# Equilibrium Informativeness in Veto Games\*

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#### **Abstract**

In a veto game a biased expert recommends an action that an uninformed decision maker can accept or reject for an outside option. The arrangement is ubiquitous in political institutions, corporations, and consumer markets but has seen limited use in applications due to a poor understanding of the equilibrium set and an ensuing debate over selection. We develop a simple method to construct every veto equilibrium and identify the most informative equilibrium in a setting that spans prior work. We show that Krishna and Morgan's (2001) equilibrium is maximally informative and strengthen Dessein's (2002) comparison of full delegation and veto. In an application we study the relationship between a patient and a doctor with a financial incentive to overtreat, and in contrast with existing literature show that the doctor's bias harms the patient *both* through excessive treatment and information loss, that the latter can be substantial, and that insurance benefits both parties by improving communication.

Keywords: veto, cheap talk, physician-induced demand, non-compliance JEL Classification: D82, I10

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## 1 Introduction

When an expert with an upward bias advises an uninformed decision maker, the effect of the bias depends on how the decision maker incorporates the advice. If the decision maker follows the advice exactly (i.e., full delegation), the bias induces higher actions than the decision maker prefers but allows all the expert's information to be transmitted. On the other hand, if the decision maker draws inference from the advice and then acts optimally (i.e., cheap talk), Crawford and Sobel (1982) (hereafter CS) shows that on average the actions are those preferred by the decision maker but communication is noisy.

Alternatively, the decision maker may draw inference but is restricted in his actions, as is the case in veto games in which the decision maker's only options are to accept the expert's proposal or to reject in lieu of an exogenous outside option. The veto terminology owes to a literature examining the "closed rule" governing legislative committees (Gilligan and Krehbiel, 1987, hereafter GK; Krishna and Morgan, 2001, hereafter KM), in which the full legislature may either accept a committee's bill without amendments or reject it entirely. However the veto arrangement is exceedingly common across a variety of settings beyond legislatures. In elections, constituents vote to approve or disapprove particular policies such as the issuance of municipal debt, but cannot for instance write in an alternative monetary amount for the bond; the board of directors of a corporation often holds the power to approve or disapprove proposals but not to unilaterally enact a proposal of their own; and in healthcare, a patient may either accept or reject a doctor's orders but may not self-prescribe treatment.<sup>1</sup>

Aside from describing a ubiquitous institutional arrangement, the veto model also captures both of the known inefficiencies associated with biased experts. That is, in contrast to the benchmark CS and full delegation models, veto equilibria identified in the literature exhibit both noisy communication and excessive actions (see Figure 1). Yet despite its appeal the veto approach has seen limited application in economic modeling due to an ongoing debate, starting with GK and KM, about equilibrium selection. In sender-receiver games it is common to focus on the most informative equilibrium, for instance in the canonical CS environment the equilibrium set is well-understood and the most informative equilibrium is easily identified.<sup>2</sup> By contrast, the veto equilibrium set is yet to be characterized. While KM's equilibrium is the most informative found to date,

<sup>&</sup>lt;sup>1</sup>For additional examples of the veto arrangement see Marino (2007) and Mylovanov (2008).

<sup>&</sup>lt;sup>2</sup>Chen et al. (2008) propose instead a perturbation-based criterion and show that it too selects the most informative CS equilibrium.

Figure 1: Modes of Communication

Communication	Excessive	Noisy
Protocol	Action	Communication
Cheap Talk	No	Yes
Full Delegation	Yes	No
Veto Game	Yes	Yes

they note that "it appears to be difficult to characterize explicitly the most informative [veto] equilibrium," (p. 445) and this has remained an open question.

In this paper we describe the full set of veto equilibria. Doing so directly is difficult, however we employ Holmstrom's (1984) observation that, by the revelation principle, a sender that best responds in a veto game also best responds by truthfully revealing her type in a suitably constructed reporting game. Similar to Melumad and Shibano (1991), we show that every equilibrium is described by an interval partition of states into pooling and separating intervals, and that for each partition there is at most one action profile consistent with equilibrium, up to an initial condition. Then, we show that every equilibrium partition can be constructed by a simple method, starting with the initial condition and sequentially choosing interval endpoints which satisfy constraints that depend only on the previous endpoint. We demonstrate that a similar method can be used to construct cheap-talk equilibria as in CS, but by comparison the method for veto games involves many more free parameters and, in contrast to CS, can generate an equilibrium set that is infinite.

We then use insights from constructing the set to identify the most informative equilibrium in the commonly studied setting with a uniform distribution and quadratic loss functions. The key idea is that the status quo is attractive for some states but not others. The sender perfectly informs the receiver for states in which the receiver prefers the sender's optimal action to the status quo, but for states in which the status quo is viable, the sender communicates strategically by pooling and this results in an information loss. Using our equilibrium set characterization, we describe the minimal region of states over which pooling must occur, and then use this result to identify the most informative equilibrium for a wide range of values of the status quo.

To demonstrate the practical implications of our results, we apply the veto model to study the interaction between an informed doctor with a financial incentive to overtreat and an uninformed patient with the option to reject treatment. By focusing on the strategic behavior of both sides of the market we address a divide in the health literature where "papers on insurance and demand tend to view the supply side as competitive and accommodating; papers on supply tend to view patients as passively accepting provider recommendations" (McGuire, 2012; p.339). In contrast to the workhorse physician-induced demand framework (i.e. full delegation) in which the doctor's bias leads only to overtreatment, in the veto model the patient is additionally harmed by the information loss stemming from the doctor's strategic misdiagnosis to forestall rejection. We show the utility loss from the latter communication effect can add substantially to the effect of a higher expected treatment, comprising up to roughly one quarter of the patient's welfare loss, and thus empirical studies focusing only on treatment level underestimate the welfare effect of financial incentives.

We also examine the role of health insurance in which the patient's ex-post cost of treatment is reduced by paying an upfront actuarially fair premium. While a standard approach predicts extra treatment due to the patient's moral hazard, in the veto equilibrium the treatment level is determined solely by the doctor's bias, and thus the sole effect of insurance is to align doctor and patient preferences and improve communication, leading to a Pareto improvement. In this way, even risk neutral patients find insurance valuable as a means to reduce the doctor's incentive to strategically misdiagnose.

While our main goal is to enable comparative statics in veto environments, by characterizing the equilibrium set and identifying its most informative element we also shed light on several issues in the existing theoretical literature. For example, GK and KM compare the informativeness of veto versus pure cheap talk and obtain conflicting results by focusing on different veto equilibria. The veto equilibrium of GK involves simpler strategies while the veto equilibrium of KM is more complex but also more informative, and the debate about the appropriate selection criterion remains active, with recent experimental work by Battaglini et. al. (2016) providing some support for GK's approach in a veto setting with multiple senders. However the inherent difficulty in this type of analysis is that comparing the GK and KM equilibria omits other equilibria that, for any particular selection criterion, may outperform both. Since we describe the full equilibrium set we help address this concern.

We demonstrate that the KM equilibrium is not simply more informative than GK's equilibrium and all cheap talk equilibria, but is in fact *the* most informative veto equilibrium. This in turn enables comparisons of outcomes other than informativeness,

such as sender and receiver payoffs, between veto games and other mechanisms. In particular, we strengthen Dessein's (2002) (hereafter DE) result that the receiver prefers full delegation to a veto game. DE uses the KM equilibrium since it "is thus far the most [informative] equilibrium identified in the literature" (p. 828), and we prove that there are no more informative equilibria. Our methodology can also directly address the apparent discrepancy of DE's result with Marino's (2007), who shows instead that the receiver can prefer veto to full delegation. While Marino suggested this discrepancy is due to the status quo being low in his model and high in DE, we show that in fact it is the distributional assumptions that are crucial, and that even at DE's high status quo his result is easily reversed when the distribution is appropriately adjusted.

Veto games have also been studied from a mechanism design perspective. Whereas the aforementioned papers make pairwise comparisons, this literature focuses on how a receiver would optimally design the communication protocol if he had the power to do so. In particular, an equilibrium of any communication game corresponds to an equilibrium of a constrained delegation game in which the receiver commits to accept a set of actions and then the sender chooses from among this set for each state. Alonso and Matouschek (2008) describes the optimal such delegation set for the receiver, and then Mylovanov (2008) shows that this constrained delegation outcome can be implemented without full commitment but rather in an equilibrium of a veto game when the status quo is appropriately chosen. By contrast we focus on a positive analysis rather than a normative one. We consider settings, such as a patient visiting a doctor or a voter participating in a referendum, in which the receiver does not have the power to design the mechanism. We model veto games and the status quo as the given institutional arrangement, identify the most informative equilibrium, and study its properties and the effects of policy changes.

Our doctor-patient application is related to the work of Pitchik and Schotter (1987) and De Jaegher and Jegers (2001) who analyze a cheap talk game in which a doctor makes a recommendation to a patient who can obtain any available treatment. In these models the doctor prefers the maximal action regardless of the state, thus departing from the CS paradigm and resulting in substantially different equilibria and comparative statics.

The rest of the paper is organized as follows. In Section 2 we introduce the general model and describe the equilibrium set. In Section 3 we then restrict attention to the uniform quadratic setting and identify the most informative equilibrium across a range of values of the status quo, including both for intermediate values as considered by GK and KM, and for lower values which we use in our application to the healthcare setting

## 2 General Model and Equilibrium Set

An uncertain state  $\theta$  with distribution F and support [0,1] is observed by a sender but not by a receiver. The sender proposes an action and the receiver then chooses either to accept the proposed action or to reject in lieu of an exogenous outside option  $\hat{a}$ . Formally, let  $m:[0,1] \to \mathbb{R}$  and  $x:\mathbb{R} \to \{0,1\}$  denote the pure strategies<sup>3</sup> of the sender and receiver and let

$$a(\theta) = \begin{cases} m(\theta) & \text{if } x(m(\theta)) = 1\\ \hat{a} & \text{if } x(m(\theta)) = 0 \end{cases}$$

denote the action induced by strategies m and x in any state  $\theta$ . We refer to the function  $a(\theta)$  as the action profile, with  $A \equiv \{a(\theta)|\theta \in [0,1]\}$  the set of accepted actions, and for each accepted action define  $\tau(a) \equiv \{\theta \mid a(\theta) = a\}$  as the set of states for which each action is induced. Also since  $a(\theta)$  may have jump discontinuities it is useful to define  $a^-(\theta) \equiv \lim_{\delta \to 0} a(\theta - \delta)$  and  $a^+(\theta) \equiv \lim_{\delta \to 0} a(\theta + \delta)$ .

Sender and receiver preferences  $u_s(a,\theta)$  and  $u_r(a,\theta)$  are single-peaked at  $a_s(\theta)$  and  $a_r(\theta)$ , with  $a_s(\theta) > a_r(\theta)$  and  $a_s$  and  $a_r$  continuous and strictly increasing for all states  $\theta$ . We make the following additional assumptions. First, for the veto to be viable in that it is rationalizable for the receiver to reject at least one of the sender's preferred actions, it is sufficient to assume that  $\hat{a} \in (a_r(0), a_r(1))$ . Also, we define the state  $\hat{\theta} \equiv a_s^{-1}(\hat{a})$  for which the sender's preferred action is the veto, and assume that  $\lim_{\theta \to -\infty} a_s(\theta) < \hat{a}$  so that  $\hat{\theta}$  is well-defined. Finally, for the sender's single-peaked preferences we will need to identify the action on the other side of the peak that gives the same utility. Specifically for  $a \le a_s(\theta)$  let  $a_s^+(a,\theta)$  be the action above  $a_s(\theta)$  that gives the sender the same payoff as a, and conversely if  $a \ge a_s(\theta)$  let  $a_s^-(a,\theta)$  be the action below  $a_s(\theta)$  that gives the sender the same payoff as a. In order for these to be well-defined we assume  $\lim_{a \to \infty} u_s(a,\theta) = \lim_{a \to -\infty} u_s(a,\theta) = -\infty$ .

<sup>&</sup>lt;sup>3</sup>CS shows there are only pure strategy equilibria in cheap talk games, but the argument does not immediately translate to veto games. While we acknowledge it is possible that mixed strategy equilibria exist, we do not study them here.

<sup>&</sup>lt;sup>4</sup>To accommodate the fact that  $\hat{\theta}$  may be negative, and thus outside the support of F, let  $u_i(a, \theta)$  be defined for  $\theta \in \mathbb{R}$  for i = r, s. Thus,  $\hat{\theta}$  can take a value with zero likelihood.

We call  $a(\theta)$  an equilibrium action profile if there exist strategies and beliefs that constitute a perfect Bayesian equilibrium and induce  $a(\theta)$ . In particular the profile must (i) satisfy the informed sender's incentive constraint, (ii) generate beliefs for each accepted action that make accepting a best response for the receiver, and (iii) accommodate beliefs for rejected actions off the equilibrium path that make rejecting a best response for the receiver. We look for the set of profiles  $a(\theta)$  that satisfy these three conditions.

**Lemma 1** The profile  $a(\theta)$  is an equilibrium profile if and only if

- (i)  $a(\theta)$  is weakly increasing;
- (ii) if  $a(\theta)$  is strictly increasing and continuous on an open interval, then  $a(\theta) = a_s(\theta)$  on this interval;
- (iii) if  $a(\theta)$  is discontinuous at  $\theta$  then  $u_s(a^-(\theta), \theta) = u_s(a^+(\theta), \theta)$  and

$$a(\theta') = \begin{cases} a^{-}(\theta) & \text{if } \theta' \in \left[a_s^{-1}(a^{-}(\theta)), \theta\right) \\ a^{+}(\theta) & \text{if } \theta' \in \left(\theta, a_s^{-1}(a^{+}(\theta))\right) \end{cases};$$

- (iv) if  $\hat{a} \ge a_s(0)$  then  $a(\hat{\theta}) = \hat{a}$ , else if  $\hat{a} < a_s(0)$  then  $a(0) \in [\hat{a}, a_s^+(\hat{a}, 0)]$ ;
- (v)  $E_{\tau(a)}[u_r(a,\theta) u_r(\hat{a},\theta)] \ge 0$  for all  $a \in A$ .

**Proof** The sender's incentive constraint requires that in each state her action is her most preferred in  $A \cup \{\hat{a}\}$ . A necessary condition is that she cannot improve by mimicking marginally higher or lower types, thus  $u_s(a^-(\theta),\theta) = u_s(a^+(\theta),\theta)$  for all  $\theta$ . The set of profiles  $a(\theta)$  that satisfy this local indifference condition is described in Proposition 1 of Melumad and Shibano (1991), and is restated here in conditions (i)-(iii). Condition (iv) accounts for the fact that in contrast to delegation games, in veto games a sender may not only induce the actions taken by other types but also the veto action. Thus, if the veto is his preferred action the sender must induce it in equilibrium so that  $a(\hat{\theta}) = \hat{a}$  when  $\hat{\theta} \in [0,1]$ . The case  $\hat{\theta} > 1 \leftrightarrow \hat{a} > a_s(1)$  is not possible since  $\hat{a} < a_r(1) < a_s(1)$  by assumption. When  $\hat{\theta} < 0 \leftrightarrow \hat{a} < a_s(0)$  the veto is lower than the preferred action of even the lowest sender type, that type must induce an action that makes him at least as well off as the veto. Conditions (i)-(iv) thus cover the sender's incentive constraint, and condition (v) describes the receiver's incentive constraint, ensuring that all accepted actions are best responses.

Finally for sufficiency it must also be shown that the sender would not deviate to some  $a' \notin A \cup \{\hat{a}\}$ . This is easily accomplished by having the receiver reject every off the path a' and hold off the path beliefs (which are unrestricted) that  $\theta = a_r^{-1}(\hat{a})$  for all off the path recommendations.

An implication of Lemma 1 is that all equilibrium profiles partition the set of states into separating and pooling intervals, in which the sender obtains her preferred action in all separating intervals, all pooling intervals include the state for which the pooling action is optimal for the sender, and upward jumps in a make the sender at that state indifferent between the lower and higher action. We now use these observations to demonstrate that every equilibrium can be constructed by a simple method of sequentially choosing the endpoints of intervals that partition the set of states [0,1]. Proposition 1 describes the equilibrium set when  $\hat{a} < a_s(0)$ , in which case we begin by specifying  $a_0 \equiv a(0)$  and intervals are constructed starting with  $[0,\theta_1)$  and moving to the right. Proposition 2 then describes the equilibrium set when  $\hat{a} \geq a_s(0)$ , where condition (iv) of Lemma 1 implies that  $a(\hat{\theta}) = \hat{a}$ . In this case, which includes that considered in GK and KM, the interval  $(\theta_0, \theta_1)$  contains  $\hat{\theta}$  and is interior, thus intervals are constructed sequentially both moving to the right and to the left.

First we define two objects of interest. For a pooling interval with action  $a > \hat{a}$ , the receiver's posterior must be sufficiently high in order to accept. Thus, for an interval with left endpoint  $\theta_i$  we let  $\bar{\theta}(a,\theta_i)$  be the smallest right endpoint for which the receiver accepts (i.e., for which  $E_{(\theta_i,\bar{\theta})}[u_r(a,\theta) - u_r(\hat{a},\theta)] \ge 0$ ) and be equal to one if no such endpoint exists. Similarly for a pooling interval with action  $a < \hat{a}$  and right endpoint  $\theta_{i+1}$ , let  $\underline{\theta}(a,\theta_{i+1})$  be the largest left endpoint for which the receiver accepts and be equal to zero if no such endpoint exists.

**Proposition 1** If  $\hat{a} < a_s(0)$ , then  $a(\theta)$  is an equilibrium profile if and only if there exists a strictly increasing (possibly infinite) sequence of interval endpoints  $\{\theta_i\}_{i\in\mathbb{N}}$  with  $\theta_0 = 0$  and  $\sup\{\theta_i\}_{i\in\mathbb{N}} = 1$  such that

- (i) on the interval  $[\theta_0, \theta_1)$  there is a pooling action  $a_0 \in [\hat{a}, a_s^+(\hat{a}, 0)]$ ,
- (ii) for any subsequent interval  $(\theta_i, \theta_{i+1})$ 
  - if  $a^-(\theta_i) < a_s(\theta_i)$  then  $(\theta_i, \theta_{i+1})$  is pooling on action  $a_s^+(a^-(\theta_i), \theta_i)$ ,
  - if  $a^-(\theta_i) = a_s(\theta_i)$  then  $(\theta_i, \theta_{i+1})$  is separating on  $a_s(\theta)$  whenever  $(\theta_{i-1}, \theta_i)$  was pooling and  $(\theta_i, \theta_{i+1})$  is pooling on  $a_s(\theta_i)$  whenever  $(\theta_{i-1}, \theta_i)$  was separating,

(iii) in any interval  $(\theta_i, \theta_{i+1})$  with a pooling action  $a_i \neq \hat{a}$ ,  $\theta_{i+1} \geq \max (a_s^{-1}(a_i), \bar{\theta}(a_i, \theta_i))$ .

**Proof** First we demonstrate that conditions (i), (ii), and (iii) are necessary. For condition (i), if  $a_0 < \hat{a}$  or  $a_0 > a_s^+(\hat{a}, 0)$ , then at  $\theta = 0$  the sender strictly improves by deviating to  $\hat{a}$ . Also the interval  $[0, \theta_1)$  is pooling, else  $a_0 = a_s(0)$  by Lemma 1(ii), the receiver perfectly infers  $\theta = 0$  and rejects because  $a_r(0) \le \hat{a} < a_s(0)$ . For condition (ii), if the preceding interval ends on an action below the sender's preferred action, then by Lemma 1(iii) the next interval starts with an upward jump above the sender's preferred action, and thus cannot be separating. If the preceding interval ends with the sender's preferred action then by the local indifference condition the next interval also begins with the sender's preferred action. Then, either the preceding interval is pooling and the interval that follows it separating or vice versa, otherwise the boundary  $\theta_i$  between the two intervals is not a true boundary since both the pooling or the separating intervals simply continue. Condition (iii) follows from conditions (iii) and (v) of Lemma 1.

For sufficiency, observe that conditions (i) and (ii) of Lemma 1 are implied by condition (ii) of the Proposition, condition (iii) of the Lemma is implied by conditions (ii) and (iii) of the Proposition, condition (iv) of the Lemma is implied by condition (i) of the Proposition, and condition (v) of the Lemma is implied by condition (iii) of the Proposition.  $\blacksquare$ 

It can be seen that Proposition 1 implies an method by which any equilibrium must be constructed when  $\hat{a} < a_s(0)$ . First, from condition (i) choose initial action  $a_0 \in [\hat{a}, a_s^+(\hat{a}, 0)]$ and label the initial interval  $[\theta_0, \theta_1)$  pooling. Next choose the right endpoint  $\theta_1$  of the initial interval, observing the constraint in condition (iii) if  $a_0 > \hat{a}$ . At this point  $a(\theta)$ is specified for  $\theta \in [0, \theta_1)$ . If  $\theta_1 = 1$  then we have found an equilibrium described by  $(a_0, \theta_1 = 1)$ , else we proceed to the next interval  $(\theta_1, \theta_2)$ . Having fixed  $a_0$  and  $\theta_1$ , condition (ii) then determines whether  $(\theta_1, \theta_2)$  is pooling or separating and in turn the resulting actions. The next choice is endpoint  $\theta_2$ , which must be large enough to satisfy the constraint in condition (iii) if the interval is pooling, and depends only on  $\theta_1$  and  $a_1$ . If  $\theta_2 = 1$  then we have found an equilibrium described by  $(a_0, \theta_1, \theta_2 = 1)$ , else we proceed to the next interval ( $\theta_2$ ,  $\theta_3$ ). We then continue to repeat the previous step until an endpoint  $\theta_i = 1$  is selected or approached in the limit. By making a sequence of choices  $(a_0, \theta_1, ..., 1)$  according to this process, the equilibrium action profile is uniquely identified at all states except for a measure zero set of boundary points  $\theta_i$ , at which either  $a^-(\theta_i)$  or  $a^+(\theta_i)$  is consistent with equilibrium. The set of all equilibria is thus contained by the set of sequences  $(a_0, \theta_1, ..., 1)$  that can be constructed using the above process.<sup>5</sup>

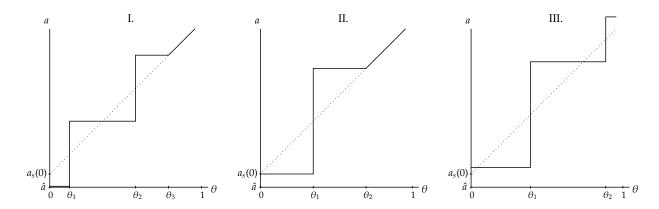


Figure 2: Examples of equilibrium action profiles when  $\hat{a} < a_s(0)$ . The solid line is action profile  $a(\theta)$ , the dotted line is the preferred action of the sender  $a_s(\theta)$ , and the horizontal axis corresponds to the status quo  $\hat{a}$ .

Figure 2 depicts several equilibrium profiles, each satisfying the sender's local indifference condition by having separating intervals coincide with  $a_s(\theta)$  and by reflecting the action profile above  $a_s(\theta)$  at the boundaries of two pooling intervals. Note also that all interior pooling intervals intersect  $a_s(\theta)$ , i.e. they include the sender for whom that action is optimal. Each of the three profiles starts with a pooling interval  $[0, \theta_1)$  according to Proposition 1. The smallest allowable length of the first pooling interval depends on the starting value  $a_0$ , and as  $a_0$  increases in going from I to III, so does the value of  $\theta_1$ . The second interval in each example is also pooling, and this will be shown to always be the case, however the third interval may be pooling as in I and III or separating as in II. In addition, note that while the interval  $[0, \theta_1)$  is smaller in I than in II, the total number of states covered by pooling intervals is larger in I than in II, and we will demonstrate that sometimes an equilibrium with the structure of II may be more informative than that of type I. That is, constructing an equilibrium by first choosing the smallest allowable  $\theta_1$ , and then the smallest allowable  $\theta_2$ , and so forth does not necessarily produce the most informative equilibrium.

Next we describe the equilibrium set when  $\hat{a} \ge a_s(0)$ , the construction of which is very similar to Proposition 1 except that the interval  $[\theta_0, \theta_1)$  is now interior.

<sup>&</sup>lt;sup>5</sup>This method may also include some profiles that are not equilibria. For example, if the right endpoint necessary to satisfy condition (iii) for the right-most interval is greater than one, then the constructed partition is not an equilibrium. The most informative equilibrium we identify later in fact outperforms this larger set of profiles.

**Proposition 2** If  $\hat{a} \ge a_s(0)$ , then  $a(\theta)$  is an equilibrium profile if and only if there exists a strictly increasing (possibly infinite) sequence of interval endpoints  $\{\theta_i\}_{i\in\mathbb{Z}}$  with  $0 = \inf\{\theta_i\}_{i\in\mathbb{Z}} \le \theta_0 \le \sup\{\theta_i\}_{i\in\mathbb{Z}} = 1$  such that

- i. the interval  $[\theta_0, \theta_1)$  contains the state  $\hat{\theta} \equiv a_s^{-1}(\hat{a})$  and has pooling action  $a_0 = \hat{a}$ ;
- ii. for any subsequent  $(i \ge 1)$  interval  $(\theta_i, \theta_{i+1})$ 
  - if  $a^-(\theta_i) < a_s(\theta_i)$  then  $(\theta_i, \theta_{i+1})$  is pooling on action  $a_s^+(a^-(\theta_i), \theta_i)$ ,
  - if  $a^-(\theta_i) = a_s(\theta_i)$  then  $(\theta_i, \theta_{i+1})$  is separating on  $a_s(\theta)$  whenever  $(\theta_{i-1}, \theta_i)$  was pooling and  $(\theta_i, \theta_{i+1})$  is pooling on  $a_s(\theta_i)$  whenever  $(\theta_{i-1}, \theta_i)$  was separating

and for any preceding  $(i \le -1)$  interval  $(\theta_i, \theta_{i+1})$ 

- if  $a^+(\theta_{i+1}) > a_s(\theta_{i+1})$  then  $(\theta_i, \theta_{i+1})$  is pooling on action  $a_s^-(a^+(\theta_{i+1}), \theta_{i+1})$ ,
- if  $a^+(\theta_{i+1}) = a_s(\theta_{i+1})$  then  $(\theta_i, \theta_{i+1})$  is separating on  $a_s(\theta)$  whenever  $(\theta_{i+1}, \theta_{i+2})$  was pooling and  $(\theta_i, \theta_{i+1})$  is pooling on  $a_s(\theta_{i+1})$  whenever  $(\theta_{i+1}, \theta_{i+2})$  was separating
- *iii.* in any interval  $(\theta_i, \theta_{i+1})$  with a pooling action  $a_i \neq \hat{a}$ ,

$$\theta_{i+1} \ge \max \left( a_s^{-1}(a_i), \ \bar{\theta}(a_i, \theta_i) \right) \quad \text{if } i \ge 1, \text{ and}$$

$$\theta_i \le \min \left( a_s^{-1}(a_i), \underline{\theta}(a_i, \theta_{i+1}) \right) \quad \text{if } i \le -1.$$

**Proof** That  $a(\hat{\theta}) = \hat{a}$  in condition (i) is given by condition (iv) of Lemma 1. There must also be pooling to the right of  $\hat{\theta}$ , otherwise  $a(\theta) = a_s(\theta)$  at slightly higher states and then by continuity  $a_r(\theta) < \hat{a} < a(\theta)$  so the receiver would reject. Conditions (ii) and (iii) are necessary by the same arguments as in Proposition 1. The argument for sufficiency is also similar. The receiver's beliefs are unrestricted for an off the path action a' and again if the posterior is  $\theta = a_r^{-1}(\hat{a})$  then the receiver rationally rejects. For the sender, any off the path message induces the status quo action  $\hat{a}$ , which is now on the equilibrium path and thus already demonstrated to not constitute an improvement.

Largely the same procedure applies here as in Proposition 1, in that interval endpoints are chosen sequentially, working outward from the initial interval and depend solely on the last chosen endpoint. Importantly, decisions about interval endpoints to the right and left of the initial interval  $[\theta_0, \theta_1)$  are independent of one another. That is, suppose interval  $(\theta_2, \theta_3)$  is pooling on  $a_2$ . By condition (iii), in order to induce the receiver to accept, the right endpoint  $\theta_3 \ge \bar{\theta}(a_2, \theta_2)$ . Thus, it cannot be guaranteed that  $\theta_3$  is sufficiently large until  $\theta_2$  is chosen and similarly  $\theta_2$  cannot be fixed until  $\theta_1$  is chosen. However this is where the chain ends, because on the interval  $[\theta_0, \theta_1)$  the action  $\hat{a}$  cannot be rejected, and thus any  $\theta_1 > \hat{\theta}$  is sufficiently large regardless of the value of  $\theta_0$ . Thus, if the sequences  $(0, ..., \theta_{-1}, \theta_0)$  and  $(\theta_1, \theta_2, ..., 1)$  are chosen independently by the procedure above, they are mutually consistent and constitute an equilibrium.

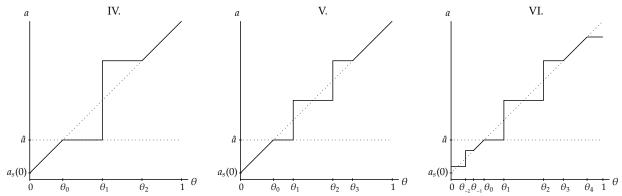


Figure 3: Examples of equilibrium action profiles when  $\hat{a} \ge a_s(0)$ . The solid line is action profile  $a(\theta)$ , the dotted line is the preferred action of the sender  $a_s(\theta)$ , and the horizontal axis corresponds to the status quo action  $\hat{a}$ .

In Figure 3, in each of the three examples  $\theta_0 = \hat{\theta}$  and all equilibria are anchored at  $a(\hat{\theta}) = \hat{a}$  according to Proposition 2. The equilibrium in IV is of the type studied by GK and the equilibrium in V is of the type studied by KM. These two equilibria start and end with separating intervals but differ in the way pooling intervals are constructed for intermediate states. It is visually apparent that V constitutes a finer partition of the states than does IV, and potentially these two equilibria may be easily compared. However, there exist many other equilibria, such as that in VI, which look quite different from IV and V, having many more intervals and not necessarily centered around the sender's preferred actions. Comparisons to these type of equilibria appear less straightforward.

## 2.1 Size of Equilibrium Set versus Crawford and Sobel (1982)

A common feature of sender-receiver games is that there may be more than one equilibrium, for example it is well known that there are multiple equilibria in the canonical CS cheap talk framework. However, while there are finitely many equilibria in CS, there is potentially a continuum of equilibria in veto games. To get a sense for the difference, first consider constructing the equilibrium set in pure cheap talk in the following manner. First, choose  $\theta_1 > 0$ , the right endpoint of initial pooling interval  $[0, \theta_1)$ . Given

 $\theta_1$ , the only consistent pooling action on this interval is  $a_0 = \arg\max_a E_{[0,\theta_1)}[u_r(a,\theta)]$  since the receiver's actions are unconstrained, and since preferences are single-peaked and monotonic  $a_0$  is unique. Next, with  $a_0$  and  $\theta_1$  fixed, the interval  $(\theta_1, \theta_2)$  must be pooling on an action  $a_1$  that makes the sender at  $\theta_1$  indifferent, thus the next action is  $a_1 = a_s^+(a_0, \theta_1)$ . Furthermore, for the receiver to take action  $a_1$  on interval  $(\theta_1, \theta_2)$ , it must be that  $a_1 = \arg\max_a E_{(\theta_1,\theta_2)}[u_r(a,\theta)]$ , again guaranteed to be unique. Thus, for a given  $\theta_1$ , there is at most one endpoint  $\theta_2$  that satisfies both of these conditions. Extending this logic, there is at most one value of  $\theta_3$  consistent with  $\theta_2$  and so forth, and thus there is at most a single equilibrium for any chosen  $\theta_1$ . And as established in CS, there is only a finite set of values of  $\theta_1$  that is consistent with an equilibrium.

By contrast, in veto games for a chosen endpoint  $\theta_1$  there is potentially a continuum of consistent values for  $\theta_2$ , and for the pair  $(\theta_1, \theta_2)$  there is potentially a continuum of consistent values for  $\theta_3$ , et cetera. And in fact, in the uniform-quadratic setting that we consider shortly, it can be shown that there is also a continuum of interval endpoints  $\theta_1$  consistent with equilibrium. Thus, while under CS the equilibrium set is finite and can be indexed by a single parameter  $\theta_1$ , the set of veto equilibria cannot be indexed in this way and can be uncountably infinite.

## 3 Informativeness

The multiplicity of equilibria makes the veto model difficult for use in applications and is at the root of a long-standing debate, starting with GK and KM, about which equilibria should be studied. Since our goal is to make the veto model amenable to comparative statics, welfare, and other similar analyses we now focus on equilibrium selection. At the current level of generality the exercise is intractable and therefore we restrict attention to the constant bias uniform-quadratic specification which is commonly used in the literature (e.g., CS, GK, and KM). Payoffs to the sender and receiver are

$$u_s = -(a - (\theta + b_s))^2$$
 and  $u_r = -(a - (\theta - b_r))^2$ ,  $b_s, b_r \ge 0$ , (1)

with preferred actions  $a_s(\theta) = \theta + b_s$  and  $a_r(\theta) = \theta - b_r$ . In a cheap talk environment such as CS,  $b_r$  is typically normalized to zero since the only relevant quantity is  $b_s + b_r$ . This assumption is restrictive here due to the existence of a status quo action  $\hat{a}$ , which interacts differently with  $b_s$  and  $b_r$ . For example, we will show that the average equilibrium action strictly increases in  $b_s$  but is constant in  $b_r$ , and use this fact to study the role of health

insurance in our doctor-patient application. Finally, we assume  $b_s + b_r \le \frac{1}{4}$ , for otherwise the biases are so high that any equilibrium must pool on a single action.

Our aim is to perform a positive analysis of the veto game, taking the institutional arrangement as given and studying equilibrium properties. We use equilibrium informativeness as a selection criterion, defined as follows:

**Definition 1** Let  $z(\theta) \equiv a(\theta) - \theta$  denote the realized bias at state  $\theta$ . Then the informativeness of an equilibrium profile is  $-Var(z(\theta))$ .

We will refer to both  $a(\cdot)$  and  $z(\cdot)$  as the equilibrium profile, depending on context. To motivate the informativeness criterion, note that with some standard algebraic manipulation the ex-ante preferences of the sender and receiver can be expressed as

$$E[u_s] = -Var(z) - (E[z] - b_s)^2$$
 and  $E[u_r] = -Var(z) - (E[z] + b_r)^2$ , (2)

decomposing into preferences over informativeness and expected bias. Previous work on sender-receiver games has focused on informativeness because of its welfare properties. For example, in CS all equilibria have the same expected bias, and therefore the most informative equilibrium is also Pareto dominant. In veto games, the expected bias can vary across equilibria but it is still the case that an increase in informativeness is a Pareto improvement. Veto models of GK, KM, DE, and Marino (2007) have thus focused on the most informative equilibrium, relying on the Pareto argument and other justifications.<sup>6</sup> We also study informativeness for its Pareto properties, and in addition in order to be able to address some of the discrepancies in the aforementioned literature. However, we acknowledge that because veto equilibria are not Pareto ranked, other criteria may also be of interest depending on the setting.<sup>7</sup>

The informativeness of an equilibrium is closely related to how many states are covered by pooling rather than separating intervals. For instance, the full delegation profile  $a(\theta) = \theta + b_s$  is maximally informative with Var(z) = 0, however it is not an equilibrium since by Propositions 1 and 2 every equilibrium starts with a pooling interval. In fact, as we now demonstrate in the following two lemmas, there is a minimal region of states around where the status quo is viable in which only pooling intervals can be supported. As we will argue, finding the most informative equilibrium

<sup>&</sup>lt;sup>6</sup>GK states that "the acquisition of information is the raison d'être for legislative committees."

<sup>&</sup>lt;sup>7</sup>For example, it is common in the mechanism design literature to focus on the equilibrium that maximizes the designer's payoff, whether the designer is the receiver (e.g., Holmstrom, 1984; Alonso and Matouschek, 2008) or the sender (Kamenica and Gentzkow, 2011). Our equilibrium characterization results in Propositions 1 and 2 can be useful for these lines of inquiry.

can reduce to identifying the most informationally efficient way to cover this pooling region.

Recall that  $\hat{\theta} \equiv \hat{a} - b_s$  is the (possibly negative) state at which the sender's preferred action is the status quo.

**Lemma 2** If  $\hat{a} < b_s$ , in any equilibrium there is a minimal region T with no separating intervals, such that

- (i) if  $a(0) = \hat{a}$  then  $T = (0, \hat{\theta} + 4(b_s + b_r))$  is covered by at most three pooling intervals;
- (ii) if  $a(0) > \hat{a}$  then  $T = (0, 2\hat{\theta} + 4(b_s + b_r))$  is covered by at most two pooling intervals.

**Lemma 3** If  $\hat{a} \ge b_s$ , in any equilibrium there is a minimal region  $T = (\hat{\theta}, \hat{\theta} + 4(b_s + b_r))$  with no separating intervals, and it is covered by at most three pooling intervals.

The proofs, both of which can be found in the Appendix, are constructive and follow directly from the equilibrium structure in Propositions 1 and 2. To get some intuition for why a minimal pooling region must exist, suppose there is a separating state  $\theta \in (\hat{\theta}, 1]$  so that  $a(\theta) = \theta + b_s$ . The receiver's best response is to accept only if her preferred action is closer to the prescribed action than to the status quo:  $\theta - b_r \ge \frac{1}{2}(\hat{a} + (\theta + b_s)) \Rightarrow \theta \ge \hat{\theta} + 2(b_s + b_r)$ . The pooling region must thus extend at least to  $\hat{\theta} + 2(b_s + b_r)$  to satisfy the receiver's incentives, and in fact it must extend even farther to also satisfy the sender's incentives. That is, if type  $\hat{\theta} + 2(b_s + b_r)$  induced her preferred action then lower types would mimic, reducing the receiver's posterior and causing to receiver to reject. Accounting for this incentive to mimic then further extends the boundary.

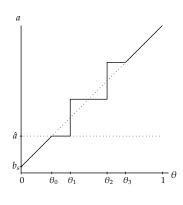
We now describe the most informative equilibrium, initially focusing on an intermediate range of statuses quo as in GK, KM, and DE (ensuing Proposition 3) and then a low range of statuses quo corresponding to Marino (2007) (ensuing Proposition 4). The low range is of particular interest for the application to the doctor patient relationship in the following section. In an earlier version of the paper<sup>8</sup> we also describe the most informative equilibrium for the remaining high statuses quo, including the equilibrium studied by Mylovanov (2008) using similar techniques. For convenience, we include a graphical depiction of the action profile  $a(\theta)$  and corresponding bias profile  $z(\theta) = a(\theta) - \theta$  in each proposition.

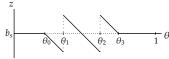
<sup>&</sup>lt;sup>8</sup>Available at https://papers.ssrn.com/sol3/papers.cfm?abstract\_id=2359136.

#### 3.0.1 Intermediate status quo

**Proposition 3** *If*  $\hat{a} \in [b_s, 1-3b_s-4b_r]$  *then the unique most informative equilibrium is separating for high and low states and has three pooling intervals for intermediate states, with partition*  $(0, \theta_0, \theta_1, \theta_2, \theta_3, 1) = (0, \hat{a} - b_s, \hat{a} + b_r, \hat{a} + 2b_s + 3b_r, \hat{a} + 3b_s + 4b_r, 1)$  *and actions:* 

$$a^{*}(\theta) = \begin{cases} \theta + b_{s} & \text{if } \theta \in [0, \theta_{0}) \\ \hat{a} & \text{if } \theta \in (\theta_{0}, \theta_{1}) \\ \hat{a} + 2(b_{s} + b_{r}) & \text{if } \theta \in (\theta_{1}, \theta_{2}) \\ \hat{a} + 4(b_{s} + b_{r}) & \text{if } \theta \in (\theta_{2}, \theta_{3}) \\ \theta + b_{s} & \text{if } \theta \in (\theta_{3}, 1] \end{cases}$$





with  $E[z^*] = b_s$ , and  $Var[z^*] = \frac{4}{3}(b_s + b_r)^3$ .

**Proof Sketch** Recall that  $\hat{\theta} \equiv \hat{a} - b_s$ , define  $\bar{\theta} \equiv \hat{a} + 3b_s + 4b_r$ , and observe that the proposed profile  $z^*$  is separating for low states  $[0, \hat{\theta})$  and high states  $(\bar{\theta}, 1]$ . Since in the separating intervals  $z^*(\theta) = b_s$  it follows that

$$Var(z^{*}) = \int_{0}^{1} (z^{*}(\theta) - b_{s})^{2} d\theta = \int_{\hat{\theta}}^{\bar{\theta}} (z^{*}(\theta) - b_{s})^{2} d\theta,$$

and therefore it suffices to show that  $z^*$  outperforms any other profile only over  $(\hat{\theta}, \bar{\theta})$ .

Meanwhile, in this region every candidate equilibrium profile  $z(\theta)$  must satisfy certain properties. In particular, the profile begins with  $z(\hat{\theta}) = b_s$  (Proposition 2(i)), is covered by no separating intervals and at most three pooling intervals (Lemma 3), and has at most one state  $t \in (\hat{\theta}, \bar{\theta})$  for which  $z(t) = b_s$  (proof of Lemma 3). With this in mind, consider the candidate  $z(\theta)$  with three pooling intervals depicted in Figure 4. Over the region  $(\hat{\theta}, t)$  the realizations of z are uniformly distributed on an interval of length  $t - \hat{\theta}$ , centered at  $b_s$ , while over the region  $(t, \bar{\theta})$  the realizations of z are uniformly distributed on an interval of length  $\bar{\theta} - t$ , also centered at  $b_s$ . As depicted in the figure, increasing t to bring the lengths of these two intervals closer in size creates a mean-preserving contraction in the distribution of z over  $(\hat{\theta}, \bar{\theta})$  and thus strictly increases

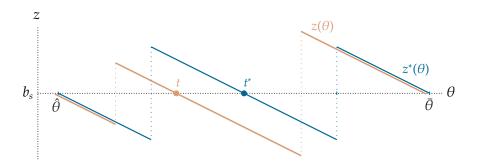


Figure 4: A graph of biases for candidate profile  $z(\theta)$  (in tan) and proposed profile  $z^*(\theta)$  (in blue) over the range  $(\hat{\theta}, \bar{\theta})$ . Since  $z^*$  is symmetric around  $t^*$  it is shown to be a mean-preserving contraction of z over this range, and is thus more informative.

informativeness. The optimal value is in fact  $t = \frac{1}{2}(\hat{\theta} + \bar{\theta})$ , which corresponds to the proposed profile  $z^*$ .

This argument demonstrates that  $z^*$  outperforms any z with three pooling intervals in  $(\hat{\theta}, \bar{\theta})$  which has  $z(\bar{\theta}) = b_s$ . To complete the proof, we must also consider candidate profiles for which  $z(\bar{\theta}) \neq b_s$  and profiles that cover  $(\hat{\theta}, \bar{\theta})$  with one or two pooling intervals. In these situations, we first construct an auxiliary profile  $\tilde{z}$  that has lower variance than the candidate z and a mean  $E[\tilde{z} \mid \theta \in (\hat{\theta}, \bar{\theta})] = b_s$ , after which we make a similar mean preserving spread argument with respect to  $\tilde{z}$ . The full proof can be found in the Appendix.

The range of intermediate values of the status quo  $\hat{a}$  in Proposition 3 includes the range considered by GK, KM, and DE, and the most informative equilibrium  $z^*$  matches the equilibrium in KM's Proposition 8, which has thus far been used as a means to compare across different communication games. By identifying this equilibrium as the *most* informative equilibrium in veto games we aid these comparisons. For instance, while KM indirectly prove that the most informative veto equilibrium is more informative than that under cheap talk, without identifying this equilibrium they do not know its other properties and thus cannot compare sender or receiver payoffs across mechanisms, which the present work now allows. In addition, DE finds that full delegation is better for the receiver than KM's particular veto equilibrium, which DE uses

<sup>&</sup>lt;sup>9</sup>GK focuses instead on a different equilibrium with two pooling intervals as depicted in Figure 3(IV), and a broader debate about equilibrium selection in this environment has persisted (Krehbiel, 2001; Battaglini, 2016), in part centered on assumptions for beliefs off the equilibrium path. While we do not make explicit restrictions in the present work, it should be noted that the most informative equilibrium is supported by beliefs surviving several reasonable refinements, including monotonicity and the Cho and Kreps (1987) intuitive criterion.

since it "is thus far the most [informative] equilibrium identified in the literature" (p. 828). We strengthen this result by showing there are in fact no other more informative equilibria.<sup>10</sup>

The equilibrium identified in Proposition 3 can also shed light on the discrepancy between DE's result that full delegation outperforms the veto arrangement for the receiver and Marino's (2007) finding that the opposite can hold. While it has been suggested in the literature (Marino, 2007; Mylovanov, 2008) that this finding results from the fact that in Marino the status quo is favorable for veto games and in DE it is not, we show it is the distributional assumptions that are crucial. In particular, using a similar argument to Marino (2007), suppose that  $\theta$  is uniformly distributed on each interval identified in Proposition 3, but that most of the mass is on the first pooling interval  $(\hat{a}-b_s, \hat{a}+b_r)$ . It is easy to verify that  $a^*$  still constitutes an equilibrium, since the informed sender's incentive constraint does not depend on distributional assumptions while the receiver's constraints in each interval depend only on conditional expectations, which have remained unchanged. Since veto outperforms full delegation for the receiver on  $(\hat{a}-b_s, \hat{a}+b_r)$ , it is the preferred mechanism whenever this interval has sufficient probability mass. Therefore even in the intermediate range of statuses quo studied in DE, veto can outcome perform full delegation with suitable distributional assumptions.

#### 3.0.2 Low status quo

Next we explore values of the status quo that are strictly below the preferred action of even the lowest type of sender ( $\hat{a} < b_s$ ). A low status quo was considered by Marino (2007) and is plausible in many situations. For instance a car owner's outside option may be to perform no repairs while a mechanic, even when observing no necessity for repair, may prefer the owner to pay for a small level of service. Similarly, a patient's outside option may be no treatment at all while a doctor, even if observing a fully healthy patient, may prefer the patient undertake some further costly diagnostics.

In this parameter range much of the logic of equilibrium construction remains the same as previously. The major departure is that now there is no sender type whose

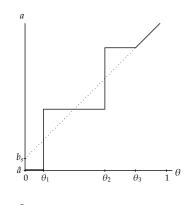
<sup>&</sup>lt;sup>10</sup>While the receiver strictly prefers full delegation to the most informative veto equilibrium  $a^*$  in Proposition 3, it should be noted that  $a^*$  is not the best veto equilibrium for the receiver. For example, consider the profile a' which mimics  $a^*$  for states  $(0, 1 - 2(b_s + b_r))$  and pools on action  $1 - b_s - 2b_r$  for states  $(1 - 2(b_s + b_r), 1)$ . The profile a' is less informative than  $a^*$  but has a lower expected action, and it is easily verified that a' constitutes an equilibrium and gives the receiver a strictly higher payoff than  $a^*$ . In fact, the receiver's payoff under a' is exactly equal to his payoff under full delegation.

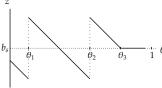
preferred action is the status quo, thus as opposed to the previous case in which  $a(\hat{\theta}) = \hat{a}$ , there is no longer a fixed initial condition from which to start equilibrium construction. Instead, the initial interval is now  $[0, \theta_1)$  with associated pooling action  $a_0$  and we examine two families of equilibria, those in which  $a_0 = \hat{a}$  and those in which  $a_0 > \hat{a}$ . We show there is a threshold for  $\hat{a}$  above which the most informative equilibrium is of the former type and below which it is of the latter type.

**Proposition 4** *If*  $\hat{a} \in [-b_r, b_s]$ , then there exists an  $\underline{a} < b_s$  such that the unique most informative equilibrium is separating for high states and,

(i) if  $\hat{a} \in (\underline{a}, b_s]$ , has three pooling intervals for all lower states with partition  $(0, \theta_1, \theta_2, \theta_3, 1) = (0, \hat{a} + b_r, \hat{a} + 2b_s + 3b_r, a + 3b_s + 4b_r, 1)$  and actions:

$$a_1^*(\theta) = \begin{cases} \hat{a} & \text{if } \theta \in (0, \theta_1) \\ \hat{a} + 2(b_s + b_r) & \text{if } \theta \in (\theta_1, \theta_2) \\ \hat{a} + 4(b_s + b_r) & \text{if } \theta \in (\theta_2, \theta_3) \\ \theta + b_s & \text{if } \theta \in (\theta_3, 1) \end{cases}$$



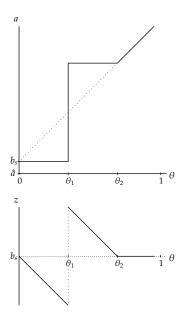


with 
$$E[z_1^*] = b_s + \frac{1}{2}(b_s - \hat{a})^2$$
 and  $Var(z_1^*) = \frac{4}{3}(b_s + b_r)^3 - \frac{1}{3}(b_s - \hat{a})^3 - \frac{1}{4}(b_s - \hat{a})^4$ ;

(ii) if  $\hat{a} \in [-b_r, \underline{a})$ , has two pooling intervals for all lower states with partition  $(0, \theta_1, \theta_2, 1) = (0, b_s + 2b_r + \hat{a}, 2b_s + 4b_r + 2\hat{a}, 1)$  and actions:

<sup>&</sup>lt;sup>11</sup>Recall from Proposition 2 that there is no equilibrium with  $a_0 < \hat{a}$  because the sender of type 0 would deviate to induce  $\hat{a}$ .

$$a_2^*(\theta) = \begin{cases} b_s & \text{if } \theta \in (0, \theta_1) \\ 3b_s + 4b_r + 2\hat{a} & \text{if } \theta \in (\theta_1, \theta_2) \\ \theta + b_s & \text{if } \theta \in (\theta_2, 1) \end{cases}$$



with 
$$E[z_2^*] = b_s$$
 and  $Var(z_2^*) = \frac{2}{3}(b_s + 2b_r + \hat{a})^3$ .

**Proof Sketch** For part (i), in contrast to Proposition 3 separating intervals now contribute to the variance of  $z_1^*$  since in the proposed equilibrium  $E[z_1^*] > b_s$ , and a different technique is used for the proof. We first fix a candidate profile z and construct an auxiliary profile  $\tilde{z}$  that mimics z for low states and uniformly shifts z up for high states, so that  $E[\tilde{z}] = E[z_1^*]$ . We show that  $\tilde{z}$  is more informative than z, and that  $\tilde{z}$  is a mean preserving spread of  $z_1^*$ . For part (ii) since  $E[z_2^*] = b_s$  we follow the same approach as in Proposition 3, showing that it is sufficient to focus on states  $(0, \theta_2)$  and that  $z_2^*$  is best on this range. Finally, a direct comparison of the expressions for informativeness in parts (i) and (ii) implies the existence of a threshold value for  $\hat{a}$  that determines which of the two equilibria is most informative overall.

## 4 An application: the doctor-patient interaction

In this section we apply the veto model to the doctor-patient relationship, in which information asymmetry problems may naturally arise. Estimates of avoidable clinical care are up to \$700 billion annually in the United States, <sup>12</sup> and one often-cited cause is the

<sup>&</sup>lt;sup>12</sup>Berwick and Hackbarth (2012) and Institute of Medicine (2010), "The healthcare imperative: lowering costs and improving outcomes: workshop series summary". Washington, D.C.: The National Academies Press.

financial incentive of doctors to prescribe more treatment than is medically prudent.<sup>13</sup> In addition, patients cannot self-prescribe and thus have limited authority: they cannot take any treatment they wish as in CS's model of cheap talk or commit ex-ante to a set of treatments they are willing to accept, as in optimal delegation (Holmstrom, 1984; Alonso and Matouschek, 2008). Instead a patient is typically only able to accept the doctor's prescription or reject it outright.<sup>14</sup> Therefore the institutional details of the doctor-patient relationship closely match the veto model.

Furthermore, the veto model predicts outcomes of the doctor-patient relationship better than competing models in the literature, which often focus on only one side of the market while treating the other as a passive participant (McGuire, 2012). For example, a popular approach in the health economics literature is the physician-induced demand framework, in which the patient in effect delegates decision-making to the doctor. Following this approach, communication is unmodeled and a patient's willingness to accept a proposed treatment does not depend on the doctor's financial incentives. Consequently the model predicts overtreatment but cannot explain non-compliance. Alternatively, a pure communication model like CS in which the receiver is unrestricted in his choice of actions allows for non-compliance but does not predict overtreatment. By contrast the veto arrangement results in both.

We now slightly modify the model from Section 2 to accommodate the ensuing comparative statics. An informed doctor observes the patient's health state  $\theta \sim U[0,1]$ , with higher values corresponding to more serious illnesses, and prescribes a treatment  $m \geq 0$ . The patient can either accept m or reject in favor of the status quo  $\hat{a} = 0$ , which represents no treatment.<sup>16</sup> The payoffs for the doctor (sender) and patient (receiver) respectively are

$$u_s = -\frac{1}{2}(\theta - a)^2 + b_s a$$
 and  $u_r = -\frac{1}{2}(\theta - a)^2 - b_r a$ ,  $b_s, b_r \ge 0$ . (3)

The first term in each payoff reflects the medical prudence of a treatment, on which both the doctor and patient agree. The second term captures financial incentives, whereby

<sup>&</sup>lt;sup>13</sup>Emanuel and Fuchs (2008).

<sup>&</sup>lt;sup>14</sup>Brownlie et al. (2008) and Chen and Vargas-Bustamante (2013) show evidence that patients do reject treatment when they lack trust in their doctor, in particular when suspecting a financial motive.

<sup>&</sup>lt;sup>15</sup>The physician-induced demand hypothesis posits that the doctor can change the patient's preferences for treatment and thus induce any prescribed action to be accepted. See Evans (1974), McGuire (2000), and Chandra et al. (2012).

<sup>&</sup>lt;sup>16</sup>The implicit assumption is that there is a positive likelihood that the patient is healthy enough that no treatment is her preferred option.

the patient pays  $b_r$  per unit of treatment while the doctor earns a constant profit margin  $b_s$ , with  $b_s \leq b_r$  to reflect the fact that treatment provision is costly on the margin. During any particular interaction the doctor and patient take these rates of payment as given, possibly being negotiated in advance between the provider and an (unmodeled) insurer, and no other transfers are permitted. The utility functions above allow for a natural interpretation of the bias parameters and are affine transformations of the original specification, thus sharing maximizers.<sup>17</sup> The status quo  $\hat{a} = 0$  captures the fact that the patient cannot self-prescribe, and corresponds to the patient's lowest feasible preferred treatment.<sup>18</sup>

The current status quo is in the range covered by Proposition 4, which identifies a profile  $a_1$  with three pooling intervals and a profile  $a_2$  with two pooling intervals as the two candidates for the most informative equilibrium. To identify which of the two is more informative, we plug in  $\hat{a} = 0$  and obtain

$$Var_1 - Var_2 = \frac{4}{3}(b_s + b_r)^3 - \frac{1}{3}b_s^3 - \frac{1}{4}b_s^4 - \frac{2}{3}(b_s + 2b_r)^3 = -4b_r^3 - 4b_r^2b_s + \frac{1}{3}b_s^3 - \frac{1}{4}b_s^4 < 0,$$

where the inequality follows from  $b_s \leq b_r$ . Thus the equilibrium with three pooling intervals is optimal and is described by profile

$$a(\theta) = \begin{cases} 0 & \text{if } \theta \in (0, b_r) \\ 2(b_s + b_r) & \text{if } \theta \in (b_r, 2b_s + 3b_r) \\ 4(b_s + b_r) & \text{if } \theta \in (2b_s + 3b_r, 3b_s + 4b_r) \end{cases}.$$

$$\theta + b_s & \text{if } \theta \in (3b_s + 4b_r, 1)$$

The patient is untreated for mild illnesses and accepts a single intermediate treatment  $a = 2(b_s + b_r)$  and all high treatments starting with  $a \ge 4(b_s + b_r)$ . When the doctor's preferred treatment falls in the range rejected by the patient, he must choose either to overstate beyond his financial incentive or to understate and induce a smaller treatment. For example, when the doctor prescribes treatment  $a = 2(b_s + b_r)$ , for illnesses  $\theta \in (b_r, b_s + 2b_r)$  it is higher than what he prefers and for illnesses  $\theta \in (b_s + 2b_r, 2b_s + 3b_r)$  it is lower than what he prefers. Because of the patient's ability to reject, the doctor is thus prevented from customizing his diagnosis.

<sup>&</sup>lt;sup>17</sup>Specifically,  $u_s^{(3)}(a, \theta) = \frac{1}{2}u_s^{(1)}(a, \theta) + \frac{1}{2}b_s(b_s + 2\theta)$  and  $u_r^{(3)}(a, \theta) = \frac{1}{2}u_r^{(1)}(a, \theta) - \frac{1}{2}b_r(-b_r + 2\theta)$ , with  $u^{(1)}$  and  $u^{(3)}$  corresponding to the utility functions in lines (1) and (3), respectively.

<sup>&</sup>lt;sup>18</sup>The patient would actually prefer negative treatments for states  $(0, b_r)$ , but we assume the patient cannot be a net seller of treatment.

## 4.1 Equilibrium Properties

One objective of this application is to compare the predictions made by the present approach which explicitly models communication with the more common physician-induced demand framework, which models the treatment decision as fully delegated to the doctor. For this comparison, we decompose the effect of the divergence in financial incentives into the effect on treatment level and treatment informativeness. Let  $\bar{a}(\theta) \equiv \theta + E[z(\theta)]$  be a fully informative profile with the same expected treatment level as the equilibrium profile. Also, let  $U_r(a(\theta)) \equiv E[u_r(a(\theta), \theta)]$  be the patient's expected utility from treatment profile  $a(\theta)$ . Recalling that  $a_r(\theta) = \theta - b_r$  is the patient's preferred profile, the patient's utility loss relative to first best can be expressed as

$$\underbrace{U_r(a_r) - U_r(a)}_{\text{Patient welfare loss}} = \underbrace{U_r(a_r) - U_r(\bar{a})}_{\text{Loss from}} + \underbrace{U_r(\bar{a}) - U_r(a)}_{\text{Loss from}}.$$
Patient welfare loss
$$\underbrace{U_r(a_r) - U_r(\bar{a})}_{\text{Loss from}} + \underbrace{U_r(\bar{a}) - U_r(a)}_{\text{Loss from}}.$$

$$\underbrace{U_r(\bar{a}) - U_r(a)}_{\text{Loss from}}.$$

$$\underbrace{U_r(\bar{a}) - U_r(a)}_{\text{Informativeness}}.$$

$$\underbrace{U_r(\bar{a}) - U_r(a)}_{\text{Informativeness}}.$$

$$\underbrace{U_r(\bar{a}) - U_r(a)}_{\text{Informativeness}}.$$

$$\underbrace{U_r(\bar{a}) - U_r(a)}_{\text{Informativeness}}.$$

The first term on the right hand side describes the difference in the patient's utility in moving from first best profile  $a_r$  to profile  $\bar{a}$  which is fully separating but has the same average treatment as the equilibrium profile. In the second line, the particular expression for this term is derived by plugging  $E[\bar{z}(\theta)] = E[z(\theta)] = b_s + \frac{1}{2}b_s^2$  (by Proposition 4(i)) into the expected utility decomposition in Equation (2). Note that since  $E[z(\theta)] = b_s + \frac{1}{2}b_s^2$ , the average treatment level depends only on  $b_s$  and not on  $b_r$ , an observation that is key in our upcoming discussion of the role of health insurance. Note also that since  $E[z(\theta)] > b_s$ , the expected level of equilibrium treatment is *higher* than if the decision were fully delegated to the doctor, which is detrimental to both the doctor and patient.

Strategic communication also gives rise to a loss of informativeness, as measured by the move from profile  $\bar{a}$  to equilibrium profile a in the second term of the decomposition. Again the particular algebraic expression is derived from the statement of Proposition 4(i) and Equation (2). It is easily verified from the expression that the loss from informativeness grows in both parameters but faster in the doctor's bias than the patient's. Thus we see that both average treatment and treatment informativeness are affected differentially by  $b_s$  and  $b_r$ . In contrast to standard communication models which tend to normalize one parameter and implicitly focus only on the sum, in veto games the presence of the outside option  $\hat{a}$  makes such a reduction restrictive.

**Remark 1** The physician-induced demand framework (equivalently, full delegation) strictly understates the negative effect of financial incentives on patient welfare. Patient welfare loss due to informativeness as a proportion of total welfare loss (due to informativeness and treatment level) grows with  $b_s$  and  $b_r$ , up to approximately 25%.

Remark 1 is important for interpreting the results from empirical studies of doctors' and patients' incentives which often embed the physician-induced demand mechanism (e.g., Clemens and Gottlieb, 2014; Gruber and Owings, 1996; McGuire and Pauly, 1991). Measuring the change in average treatment level alone understates the true impact of financial incentives by ignoring informativeness. In this sense, the estimated welfare loss in a physician-induced demand model is a lower bound for the true welfare loss.

## 4.2 Policies Affecting Financial Incentives

We highlight some comparative statics that correspond to commonly studied policy questions and track their impact on the utilities of the doctor and the patient, the average treatment level, and the informativeness of the treatment plan. Figure 5 summarizes the findings while the results are explored in more detail below.

Policy	$U_r$	$U_s$	Treatment Level	Informativeness
Insurance (lower out of pocket costs and higher premium)	<b>↑</b>	<b>↑</b>	same	<u> </u>
Increase in reimbursement rate with or without pass-through	<b>↓</b>	↑ or ↓	1	<b>↓</b>

Figure 5: Effects of Various Policies

## 4.2.1 Reducing patient out-of-pocket cost through insurance

We consider a simple insurance contract in which a consumer has coinsurance  $\gamma \in [0, 1]$  and so pays an out-of-pocket cost  $\gamma b_r$  per unit of treatment and an actuarially fair premium  $F(\gamma)$  that satisfies the zero expected profit condition  $F(\gamma) = (1 - \gamma)b_r E[a(\theta|\gamma)]$ .

**Proposition 5** When the patient holds more insurance (lower coinsurance  $\gamma$ ), informativeness strictly increases and the average treatment level remains unchanged. Therefore, insurance is Pareto improving and full insurance ( $\gamma = 0$ ) is preferred by both the doctor and patient.

**Proof** As established in Proposition 4(i), the average treatment level is unaffected by the patient's parameter and since the contract is actuarially fair, his total expenditure does not change with  $\gamma$ .<sup>19</sup> Meanwhile, more insurance reduces the patient's bias parameter, which by the same proposition improves informativeness.

An interesting aspect of this analysis is that the moral hazard, which is thought to accompany the purchase of insurance, is not present here since the average treatment does not depend on the bias parameter of the patient. Indeed, moral hazard would occur in a pure cheap talk model in which the patient is free to choose among all actions. Under the veto arrangment, the patient receives a take-it-or-leave-it offer from the doctor, and thus even though his marginal willingness to receive treatment increases, he may not be able to act on this. The invariance of expected treatment with respect to the amount of purchased insurance is consistent with empirical evidence of a rather muted effect of copayments on the quantity of treatment received (Feldstein, 1973; Gibson et al., 2005; Goldman et al., 2007; Manning et al., 1987; McGuire, 2012; Newhouse et al., 1993). The role of insurance in this framework is thus solely to commit the patient to have preferences for treatment closer to the doctor's by reallocating spending from ex-post to ex-ante. This improves informativeness and demonstrates a value for insurance beyond its traditional role of reducing risk.

#### 4.2.2 A change in reimbursement rates

The parameter  $b_s$  represents the doctor's profit margin which in particular depends on the reimbursement rates negotiated with insurers,<sup>20</sup> and here we explore how changing these rates affects average treatment level, informativeness, and welfare. First, by

<sup>&</sup>lt;sup>19</sup>The only possible exception is that when  $\gamma$  becomes small enough, the equilibrium profile  $a_2^*$  from Proposition 4(ii) becomes more informative than  $a_1^*$ . If such a regime change occurs, informativeness still increases continuously as  $\gamma$  decreases, while at the regime switch E[z] jumps down from  $b_s + \frac{1}{2}b_s^2$  to  $b_s$ , which is better for both the doctor and patient. Thus, regardless of whether a regime switch occurs, a lower  $\gamma$  is a Pareto improvement.

<sup>&</sup>lt;sup>20</sup>The comparative statics similarly apply to other common factors affecting doctors' margins, including equity stakes in hospitals and labs and payments from pharmaceuticals on the revenue side or expenditures related to time, staff, malpractice insurance, technology or other sources on the cost side.

Proposition 4(i) higher reimbursement leads to more treatment and less informativeness, and by these two effects patient welfare declines.<sup>21</sup> The doctor is harmed by the loss of informativeness, and also from the increased treatment level, since by Proposition 4(i) the equilibrium treatment exceeds his preferred level by  $\frac{1}{2}b_s^2$ . In addition, there is a direct effect for the doctor that countervails the previous two. To see this, note that as reimbursement  $b_s$  increases, even if the treatment profile  $a(\theta)$  remains unchanged, the doctor receives a higher total payment and is thus better off.<sup>22</sup> The net effect for the doctor is ambiguous, and by inspection it can be verified that he is better off as long as  $b_s$  and  $b_r$  are sufficiently small.

Next, if the zero-profit insurer accounts for the increased reimbursement rate by raising the patient's premium there are no additional effects on total surplus though there is a lump sum transfer of money from the patient to the insurer. If instead the insurer passes through the increased reimbursement with a corresponding increase in the out-of-pocket payment an additional distortion arises. Treatment now becomes less informative with the level remaining unchanged, a Pareto loss.

**Proposition 6** An increase in the doctor's profit margin  $b_s$  always makes the patient worse off and makes the doctor worse off if and only if  $b_s$  and  $b_r$  are sufficiently large. In addition, offsetting a higher  $b_s$  with higher  $b_r$  results in a further Pareto loss.

These findings are relevant for the effectiveness of prospective payment policies that offer doctors a fixed fee based on the classification of a patient or condition with little or no marginal reimbursement, as is found in accountable care organizations and capitation payment systems. Such policies have generally been found to reduce utilization and costs, <sup>23</sup> and this is in line with the prediction of our model. Our approach suggests that reduced marginal reimbursement systems have the additional benefit of improved informativeness.

<sup>&</sup>lt;sup>21</sup>It has been suggested that in some contexts doctors reduce treatment when their reimbursement rates increase (see Chandra et al., 2012). This phenomenon, sometimes referred to as income targeting, stems from the idea that when doctors have diminishing marginal utility over money, the income effect of a higher reimbursement rate outweighs the substitution effect. In contrast, the doctor in our model has a constant marginal utility for money and thus no income effect is possible.

<sup>&</sup>lt;sup>22</sup>Observe that the relationship between the sender's utility function in Equations (1) and (3) is given by  $E[u_s^{(3)}] = \frac{1}{2}E[u_s^{(1)}] + \frac{1}{2}b_s(b_s + 1)$ . This last term measures the direct effect for the doctor, which does not exist in the original quadratic loss specification.

<sup>&</sup>lt;sup>23</sup>See Christianson and Conrad (2011) and Cutler and Zeckhauser (2000) for literature reviews on provider responses to payment systems.

The comparative statics also provide an explanation for the fact that increased reimbursement rates do not increase the provision of preventive care (Town et al., 2005). One may interpret the health status  $\theta$  as the probability of a serious illness, with preventive care corresponding to the doctor's preferred treatment for low  $\theta$ . In equilibrium the patient receives no treatment for states  $[0, b_r]$ , and this remains the case even as  $b_s$  rises.

#### 5 Conclusion

The veto rule is a common institutional arrangement that, in our estimation, has been under-utilized in applications due to a lack of tractability. While several equilibria have been studied in the literature (GK, KM), and the metric of informativeness has been widely accepted in similar communication games, identifying the set of veto equilibria and finding the most informative element of this set has remained an unsolved problem. In this paper we describe the equilibrium set and identify the most informative equilibrium in a setting that includes previous work. The key to finding the most informative equilibrium in fact arises from the characterization of the set; namely there is a subset of states over which any equilibrium profile must have pooling regions, and the proposed equilibrium is constructed to be optimal over this range.

In finding the most informative equilibrium we contribute to a literature that compares communication protocols. We strengthen the result in DE that for intermediate values of the status quo the receiver prefers full delegation to veto. In addition, we extend the work of KM, who showed indirectly that the most informative equilibrium in veto games is more informative than that in cheap talk. Since we explicitly characterize the most informative veto equilibrium, we enable other comparisons with the most informative cheap talk equilibrium, including receiver and sender welfare.

The main contribution of our analysis is to facilitate the use of the veto model in applications, and to demonstrate the practical importance of the results we study the doctor-patient relationship. The institutional setting fits the veto environment quite well: doctors are more informed than patients, typically prefer more treatment, and have control since patients cannot self-prescribe. In addition the predictions of the veto model are more in line with empirical evidence than those of previous models in the health literature. For example, under veto patients are overtreated on average *and* are able to reject the doctor's recommendations. In contrast, the physician-induced demand framework predicts overtreatment without allowing non-compliance while pure cheap talk allows non-compliance but no excessive treatment.

By explicitly modeling communication the veto framework provides new insights compared to the workhorse physician-induced demand model. The doctor's financial incentive leads not only to excessive treatment but also to information loss as the doctor strategically misdiagnoses to avoid rejection, and patient welfare is potentially affected more by the information loss than by overtreatment. Thus by focusing only on average treatment levels and ignoring information loss, the estimated harm to patients from increasing doctors' financial incentives (e.g. higher reimbursement rates, allowing ownership of diagnostic labs, increasing cost of malpractice insurance, etc.) is substantially understated. Furthermore, the patient's preference for treatment affects only the informativeness of communication but not the average treatment level. Consequently a patient with a lower co-insurance payment receives the same amount of treatment but on average the treatment is better suited for the illness as communication improves due to a closer alignment of incentives. Thus even risk neutral patients find insurance valuable as a means to reduce the doctor's incentive to misdiagnose.

# 6 Appendix

This appendix is organized as follows. First we prove Lemmas 3 and 2 which establish the minimal pooling regions for equilibria at different values of the status quo  $\hat{a}$ . Then we identify the most informative equilibrium for intermediate (Proposition 3) and low (Proposition 4) values of the status quo, the proofs of which share a common approach.

#### 6.1 Proof of Lemma 3

We first recall several facts from Proposition 2. We refer to  $a_s(\theta) = \theta - b_s$  as the sender's diagonal and observe that every interior pooling interval must intersect it. Further, if the right endpoint of a pooling interval is below the sender's diagonal then the next interval is also pooling and at its left endpoint is above the diagonal by the same amount. Since  $a(\hat{\theta}) = \hat{a}$  is on the diagonal and the initial interval pools to the right, it must end below the diagonal and the second interval  $(\theta_1, \theta_2)$ , if it exists, must also be pooling.

It is also helpful to compute several expressions for interval endpoints and actions. The pooling action on interval ( $\theta_1$ ,  $\theta_2$ ) must satisfy the sender's indifference condition at  $\theta_1$ , thus

$$\theta_1 + b_s = \frac{1}{2}(\hat{a} + a_1) \implies a_1 = 2\theta_1 + 2b_s - \hat{a}.$$
 (4)

Similarly, if  $(\theta_2, \theta_3)$  is also pooling then the sender's indifference at  $\theta_2$  implies

$$\theta_2 + b_s = \frac{1}{2}(a_1 + a_2) \implies a_2 = 2(\theta_2 - \theta_1) + \hat{a}.$$
 (5)

In addition, while the right endpoint  $\theta_1$  of the first pooling interval is unconstrained since the receiver must accept  $\hat{a}$  when it is prescribed, in the second pooling interval her posterior must be sufficiently high to accept, thus

$$\frac{1}{2}(\theta_1 + \theta_2) - b_r \ge \frac{1}{2}(a_1 + \hat{a}) \quad \Rightarrow \quad \theta_2 \ge \theta_1 + 2b_s + 2b_r, \tag{6}$$

If there is no third interval  $(\theta_2, \theta_3)$  then the result of the lemma is immediate. If the third interval is separating then  $\theta_2 = a_1 - b_s$ , and plugging this into the receiver's posterior condition for  $(\theta_1, \theta_2)$  obtains

$$\frac{1}{2}(\theta_1 + (a_1 - b_s)) - b_r \ge \frac{1}{2}(a_1 + \hat{a}) \implies \theta_1 \ge \hat{a} + b_s + 2b_r \implies \theta_2 \ge \hat{a} + 3b_s + 4b_r,$$

with the last inequality following from (6). But then  $(\hat{a} - b_s, \hat{a} + 3b_s + 4b_r) = (\hat{\theta}, \hat{\theta} + 4b_s + 4b_r)$  is covered by two pooling intervals. If instead  $(\theta_2, \theta_3)$  is pooling then by (5) and (6)

$$a_2 = \hat{a} + 2(\theta_2 - \theta_1) \ge \hat{a} + 4b_s + 4b_r$$

and in order to hit the sender's diagonal the third pooling interval must include the state  $\theta = a_2 - b_s \ge \hat{a} + 3b_s + 4b_r$ . Here  $(\hat{a} - b_s, \hat{a} + 3b_s + 4b_r)$  is covered by three pooling intervals.

That  $\theta_3 \ge \hat{a} + 3b_s + 4b_r \ge \hat{\theta} + 4(b_s + b_r)$  also implies that on the interval  $(\hat{\theta}, \hat{\theta} + 4(b_s + b_r))$  there is at most one diagonal intersection, an observation that will be used in the proofs of Propositions 3 and 4.

#### 6.2 Proof of Lemma 2

Define  $a_0 \equiv a(0)$ . For part (i) of the lemma focusing on equilibria in which  $a_0 = \hat{a}$ , observe that the result was already demonstrated above. In particular the only difference here is that  $\hat{a} - b_s < 0$ , but none of the arguments presented above assume otherwise.

However part (ii) of Lemma 2 is different since  $a_0 > \hat{a}$  and in these equilibria the status quo is never utilized. Instead observe that if there is a second pooling interval then to satisfy the sender's indifference at  $\theta_1$  it must be that

$$\theta_1 + b_s = \frac{1}{2}(a_0 + a_1) \quad \Rightarrow \quad a_1 = 2\theta_1 + 2b_s - a_0.$$
 (7)

Also, on the initial interval the receiver now has the ability to reject and to meet her constraint it must be that

$$\frac{1}{2}(0+\theta_1) - b_r \ge \frac{1}{2}(a_0 + \hat{a}) \quad \Rightarrow \quad \theta_1 \ge \hat{a} + 2b_r + a_0. \tag{8}$$

Then, meeting the receiver's posterior constraint on  $(\theta_1, \theta_2)$  requires  $\frac{1}{2}(\theta_1 + \theta_2) - b_r \ge \frac{1}{2}(a_1 + \hat{a})$ , which by plugging in (7) and (8) implies

$$\theta_2 \ge 2\hat{a} + 4b_r + 2b_s = 2\hat{\theta} + 4(b_s + b_r),$$

which concludes the proof.

## 6.3 Proofs of Informativeness Propositions 3 and 4

The approach for the proof of each proposition is to start with a candidate profile  $z(\theta)$ , transform it to a more informative profile  $\tilde{z}(\theta)$ , and then show that  $\tilde{z}(\theta)$  performs worse than the conjectured best profile. Claim 1 establishes a convenient way to compute the average value of z for any equilibrium profile, while Claims 2 and 3 describe two transformations of  $z(\theta)$  that increase informativeness and are used repeatedly in the proofs. Claim 1 relies on equilibrium properties while Claims 2 and 3 are purely mathematical.

**Claim 1** *If equilibrium profile*  $a(\theta)$  *covers interval*  $(\theta_l, \theta_h)$  *with at least two intervals then* 

$$E[z \mid \theta \in (\theta_l, \theta_h)] = b_s + \frac{1}{2} \left( a(\theta_l) - (\theta_l + b_s) \right)^2 - \frac{1}{2} \left( a(\theta_h) - (\theta_h + b_s) \right)^2.$$

**Proof of Claim:** First, we argue that if an equilibrium profile intersects the sender's diagonal at states  $\theta_l$  and  $\theta_h$  (i.e.,  $a(\theta_l) = \theta_l + b_s$  and  $a(\theta_h) = \theta_h + b_s$ ), then  $E[z(\theta) | \theta \in (\theta_l, \theta_h)] = b_s$ . To see this, observe that  $(\theta_l, \theta_h)$  is partitioned into separating and pooling intervals. Over all separating intervals the expected bias is  $b_s$  since  $z(\theta) = b_s$  at every state. Next consider the leftmost pooling interval  $[\theta_i, \theta_{i+1})$ , if it exists, which is either preceded by a separating interval or starts with  $\theta_l$ . In either case the profile begins at  $\theta_i$  on the sender's diagonal, pools to the right until  $\theta_{i+1}$ , then jumps symmetrically above the sender's diagonal and pools to the right at least until reaching the diagonal again. From  $\theta_i$  until this next diagonal intersection there are two pooling intervals symmetric around the diagonal, and on this region the conditional expected bias is  $b_s$ . Then we

again begin on the sender's diagonal and proceed the same way until we reach  $\theta_h$ , which by construction must be on the sender's diagonal, maintaining a conditional mean of  $b_s$  all along the way.

Next, we find the value of  $E[z \mid \theta \in (\theta_l, \theta_h)]$  by integrating  $z(\theta)$  between the first and last intersection of the sender's diagonal, and then accounting for the states outside of this subset. For a given equilibrium profile  $a(\theta)$ , define  $\theta \equiv \theta_l + z(\theta_l) - b_s$  and  $\bar{\theta} \equiv \theta_h + z(\theta_h) - b_s$ . Observe that if  $z(\theta_l) > b_s$  then  $\theta$  identifies the first diagonal intersection to the right of  $\theta_l$ , while if  $z(\theta_l) < b_s$  then  $\theta$  identifies the first diagonal intersection to the left of  $\theta_l$  if the pooling interval were extended. The state  $\bar{\theta}$  is similarly defined. Then,

$$\begin{split} E[z\mid\theta\in(\theta_{l},\theta_{h})] &= \int_{\underline{\theta}}^{\bar{\theta}}z(\theta)\,d\theta \,+\, \int_{\theta_{l}}^{\underline{\theta}}z(\theta)\,d\theta \,+\, \int_{\bar{\theta}}^{\theta_{h}}z(\theta)\,d\theta \\ &= (\bar{\theta}-\underline{\theta})b_{s} \,+\, Sign[\underline{\theta}-\theta_{l}]\cdot\frac{1}{2}(\underline{\theta}-\theta_{l})(\underline{\theta}-\theta_{l}+2b_{s}) \,+\, Sign[\theta_{h}-\bar{\theta}]\cdot\frac{1}{2}(\theta_{h}-\bar{\theta})(\bar{\theta}-\theta_{h}+2b_{s}) \\ &= b_{s} \,+\,\frac{1}{2}(\underline{\theta}-\theta_{l})^{2} \,-\,\frac{1}{2}(\bar{\theta}-\theta_{h})^{2}. \end{split}$$

In the second line, the first term follows from the argument above that the conditional average  $z(\theta)$  on  $(\underline{\theta}, \bar{\theta})$  equals  $b_s$ . The second and third terms in the first line integrate  $z(\theta)$  over pooling intervals between  $\theta_l$  and  $\underline{\theta}$  and  $\overline{\theta}$  and a and account for the fact that, for instance, it is possible that  $\underline{\theta} < \theta_l$ . The third line follows from the observation that  $Sign[x] \cdot x = |x|$  and a regrouping of terms. By the definitions of  $\underline{\theta}$  and  $\underline{\theta}$ , this line is equivalent to the statement of the claim.

For the following two proofs,  $|\Theta|$  denotes the Lebesgue measure of the set  $\Theta$ .

**Claim 2** If  $\Theta$  is partitioned into  $\Theta_A$  and  $\Theta_B$  so that  $E_{\Theta_A}[z(\theta)] \equiv z_A < z_B \equiv E_{\Theta_B}[z(\theta)]$ , and

$$\tilde{z}(\theta) \equiv \begin{cases} z(\theta) + \delta_1 & \text{if } \theta \in \Theta_A \\ z(\theta) - \delta_2 & \text{if } \theta \in \Theta_B \end{cases},$$

with  $z_A + \delta_1 \le z_B - \delta_2$  and  $\delta_1, \delta_2 \ge 0$ , then

$$\int_{\Theta} (z(\theta) - E_{\Theta}[z(\theta)])^2 d\theta \geq \int_{\Theta} (\tilde{z}(\theta) - E_{\Theta}[\tilde{z}(\theta)])^2 d\theta.$$

**Proof of Claim:** Figure 6 illustrates an example of the situation described in the claim. In the figure, note that even after the shift it remains that  $\tilde{z}_B > \tilde{z}_A$ . Evaluating this shift

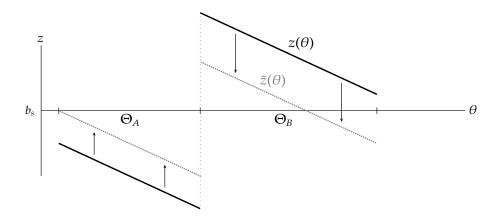


Figure 6: The variance is reduced when  $z(\theta)$  is uniformly shifted up on  $\Theta_A$  and down on  $\Theta_B$ .

below,

$$\int_{\Theta} (z(\theta) - E_{\Theta}[z(\theta)])^{2} d\theta = \int_{\Theta_{A}} (z(\theta) - z_{A})^{2} d\theta + |\Theta_{A}|(E_{\Theta}[z(\theta)] - z_{A})^{2}$$

$$+ \int_{\Theta_{B}} (z(\theta) - z_{B})^{2} d\theta + |\Theta_{B}|(E_{\Theta}[z(\theta)] - z_{B})^{2}$$

$$\geq \int_{\Theta_{A}} (\tilde{z}(\theta) - \tilde{z}_{A})^{2} d\theta + |\Theta_{A}|(E_{\Theta}[\tilde{z}(\theta)] - \tilde{z}_{A})^{2}$$

$$+ \int_{\Theta_{B}} (\tilde{z}(\theta) - \tilde{z}_{B})^{2} d\theta + |\Theta_{B}|(E_{\Theta}[\tilde{z}(\theta)] - \tilde{z}_{B})^{2}$$

$$= \int_{\Theta} (\tilde{z}(\theta) - E_{\Theta}[\tilde{z}(\theta)])^{2} d\theta.$$

In the first line, the sum of square distances to  $E_{\Theta}[z(\theta)]$  is separately taken over regions  $\Theta_A$  and  $\Theta_B$ , and within each region the sum is further decomposed into a sum of square distances to the conditional means  $z_A$  and  $z_B$  and a term to account for the difference between the conditional and unconditional means. In the second line, we switch to profile  $\tilde{z}(\theta)$ , which leaves the first and third terms in the first line unchanged. Furthermore, by construction  $z_A \leq \tilde{z}_B \leq z_B$ , and thus both the second and fourth terms in the first line are weakly smaller in the second line.

The next claim establishes a minimum amount of variance that arises in pooling regions when part of the pooling occurs below the diagonal and part above, as for example between  $\theta_1$  and  $\theta_2$  in Figure 3 parts V and VI.

**Claim 3** For an equilibrium profile  $a(\theta)$ , if there are two disjoint pooling regions  $\Theta_A$  and  $\Theta_B$ 

with non-overlapping biases (i.e.  $\sup_{\Theta_A} z(\theta) \leq \inf_{\Theta_B} z(\theta)$ ) then

$$\int_{\Theta_A \cup \Theta_B} (z(\theta) - E[z(\theta)])^2 d\theta \ge \frac{1}{12} \left| \Theta_A \cup \Theta_B \right|^3.$$

**Proof of Claim:** Since both regions are pooling, over region  $\Theta_A$  the bias z is uniformly distributed on  $\left(\sup_{\Theta_A} z(\theta) - |\Theta_A|, \sup_{\Theta_A} z(\theta)\right)$ , and over region  $\Theta_B$  the bias z is uniformly distributed on  $\left(\inf_{\Theta_B} z(\theta), \inf_{\Theta_B} z(\theta) + |\Theta_B|\right)$  over region  $\Theta_B$ . Define

$$\tilde{z}(\theta) = \begin{cases} z(\theta) + \inf_{\Theta_B} z(\theta) - \sup_{\Theta_A} z(\theta) & \text{if } \theta \in \Theta_A \\ z(\theta) & \text{if } \theta \in \Theta_B \end{cases},$$

and note that by Claim 2

$$\int_{\Theta_A \cup \Theta_B} (z(\theta) - E[z(\theta)])^2 d\theta \ge \int_{\Theta_A \cup \Theta_B} (\tilde{z}(\theta) - E[\tilde{z}(\theta)])^2 d\theta.$$

Note also that  $\tilde{z}$  is distributed uniformly on  $(\inf_{\Theta_A} z(\theta), \inf_{\Theta_A} z(\theta) + |\Theta_A \cup \Theta_B|)$ , thus

$$\int_{\Theta_A \cup \Theta_B} (\tilde{z}(\theta) - E[\tilde{z}(\theta)])^2 d\theta \geq \int_{\inf_{\Theta_A} z(\theta)}^{\inf_{\Theta_A} z(\theta) + |\Theta_A| + |\Theta_B|} (\tilde{z} - E[\tilde{z}(\theta)])^2 d\tilde{z} \geq \frac{1}{12} \left| \Theta_A \cup \Theta_B \right|^3,$$

where the final inequality is derived from the fact that the preceding integral is minimized if  $E[\tilde{z}(\theta)] = \inf_{\Theta_A} z(\theta) + \frac{1}{2} (|\Theta_A| + |\Theta_B|)$ .

Using the preceding claims we are now ready to prove Propositions 3 and 4.

## **6.3.1** Proof of Proposition 3

Proposition 3 applies to statuses quo  $\hat{a} \in [b_s, 1 - 3b_s - 4b_r]$ , and we have shown that in this case  $a_0 = \hat{a}$  in every equilibrium. With an eye towards the proof of Proposition 4 in which  $\hat{a} \in [-b_r, b_s]$ , here we find the most informative equilibrium in which  $a_0 = \hat{a}$  for any  $\hat{a} \in [-b_r, 1 - 3b_s - 4b_r]$ . The generalized statement of Proposition 3 is below:

**Generalized Proposition 3** If  $\hat{a} < 1 - 3b_s - 4b_r$ , then of the equilibria with  $a_0 = \hat{a}$  the strictly most informative equilibrium has boundaries  $(\theta_0, \theta_1, \theta_2, \theta_3) = (\max(\hat{\theta}, 0), \hat{a} + b_r, \hat{a} + b_r, \hat{a})$ 

 $2b_s + 3b_r$ ,  $\hat{a} + 3b_s + 4b_r$ ) and action profile

$$a_1^*(\theta) = \begin{cases} \theta + b_s & \text{if } \theta \in (0, \theta_0) \\ \hat{a} & \text{if } \theta \in (\theta_0, \theta_1) \\ \hat{a} + 2(b_s + b_r) & \text{if } \theta \in (\theta_1, \theta_2) \\ \hat{a} + 4(b_s + b_r) & \text{if } \theta \in (\theta_2, \theta_3) \\ \theta + b_s & \text{if } \theta \in (\theta_3, 1) \end{cases}$$

with

$$E[z_1^*] = b_s + \frac{1}{2}(\min(\hat{\theta}, 0))^2 \quad and \quad Var(z_1^*) = \frac{4}{3}(b_s + b_r)^3 - \frac{1}{3}\left|\min(\hat{\theta}, 0)\right|^3 - \frac{1}{4}\left|\min(\hat{\theta}, 0)\right|^4.$$

**Proof** That  $E[z_1^*] = b_s + \frac{1}{2}(\min(\hat{\theta}, 0))^2$  is implied by Claim 1, and the expression for variance can be computed explicitly. Recall that  $\hat{\theta} \equiv \hat{a} - b_s$ ,  $\bar{\theta} \equiv \hat{a} + 3b_s + 4b_r$ , and that Lemmas 2(i) and 3 establish that  $(\max(\hat{\theta}, 0), \bar{\theta})$  is covered by no separating and at most three pooling intervals. Also, as demonstrated in the final paragraph of the proof of Lemma 3, on  $(\hat{\theta}, \bar{\theta})$  there is at most one diagonal intersection.

First consider a candidate profile  $a(\theta)$  with zero diagonal intersections on  $(\hat{\theta}, \bar{\theta})$ . This implies that  $(\hat{\theta}, \bar{\theta})$  is covered either by a single pooling action  $\hat{a}$  or two pooling intervals  $(\max(\hat{\theta}, 0), \theta_1)$  and  $(\theta_1, \bar{\theta})$ , in which the first interval is entirely below the sender's diagonal and the second interval is entirely above it. Then,

$$Var(z) \ge \frac{1}{12}(\bar{\theta} - \max(\hat{\theta}, 0))^3 > \frac{1}{12}(4b_s + 4b_r)^3 \ge Var(z_1^*).$$

The first inequality follows from Claim 3, the second inequality comes from the parameter constraint  $\hat{a} + 3b_s + 4b_r < 1$ , and the final inequality follows from the expressions for variance confirmed above. Thus every candidate profile that does not intersect the sender's diagonal to the right of  $\hat{\theta}$  is less informative than  $a_1^*$ .

Next consider profiles with one diagonal intersection on  $(\hat{\theta}, \bar{\theta})$ . In these, the second pooling interval intersects the sender's diagonal to the right of  $\hat{\theta}$  precisely once at state t. This includes profiles that to the right of  $\hat{\theta}$  consist of two pooling intervals, or of three pooling intervals with the third interval ending above the sender's diagonal. Define

$$\tilde{z}(\theta) \equiv \begin{cases} \hat{a} + \theta & \text{if } \theta \in (\min(\hat{\theta}, 0), 0) \\ z(\theta) & \text{if } \theta \in (0, t) \\ z(\theta) + b_s - E[z \mid \theta \in (t, 1)] & \text{if } \theta \in (t, 1) \end{cases}.$$

The profile  $\tilde{z}$  alters the candidate z in two ways, as demonstrated in Figure 7. First, if  $\hat{\theta} < 0$  then the initial pooling interval is extended to the left from state  $\theta = 0$  to state  $\theta = \hat{\theta}$ , and in doing so ensures that  $\int_{\hat{\theta}}^{t} \tilde{z}(\theta) d\theta = b_{s}$ . Note there remains no probability mass for negative states, but defining  $\tilde{z}$  over this region will help in ensuing calculations. Also, observing that  $E[z \mid \theta \in (t, 1)] < b_{s}$ , the profile  $\tilde{z}$  corresponds to a uniform upward shift for states (t, 1) so that  $E[\tilde{z} \mid \theta \in (t, 1)] = b_{s}$ . Finally, note the region  $(t, \bar{\theta})$  is covered

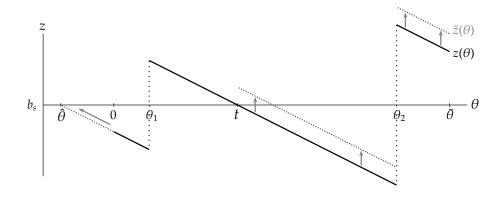


Figure 7: Alternate profile  $\tilde{z}$  extends the initial pooling interval to negative states if  $\hat{\theta} < 0$  and uniformly increases  $z(\theta)$  on  $(t, \bar{\theta})$ .

by at most two pooling intervals  $(t, \theta_2)$  and  $(\theta_2, \bar{\theta})$ , and that  $\tilde{z}$  is smaller at every point in the former region than at any point in the latter region. Then, the following sequence of inequalities follows, explained in detail below:

$$Var(z) \geq Var(\tilde{z})$$

$$= \int_{0}^{1} (\tilde{z}(\theta) - b_{s})^{2} d\theta - (E[\tilde{z}] - b_{s})^{2}$$

$$\geq \int_{\hat{\theta}}^{t} (\tilde{z}(\theta) - b_{s})^{2} d\theta + \int_{t}^{\bar{\theta}} (\tilde{z}(\theta) - b_{s})^{2} d\theta - \int_{\min(\hat{\theta}, 0)}^{0} (\hat{a} - \theta - b_{s})^{2} d\theta - (E[\tilde{z}] - b_{s})^{2}$$

$$\geq \frac{1}{12} (t - \hat{\theta})^{3} + \frac{1}{12} (\bar{\theta} - t)^{3} - \int_{\min(\hat{\theta}, 0)}^{0} (\hat{a} - \theta - b_{s})^{2} d\theta - (E[\tilde{z}] - b_{s})^{2}$$

$$\geq \frac{1}{6} \left(\frac{1}{2} (\bar{\theta} - \hat{\theta})\right)^{3} - \int_{\min(\hat{\theta}, 0)}^{0} (\hat{a} - \theta - b_{s})^{2} d\theta - (E[\tilde{z}] - b_{s})^{2}$$

$$= \frac{4}{3} (b_{s} + b_{r})^{3} - \int_{\min(\hat{\theta}, 0)}^{0} (\hat{a} - \theta - b_{s})^{2} d\theta - (E[\tilde{z}] - b_{s})^{2}$$

$$= Var(z_{1}^{*}).$$

In the first line, in which  $Var(\cdot)$  integrates only over [0,1] and not the extended region, by moving from profile z to  $\tilde{z}$  we leave the profile unchanged on (0,t) but uniformly shift it up on (t,1). Since by Claim 1,  $E[z\mid\theta\in(t,\bar{\theta})]< b_s\leq E[z\mid\theta\in(0,t)]$ , the shift brings these conditional means closer together which by Claim 2 reduces variance, thus obtaining the inequality. In the second line we employ the fact that variance can be computed by summing square distances to any number and then subtracting the square difference between that number and the true mean. In the third line, we add and subtract square distances to  $b_s$  on  $(\hat{\theta},0)$  and drop square distances on  $(\bar{\theta},1)$ , which leads to the inequality. The fourth line uses the fact that within each interval, both  $(\hat{\theta},t)$  and  $(t,\bar{\theta})$  are each spanned by two pooling intervals with non-overlapping values of  $z(\theta)$ , which by Claim 3 implies the first two terms on the right hand side. The fifth line uses the fact that  $t=\frac{1}{2}(\bar{\theta}+\hat{\theta})$  minimizes the sum of the first two terms in the previous line, and the ensuing penultimate line follows from plugging in  $\bar{\theta}-\hat{\theta}=4b_s+4b_r$ . Finally by Claim 1,  $E[\bar{z}]=E[z_1^*]$  and the last equality obtains by verifying the expression for  $Var(z_1^*)$  confirmed earlier.

#### 6.3.2 Proof of Proposition 4

The proof of Generalized Proposition 3 above also characterizes the most informative equilibrium with  $a_0 = \hat{a}$  when  $\hat{a} \in [-b_r, b_s)$ . Here we consider the remaining case  $a_0 > \hat{a}$  and establish the existence of a threshold  $\underline{a}$  below which this case gives the most informative equilibrium.

Let  $\bar{\theta} \equiv 2b_s + 4b_r + 2\hat{a}$ , and since  $\hat{a} < b_s$  Lemma 2(ii) states that every profile covers the region  $(0, \bar{\theta})$  by at most two pooling intervals  $(0, \theta_1)$  and  $(\theta_1, \bar{\theta})$ , and no separating intervals, as in the example in Figure 8. We first show that that  $z(\theta)$  is non-overlapping on these two intervals. Observe that

$$z^{-}(\bar{\theta}) = z^{+}(\theta_{1}) - (\bar{\theta} - \theta_{1})$$

$$= 2b_{s} - z^{-}(\theta_{1}) - (\bar{\theta} - \theta_{1})$$

$$= 2b_{s} - (a_{0} - \theta_{1}) - (\bar{\theta} - \theta_{1})$$

$$= 2b_{s} + 2\theta_{1} - a_{0} - \bar{\theta}$$

$$\geq 2b_{s} + 2(\hat{a} + 2b_{r} + a_{0}) - a_{0} - \bar{\theta}$$

$$= a_{0}$$

$$= z^{+}(0).$$

In the first line, the equality comes from the fact that over the pooling interval  $(\theta_1, \bar{\theta})$ 

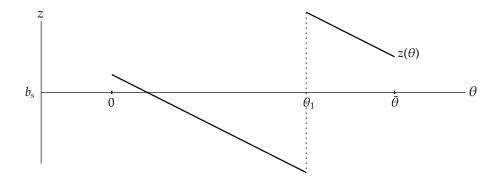


Figure 8: An example demonstrating no overlap in the distribution of z on the intervals  $(0, \theta_1)$  and  $(\theta_1, \bar{\theta})$ , resulting from the fact that  $\theta_1 \ge \frac{1}{2}\bar{\theta}$ .

the value of  $z(\theta)$  drops by the length of the interval. The second line then plugs in for the value of  $z^+(\theta_1)$ , using the fact that the profile is reflected above the line  $z = b_s$  at the interval boundary. The third equality again comes from the fact that over the pooling interval  $(0, \theta_1)$  the value of  $z(\theta)$  drops by the length of the interval and that the starting value is  $z(0) = a_0$ . The ensuing inequality follows from the fact that  $\theta_1 \ge \hat{a} + 2b_r + a_0$  in order to meet the receiver's posterior constraint, and the final two lines follow from the definitions of  $\bar{\theta}$  and  $a_0$ . Thus the values of  $z(\theta)$  are non-overlapping on intervals  $(0, \theta_1)$  and  $(\theta_1, \bar{\theta})$ , as depicted in Figure 8. Then,

$$Var(z) = \int_{0}^{\bar{\theta}} (z(\theta) - E[z])^{2} d\theta + \int_{\bar{\theta}}^{1} (z(\theta) - E[z])^{2} d\theta$$

$$\geq \frac{1}{12} \bar{\theta}^{3} + \int_{\bar{\theta}}^{1} (z(\theta) - E[z])^{2} d\theta$$

$$\geq \frac{2}{3} (b_{s} + 2b_{r} + \hat{a})^{3}$$

$$= Var(z_{2}^{*}).$$

The first inequality follows by Claim 3 since we have shown that values of  $z(\theta)$  are non-overlapping on  $(0, \theta_1)$  and  $(\theta_1, \bar{\theta})$ , and is strict if  $\theta_1 > \hat{a} + 2b_r + a_0$ . The second inequality comes from the definition of  $\bar{\theta}$  and is strict unless the region  $(\bar{\theta}, 1)$  is separating, which occurs only if  $z(\bar{\theta}) = b_s$ . It can then be seen that  $\theta_1 = \hat{a} + 2b_r + a_0$  and  $z(\bar{\theta}) = b_s$  are satisfied in this class of profiles *only* by  $z_2^*$ , thus it is strictly the most informative.

Now, we show that for every  $b_s$ ,  $b_r$  there is a threshold  $\underline{a} < b_s$  such that the three pooling interval profile  $a_1^*$  of part (i) is most informative when  $\hat{a} \ge \underline{a}$  and the two pooling

interval profile  $a_2^*$  of part (ii) is most informative when  $\hat{a} \leq \underline{a}$ . To show this we focus on  $\Delta Var(\hat{a}) \equiv Var(z_2^*, \hat{a}) - Var(z_1^*, \hat{a})$  and establish that  $\Delta Var(\hat{a})$  is monotone increasing:

$$\frac{d}{d\hat{a}} \left( \Delta Var(\hat{a}) \right) \equiv \frac{d}{d\hat{a}} \left( Var(z_2^*, \hat{a}) - Var(z_1^*, \hat{a}) \right) = 2(b_s + 2b_r + \hat{a})^2 - (b_s - \hat{a})^2 - (b_s - \hat{a})^3 \tag{9}$$

Recalling that  $\hat{a} \in [-b_r, b_s)$  in Proposition 4, observe  $\frac{d}{d\hat{a}} \Big( \Delta Var(-b_r) \Big) = 2(b_s + b_r)^2 - (b_s + b_r)^2 - (b_s + b_r)^3 > 0$  since  $b_s + b_r < 1$  and  $\frac{d}{d\hat{a}} \Big( \Delta Var(b_s) \Big) = 2(b_s + 2b_r)^2 - b_s^2 - b_s^3 \ge 2b_s^2 - b_s^3 > 0$ , where the last inequality follows from  $b_s < 1$ . Also, observe that  $\frac{d^2}{d\hat{a}^2} \Big( \Delta Var(\hat{a}) \Big) = 4(b_s + b_r + \hat{a}) + 2(b_s - \hat{a}) + 3(b_s - \hat{a})^2 \ge 0$ , and thus  $\frac{d}{d\hat{a}} \Big( \Delta Var(\hat{a}) \Big) \ge 0$  for all  $\hat{a} \in [-b_r, b_s)$ .

Now note that  $\Delta Var(b_s) > 0$ , that is at the highest status quo in this region the three pooling interval equilibrium  $a_1^*$  is strictly more informative than the two pooling interval equilibrium  $a_2^*$ . Since  $\Delta Var(\hat{a})$  is continuous there must be a nonempty set of statuses quo  $(\underline{a}, b_s)$  over which  $a_1^*$  is optimal. Then, since we showed  $\Delta Var(\hat{a})$  is monotone increasing if there exists an  $\underline{a} \in [-b_r, b_s)$  so that  $\Delta Var(\underline{a}) = 0$ , then for all  $\hat{a} \in [-b_r, \underline{a})$  the two pooling interval profile  $a_2^*$  is optimal.

Finally, note that  $\Delta Var(-b_r) < 0$  if and only if  $b_r$  is sufficiently small relative to  $b_s$ , thus  $\underline{a} \ge -b_r$  and the two pooling interval equilibrium  $z_2^*$  is optimal only if  $b_r$  is sufficiently small.

#### 6.4 Proof of Remark 1

That physician induced demand (PID) strictly understates the patient welfare loss by ignoring informativeness follows directly from the decomposition of patient welfare loss in the main text. The ratio of the loss of informativeness to the loss from treatment level is given by

$$R(b_r, b_s) \equiv \frac{\frac{4}{3}(b_s + b_r)^3 - \frac{1}{3}b_s^3 - \frac{1}{4}b_s^4}{(b_s + \frac{1}{2}b_s^2 + b_r)^2},$$

with

$$\frac{dR}{db_r} = \frac{32b_r^3 + 48b_r^2b_s(b_s + 2) + 96b_rb_s^2(b_s + 1) + 12b_s^3(5b_s + 4)}{3(2b_r + b_s(b_s + 2))^3} > 0,$$

$$\frac{dR}{db_s} = \frac{8(b_r^3(4 - 8b_s) + 6b_r^2b_s(2 - 3b_s) + 3b_rb_s^2(3 - 5b_s) + 3(1 - b_s)b_s^3)}{3(2b_r + b_s(b_s + 2))^3} > 0.$$

That both derivatives are positive follows from the fact that  $b_s \in (0, \frac{1}{3})$  and  $b_r \in (0, \frac{1}{4})$ . Given the two parameter constraints  $b_s \le b_r$  and  $3b_s + 4b_r \le 1$ , a calculation confirms that the maximum occurs where the second constraint binds and the first does not, yielding a maximum ratio of approximately  $R \approx 0.349$ , which translates to approximately 25.9% of the total welfare loss.

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