Profit with Solution 2: $\pi_2 = \frac{(-2c_H + c_L + 2q_H - q_L + \gamma_y s_L)^2}{16q_H - 8q_L}$.

Profit with Solution 3: $\pi_3 = \frac{(-c_L + q_L + \gamma_y s_L)^2}{8q_L}$.

Since $q_H > q_L$, $\pi_1 - \pi_2 = \frac{(c_L q_H - c_H q_L + \gamma_y(-q_H + q_L)s_L)^2}{4q_L(2q_H^2 - 3q_H q_L + q_L^2)} > 0$ and $\pi_1 - \pi_3 = \frac{(c_H - c_L - q_H + q_L)^2}{4(q_H - q_L)} > 0$.

For the conditions presented in Proposition 2b, offering both products (Solution 1) is a feasible optimal solution alternative ($x_H = \frac{-c_H+c_L+q_H-q_L}{2(q_H-q_L)} > 0$ and $x_L = \frac{-c_Lq_H+c_Hq_L+\gamma_y(q_H-q_L)s_L}{2(q_H-q_L)q_L} > 0$). Because this is the most profitable option, Solution 1 is the optimal best response for the leader under the conditions presented in the Proposition 2b. Following Proposition 1b, the monopolist's optimal product line would be offering only H for the same conditions which completes the proof.

Next we show how the quantities are also increasing:

- (a) The high-quality focused entrant's best response is $y_H = -((c_H + \gamma_y s_H + q_H(-1 + x_H) + q_L x_L)/(2q_H))$. Then, by solving the first-order conditions and checking for feasibility conditions, the leader's best response is characterized and compared to the monopolist's solution as follows:

$$\text{when } \frac{4q_H-q_L}{3q_H} > \frac{q_L-c_L}{q_H-c_H}, \text{ for } \frac{q_H(q_L-c_L-(q_H-c_H))}{(q_H-q_L)s_H} < \gamma_y \leq \frac{q_H-c_H}{3s_H},$$

$$x_H^* = \frac{(-c_Hq_H+c_Lq_H+(q_H-q_L)(q_H+\gamma_y s_H))}{(2q_H(q_H-q_L))} > x_H^M, x_L^* = -\frac{c_Lq_H-c_Hq_L}{2q_Hq_L-2q_L^2} > x_L^M.$$

- (b) The low-quality focused entrant's best response is $y_L = -((c_L + \gamma_y s_L + q_L(-1 + x_H + x_L))/(2q_L))$. Then, by solving the first-order conditions and checking for feasibility conditions, the leader's best response is characterized and compared to the monopolist's solution as follows:

$$\text{when } \frac{4q_H-q_L}{3q_L} > \frac{q_H-c_H}{q_L-c_L}, \text{ for } \frac{c_Lq_H-c_Hq_L}{(q_H-q_L)s_L} < \gamma_y \leq \frac{q_L-c_L}{3s_L},$$

$$x_H^* = (-c_H + c_L + q_H - q_L)/(2(q_H - q_L)) > x_H^M, x_L^* = (-c_Lq_H + c_Hq_L + \gamma_y(q_H - q_L)s_L)/(2(q_H - q_L)q_L) > x_L^M. \quad \square$$

Proof of Proposition 3: We first solve the game and identify leader's and follower's actions for the parametric sets presented in the proposition. Then, we compare them with the monopolist's solution as presented in Proposition 1. Following the fact that $x_i + y_i = D_i(p_H, p_L)$, price-demand equations (1) can be solved for prices as follows: $p_H = q_H(1 - x_H - y_H) - q_L x_L$ and $p_L = q_L(1 - x_L - x_H - y_H)$ for the high-quality entrant and $p_H = q_H(1 - x_H) - q_L(x_L + y_L)$ and $p_L = q_L(1 - x_L - y_L - x_H)$ for the low-quality entrant. Note that the capacity constraint is always satisfied by equality at the optimal solution for both firms ($s_j y_j = K_Y$ and $s_H x_H + s_L x_L = K_X$).

- (a) We solve the game for the high-quality focused entrant. Plugging in the capacity and price functions in objective function (3), and solving for the first-order conditions, we find that $y_H(x_L, x_H) = \frac{-c_H+\gamma_y s_H+q_H(-1+x_H)+q_L x_L}{2q_H}$. Plugging this back into objective function (2), and solving for the first-order conditions, we find three alternative solutions for the leader depending on the Lagrangian multipliers as follows:
 - Solution 1: Offer both products: $x_H = \frac{-c_H+c_L+q_H-q_L-\gamma_y s_H+\gamma_y s_L}{2(q_H-q_L)}$ and $x_L = -\frac{c_Lq_H-c_Hq_L-\gamma_y q_L s_H+\gamma_y q_H s_L}{2q_Hq_L-2q_L^2}$.
 - Solution 2: Offer only H when $\frac{c_Lq_H-c_Hq_L-\gamma_y q_L s_H+\gamma_y q_H s_L}{q_H} \geq 0$: $x_L = 0$ and $x_H = \frac{q_H-c_H-\gamma_y s_H}{2q_H}$.
 - Solution 3: Offer only L when $\frac{q_H(c_H-c_L-q_H+q_L+\gamma_y s_H-\gamma_y s_L)}{2q_H-q_L} \geq 0$: $x_H = 0$ and

$$x_L = -\frac{-2c_Lq_H + c_Hq_L + q_Hq_L + \gamma_yq_Ls_H - 2\gamma_yq_Hs_L}{-4q_Hq_L + 2q_L^2}.$$

We find that Solution 3 is the only feasible optimal alternative when $c_H/q_H > c_L/q_L$, $\frac{s_L}{s_H} < \frac{q_L - c_L}{q_H - c_H} < 1$, and $\frac{(q_H - c_H) - (q_L - c_L)}{s_H - s_L} \leq \gamma_y < \frac{-2c_Lq_H + (c_H + q_H)q_L}{-q_Ls_H + 2q_Hs_L}$. Solving for optimal y_H , we find $y_H = \frac{2c_Lq_H + 4q_H^2 - 3q_Hq_L + c_H(-4q_H + q_L) - 4\gamma_yq_Hs_H + \gamma_yq_Ls_H + 2\gamma_yq_Hs_L}{4q_H(2q_H - q_L)}$ that requires

$$\gamma_y < \frac{q_H(2c_L + 4q_H - 3q_L) + c_H(-4q_H + q_L)}{4q_Hs_H - q_Ls_H - 2q_Hs_L} (< \frac{-2c_Lq_H + (c_H + q_H)q_L}{-q_Ls_H + 2q_Hs_L}).$$

Then, $x_L > x_L^M \Rightarrow K^{X*} > K_M$ when $\frac{(q_H - c_H) - (q_L - c_L)}{s_H - s_L} < \gamma_y$, which completes the proof for Proposition 3a.

- (b) We solve the game for the low-quality focused entrant. Plugging in the capacity equality and price functions in objective function (3), and solving for the first-order conditions, we find that $y_L(x_L, x_H) = \frac{c_L + \gamma_y s_L + q_L(-1 + x_H + x_L)}{2q_L}$. Plugging this back into objective function (2), and solving for the first-order conditions, we find three alternative solutions for the leader depending on the Lagrangian multipliers as follows:

Solution 1: Offer both products: $x_H = \frac{-c_H + c_L + q_H - q_L - \gamma_y s_H + \gamma_y s_L}{2(q_H - q_L)}$ and $x_L = -\frac{c_Lq_H - c_Hq_L - \gamma_yq_Ls_H + \gamma_yq_Hs_L}{2q_Hq_L - 2q_L^2}$.

Solution 2: Offer only H when $\frac{c_Lq_H - c_Hq_L - \gamma_yq_Ls_H + \gamma_yq_Hs_L}{2q_H - q_L} \geq 0$: $x_L = 0$ and $x_H = \frac{-2c_H + c_L + 2q_H - q_L - 2\gamma_y s_H + \gamma_y s_L}{4q_H - 2q_L}$.

Solution 3: Offer only L when $c_H - c_L - q_H + q_L + \gamma_y s_H - \gamma_y s_L \geq 0$: $x_H = 0$ and $x_L = -\frac{c_L - q_L + \gamma_y s_L}{2q_L}$.

We find that Solution 2 is the only feasible optimal alternative when $c_H/q_H \leq c_L/q_L$, $\frac{q_L - c_L}{s_L} > \frac{q_H - c_H}{s_H}$, and $\gamma_y \leq \frac{c_Lq_H - c_Hq_L}{q_Ls_H - q_Hs_L}$.

Solving for optimal y_L , we find $y_L = \frac{c_L + \gamma_y s_L + (q_L(-2c_H + c_L - 2q_H + q_L - 2\gamma_y s_H + \gamma_y s_L))/(4q_H - 2q_L)}{2q_L}$ that requires

$$\gamma_y > \frac{4c_Lq_H - 2c_Hq_L - c_Lq_L - 2q_Hq_L + q_L^2}{2q_Ls_H - 4q_Hs_L + q_Ls_L}.$$

Then, $x_H > x_H^M \Rightarrow K^{X*} > K_M$ when $\gamma_y < \frac{c_Lq_H - c_Hq_L}{q_Ls_H - q_Hs_L}$ which completes the proof. □

Proof of Proposition 4: Following the proof of Proposition 3b, when $c_H/q_H \leq c_L/q_L$, $\frac{q_L - c_L}{s_L} > \frac{q_H - c_H}{s_H}$, and $\gamma_y \leq \frac{c_Lq_H - c_Hq_L}{q_Ls_H - q_Hs_L}$, optimal $y_L = \frac{c_L + \gamma_y s_L + (q_L(-2c_H + c_L - 2q_H + q_L - 2\gamma_y s_H + \gamma_y s_L))/(4q_H - 2q_L)}{2q_L}$

that requires $\gamma_y > \frac{4c_Lq_H - 2c_Hq_L - c_Lq_L - 2q_Hq_L + q_L^2}{2q_Ls_H - 4q_Hs_L + q_Ls_L}$. Then, we take the derivative of $K_Y = s_L y_L$ with respect to capacity cost parameter γ_y : $\partial_{\gamma_y} K_Y = -\frac{s_L(s_L + (q_L(-2s_H + s_L))/(4q_H - 2q_L))}{2q_L}$ which is positive when $\frac{s_H}{s_L} > \frac{4q_H - q_L}{2q_L}$. □

Proof of Lemma 2: Following the price–demand equations (1) and the fact that $x_i = D_i(p_H, p_L)$, these equations can be solved for prices as follows: $p_H = q_H(1 - x_H) - q_Lx_L$ and $p_L = q_L(1 - x_L - x_H)$. Note that the capacity constraint is always satisfied by equality at the optimal solution ($s(q_H)x_H + s(q_L)x_L = K_X$). We plug this fact, price, and cost functions into the objective function. The resulting problem is as follows: $\pi^M = x_H(q_H(1 - x_H) - q_Lx_L - \alpha q_H^2) + x_L(q_L(1 - x_H - x_L) -$

$\alpha q_L^2) - \gamma_x(q_H^2 x_H + q_L^2 x_L)$. Hessian $(\pi^M) = \{-2q_H, -2q_L\}, \{-2q_L, -2q_L\}$. Because this is negative definite, we use first-order conditions to find the optimal $\{x_H(q_H, q_L), x_L(q_H, q_L)\}$ which yields: $x_L = (1/2)q_H(\gamma_x + \alpha)$, $x_H = 1/2(1 - \gamma_x q_H - \gamma_x q_L - q_H \alpha - q_L \alpha)$. We continue by plugging the optimal quantities back in the profit function and solving for the optimal quality levels.

$\pi^M = (1/4)q_H(1 + (\gamma_x + \alpha)(q_H^2(\gamma_x + \alpha) - q_L^2(\gamma_x + \alpha) + q_H(-2 + q_L(\gamma_x + \alpha))))$. This function is strictly concave in q_L since $\partial_{q_L, q_L} \pi^M = -(1/2)q_H(\gamma_x + \alpha)^2$. Then, we use first-order conditions to find the optimal $q_L = q_H/2$. We continue by plugging it back in the profit function and solving for the optimal q_H .

$\pi^M = (1/4)q_H(1 + 1/4q_H(\gamma_x + \alpha)(-8 + 5\gamma_x q_H + 5q_H \alpha))$. $\partial_{q_H, q_H} \pi^M = 1/8(\gamma_x + \alpha)(-8 + 15q_H(\gamma_x + \alpha))$. Then, we need to double check the critical point to find out the optimal solution. Solving for the first-order conditions, we find two critical points ($q_H = 2/(5(\gamma_x + \alpha))$, $q_H = 2/(3(\gamma_x + \alpha))$) in addition to the boundary conditions ($q_L \geq q$ and $q_H \leq \bar{q}$).

For the conditions presented in Lemma 2, we find that $q_H^M = 2/(5(\gamma_x + \alpha))$ is the unique global maximum point. Then, $q_L^M = \frac{1}{5(\alpha + \gamma_x)}$, $x_H^M = \frac{1}{5}$, $x_L^M = \frac{1}{5}$, $p_H^M = \frac{7}{25(\alpha + \gamma_x)}$, $p_L^M = \frac{3}{25(\alpha + \gamma_x)}$, $\pi^M = \frac{1}{25(\alpha + \gamma_x)}$. □

Proof of Proposition 5: We solve the game and identify leader’s and follower’s actions for the parametric sets presented in the proposition. Then, we compare them with the monopolist’s solution as presented in Lemma 2.

Plugging in $(s(q_H)y_H = K_Y)$ and the price–demand equations (1) into Equation (7), and solving for the first-order conditions, we find that $y_H(x_L, x_H) = \frac{-q_L x_L + q_H(-1 + x_H + q_H(\gamma_y + \alpha))}{2q_H}$. We plug $y_H(x_L, x_H)$, price, and cost functions in Equation (6). The resulting problem is as follows: $\pi^{UH} = (\frac{1}{2q_H})(q_L^2 x_L^2 + q_H^3 x_H(\gamma_y - \alpha) - q_H q_L x_L(-1 + 2x_H + 2x_L + 2q_L \alpha) + q_H^2(x_H - x_H^2 + q_L x_L(\gamma_y + \alpha)))$. We solve the problem sequentially: first we find optimal quantities taking quality levels q_H and q_L as given. Then, we plug them in and solve for the optimal quality levels. First, we find the Hessian $(\pi^{UH}) = \{-q_H, -q_L\}, \{-q_L, (-4q_H q_L + 2q_L^2)/(2q_H)\}$. It is negative definite if $q_H^* > 3/2 q_L^*$. We will verify this condition later.

We continue by solving first-order conditions to find the optimal $\{x_H(q_H, q_L), x_L(q_H, q_L)\}$ which yields $x_L = (q_H \alpha)/2$, $x_H = 1/2(1 + \gamma_y q_H - q_H \alpha - q_L \alpha)$. We continue by plugging the optimal quantities back into the profit function and solving for the optimal quality levels.

$\pi^{UH} = (1/8) q_H((1 + \gamma_y q_H)^2 - 2q_H(1 + \gamma_y q_H)\alpha + (q_H^2 + 2q_H q_L - 2q_L^2)\alpha^2)$. This function is strictly concave in q_L since $\partial_{q_L, q_L} \pi^{UH} = -(q_H \alpha^2)/2$. Then, we use first-order conditions to find the optimal $q_L = q_H/2$. Here we verify the condition on the Hessian since $q_H = 2q_L > 3/2 q_L$. We continue by plugging it back into the profit function and solving for the optimal q_H .

$\pi^{UH} = (1/8) q_H((1 + \gamma_y q_H)^2 - 2q_H(1 + \gamma_y q_H)\alpha + (3q_H^2 \alpha^2)/2)$. We next show that this function is everywhere increasing (derivative with respect to q_H is positive for all q_H) for $\gamma_y \geq \alpha$ which proves that the maximum is achieved at the boundary condition.

$\partial_{q_H} \pi^{UH} = (1/16) (2 + q_H(6\gamma_y^2 q_H + \gamma_y(8 - 12q_H\alpha) + \alpha(-8 + 9q_H\alpha))) = 2 + 8q_H(\gamma_y - \alpha) + q_H^2(6(\gamma_y - \alpha)^2 + 3\alpha^2) > 0$ for all q_H when $\gamma_y \geq \alpha$ as presented in the proposition. Hence, $q_H^* = \bar{q} > q_H^M$.

Solving all functions backwards and comparing the results with Lemma 2, we find $q_L^* = \bar{q}/2 (> q_L^M)$, $x_H^* = (1/4) (2 + \bar{q}(2\gamma_y - 3\alpha)) (> x_H^M$ only if $\alpha < \frac{2\gamma_y}{3} + \frac{2}{5\bar{q}}$), $x_L^* = (\bar{q}\alpha)/2 (> x_L^M)$. □

Proof of Proposition 6: We solve the game and identify the leader’s and follower’s actions for both types of entrants as presented in the proposition. Then we compare them with the monopolist’s solution as presented in Lemma 2.

- (a) We solve the game for the high-quality focused entrant. Plugging ($s(q_H)y_H = K_Y$) and the price functions from Equation (1) into Equation (7), and solving for the first-order conditions, we find that $y_H(x_L, x_H) = -((q_L x_L + q_H(-1 + x_H + q_H(\gamma_y + \alpha)))/(2q_H))$. We plug in $y_H(x_L, x_H)$, capacity ($s(q_H)x_H + s(q_L)x_L = K_X$), price, and cost functions in Equation (8). The resulting problem is as follows: $\pi^{KH} = (1/2q_H)(q_L^2 x_L^2 - q_H^3 x_H(\gamma_y + \alpha) - q_H q_L x_L(-1 + 2\gamma_y q_L + 2x_H + 2x_L + 2q_L\alpha) + q_H^2(x_H - x_H^2 + q_L x_L(\gamma_y + \alpha)))$.

We solve the problem sequentially: first we find optimal quantities taking quality levels q_H and q_L as given. Then, we plug them in and solve for the optimal quality levels. Hessian (π^{KH}) = $\{-q_H, -q_L\}, \{-q_L, (-4q_H q_L + 2q_L^2)/(2q_H)\}$. It is negative definite if $q_H^* > 3/2 q_L^*$. We will verify this condition later.

We continue by solving first-order conditions to find the optimal $\{x_H(q_H, q_L), x_L(q_H, q_L)\}$ which yields $x_L = 1/2q_H(\gamma_y + \alpha)$, $x_H = 1/2(1 - (q_H + q_L)(\gamma_y + \alpha))$. We continue by plugging the optimal quantities back into the profit function and solving for the optimal quality levels.

$$\pi^{KH} = (1/8)q_H(1 - 2q_H(\gamma_y + \alpha) + (q_H^2 + 2q_H q_L - 2q_L^2)(\gamma_y + \alpha)^2).$$

This function is strictly concave in q_L since $\partial_{q_L, q_L} \pi^{KH} = -((q_H(\gamma_y + \alpha)^2)/2)$. Then, we use first-order conditions to find the optimal $q_L = q_H/2$. Here we verify the condition on the Hessian since $q_H = 2q_L > 3/2 q_L$. We continue by plugging it back in the profit function and solving for the optimal q_H .

$\pi^{KH} = (1/16)(q_H(2 - 4q_H(\gamma_y + \alpha) + 3q_H^2(\gamma_y + \alpha)^2))$. We next show that this function is everywhere increasing (derivative with respect to q_H is positive for all q_H) which proves that the maximum is achieved at the boundary condition.

$\partial_{q_H} \pi^{KH} = (1/16)(2 + q_H(\gamma_y + \alpha)(-8 + 9q_H(\gamma_y + \alpha)))$. Replace $a = q_H(\gamma_y + \alpha)$, then proving $F(a) = 2 + a(9a - 8)$ is positive for all a is enough. Since $F(a)$ is a strictly convex function, we can find that minimum value is achieved at $a = 4/9$ with $F(4/9) = 2/9 > 0$. Then $F(a) > 0$ for all a , which shows that $\partial_{q_H} \pi^{KH} > 0$ for all q_H , which shows that π^{KH} is everywhere increasing. Hence, $q_H^* = \bar{q} > q_H^M$ when $\gamma_y + \alpha > \frac{2}{5\bar{q}}$.

Solving all functions backwards and comparing the results with Lemma 2 when $\gamma_y + \alpha > \frac{2}{5\bar{q}}$, we find $q_L^* = \bar{q}/2 (> q_L^M)$, $x_H^* = (1/4) (2 - 3\bar{q}(\gamma_y + \alpha)) (< x_H^M)$, $x_L^* = (1/2) (\bar{q}(\gamma_y + \alpha)) (> x_L^M)$.

- (b) We solve the problem for low-quality focused entrant. Plugging $(s(q_L)y_L = K_Y)$ and price functions from equation (1) into Equation (7), and solving for the first-order conditions, we find that $y_L(x_L, x_H) = (1/2)(1 - \gamma_y q_L - x_H - x_L - q_L \alpha)$. We plug in $y_L(x_L, x_H)$, capacity $(s(q_H)x_H + s(q_L)x_L = K_X)$, price, and cost functions in Equation (8). The resulting problem is as follows: $\pi^{KL} = (1/2)(-2q_H(-1 + x_H)x_H - 2q_H^2 x_H(\gamma_y + \alpha) + q_L(x_H^2 + x_H(-1 + \gamma_y q_L - 2x_L + q_L \alpha) - x_L(-1 + \gamma_y q_L + x_L + q_L \alpha)))$. We solve the problem sequentially: first we find optimal quantities taking quality levels q_H and q_L as given. Then, we plug them in and solve for the optimal quality levels. Hessian $(\pi^{KL}) = \{1/2(-4q_H + 2q_L), -q_L\}, \{-q_L, -q_L\}$. It is negative definite.

We continue by solving first-order conditions to find the optimal $\{x_H(q_H, q_L), x_L(q_H, q_L)\}$ which yields $x_L = 1/2q_H(\gamma_y + \alpha)$, $x_H = 1/2(1 - (q_H + q_L)(\gamma_y + \alpha))$. We continue by plugging the optimal quantities back in the profit function and solving for the optimal quality levels. $\pi^{KL} = (1/8)(2q_H^3(\gamma_y + \alpha)]^2 + 2q_H^2(\gamma_y + \alpha)(-2 + q_L(\gamma_y + \alpha)) - q_L(-1 + q_L(\gamma_y + \alpha))^2 + q_H(2 - 2q_L^2(\gamma_y + \alpha)^2)$. We check the concavity of π^{KL} with respect to x_H . $\partial_{q_H, q_H} \pi^{KL} = (1/2)(\gamma_y + \alpha)(-2 + 3q_H(\gamma_y + \alpha) + q_L(\gamma_y + \alpha)) < 0$ only if $(\gamma_y + \alpha) < 2/(3q_H^* + q_L^*)$. We will verify this condition later. Next, we use first-order conditions to find the optimal $q_H = (1 + q_L(\gamma_y + \alpha))/(3(\gamma_y + \alpha))$ which ensures the feasibility ($q_H > q_L$) and concavity conditions are satisfied. We continue by plugging it back into the profit function and solving for the optimal q_L .

$\pi^{KL} = (1/(216(\gamma_y + \alpha)))(8 - 21q_L(\gamma_y + \alpha) + 42q_L^2(\gamma_y + \alpha)^2 - 37q_L^3(\gamma_y + \alpha)^3)$. We next show that this function is everywhere decreasing (derivative with respect to q_L is negative for all q_L) which proves that the maximum is achieved at the boundary condition.

$\partial_{q_L} \pi^{KL} = (1/72)(-7 + q_L(\gamma_y + \alpha)(28 - 37q_L(\gamma_y + \alpha)))$. Replace $a = q_L(\gamma_y + \alpha)$, then proving $F(a) = -7 + a(28 - 37a)$ is negative for all a is enough. Since $F(a)$ is a strictly concave function, we can find that maximum value is achieved at $a = 14/37$ with $F(14/37) = -63/37 < 0$. Then $F(a) < 0$ for all a , which shows that $\partial_{q_L} \pi^{KL} < 0$ for all q_L , which shows that π^{KL} is everywhere decreasing. Hence, $q_L^* = \underline{q} < q_L^M$ when $\gamma_y + \alpha < \frac{1}{5\bar{q}}$.

Solving all functions backwards and comparing the results with Lemma 2 when $\gamma_y + \alpha < \frac{1}{5\bar{q}}$, we find $q_H^* = \frac{1+q(\gamma_y+\alpha)}{3(\gamma_y+\alpha)} < q_H^M$, $x_H^* = (1/3) (1 - 2\underline{q}(\gamma_y + \alpha)) > x_H^M$, $x_L^* = (1/6) (1 + \underline{q}(\gamma_y + \alpha)) < x_L^M$. □

Proof of Corollary 1: We compare the solutions for the Stackelberg game as presented in the proof of Proposition 6 and the monopolist's solution as presented in Lemma 2. Remember that when $\gamma_y + \alpha > \frac{2}{5\bar{q}}$, $p_H^* = (1/4)\bar{q}(1 + 3\bar{q}(\gamma_y + \alpha))$; $p_L^* = (1/8)\bar{q}(1 + 2\bar{q}(\gamma_y + \alpha))$ and when $\frac{2}{5\bar{q}} < \gamma_y + \alpha < \frac{1}{5\bar{q}}$, $p_H^M = 7/(25(\gamma_y + \alpha))$; $p_L^M = 3/(25(\gamma_y + \alpha))$.

We first test whether $p_H^* > p_H^M$. It requires $1/4\bar{q}(1 + 3\bar{q}(\gamma_y + \alpha)) > 7/(25(\gamma_y + \alpha))$. When we replace $\bar{q}(\gamma_y + \alpha) = \beta$, the condition reduces to $3\beta^2 + \beta - (28/25) > 0 \Rightarrow (\beta + (4/5))(\beta - (7/15)) > 0 \Rightarrow \beta > 7/15 \Rightarrow (\gamma_y + \alpha) > 7/(15\bar{q})$. Combining all conditions on γ_y and α yields the condition in the corollary.

Next we test whether $p_L^* > p_L^M$. It requires $(1/8)\bar{q}(1 + 2\bar{q}(\gamma_y + \alpha)) > 3/(25(\gamma_y + \alpha))$. When we replace $\bar{q}(\gamma_y + \alpha) = \beta$, the condition reduces to $2\beta^2 + \beta - (24/25) > 0 \Rightarrow (\beta + (1/20)(5 + \sqrt{217}))(\beta - (1/20)(\sqrt{217} - 5)) > 0 \Rightarrow \beta > (1/20)(\sqrt{217} - 5) \Rightarrow (c_k + \alpha) > (1/20\bar{q})(\sqrt{217} - 5)$. Combining all conditions on γ_y and α yields the condition in the corollary. \square

Proof of Corollary 2: We compare the solutions for the Stackelberg game as presented in the proof of Proposition 6 and the monopolist's solution as presented in Lemma 2. Remember that when $\gamma_y + \alpha > \frac{2}{5\bar{q}}$, $x_H^* = (1/4)(2 - 3\bar{q}(\gamma_y + \alpha))$, $x_L^* = (1/2)(\bar{q}(\gamma_y + \alpha))$, and $y_H^* = 1/4(1 - \bar{q}(\gamma_y + \alpha))$. Remember that when $\frac{2}{5\bar{q}} < \gamma_y + \alpha < \frac{1}{5\bar{q}}$, $x_H^M = \frac{1}{5}$ and $x_L^M = \frac{1}{5}$.

$x_H^* + y_H^* + x_L^* < x_H^M + x_L^M$ when $10\bar{q}(\gamma_y + \alpha) > 7$. Combining all conditions on γ_y and α yields the condition in the corollary. \square