New and Improved?*

Eric Schmidbauer†

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Abstract

Are new versions of products necessarily better? I analyze product innovation by a firm that engages in research and development designed to improve an existing product, the outcome of which is uncertain. If the firm adopts the innovation its modified product appears to consumers as “new and improved,” but consumers do not immediately know whether or how much the product is better. I find that new products are on average improved and therefore command a pricing premium. This induces some types to exploit the innovation signal by selling new versions that are only trivially different from their older version or that require inefficiently high upgrade costs. Nevertheless, the incentive to “show off” by introducing a new product may improve total welfare by inducing more innovation adoption and thereby mitigating the standard monopoly underinvestment problem. Firms benefit ex-ante from better consumer information about quality or from committing to not exploit their informational advantage.

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†Department of Economics, University of Central Florida. 4336 Scorpius St., Orlando, FL 32816. eschmidb@ucf.edu.
1 Introduction

How do consumers update their beliefs about a “new” or “improved” version of a product before purchase? For example, how much value will the latest edition of a college textbook provide over its predecessor? Should a consumer who is told “roads change by as much as 15% every year” purchase an updated map for her GPS device? Or suppose a familiar household cleanser’s packaging states “WOW! Powerful New Formula,” but its price has increased by 10%. Is the touted improvement in performance worth the higher price?

In each of these examples consumers are likely unaware of the exact value offered by the “new” or “improved” version of the product. Facing perhaps thousands of such new products each year, consumers must discern major breakthroughs from the more common incremental improvements before making their purchase decision. For their part, although firms may devote significant resources to research and development the outcome of such efforts is highly volatile and often results in failure (Stevens and Burley, 1997). Firms must decide which research outcomes to implement and which to censor from the market, knowing that some consumers may not be willing or able to become immediately informed of the new product’s value.

This paper uses a signaling model to investigate the incentive of firms to introduce improved products and the welfare consequences of these introductions when consumers are uncertain of the quality of the improvement. Consumers form beliefs about quality knowing the product exceeds the firm’s minimal threshold for new product launch. I find this leads to a “newness premium” resulting from the information conveyed in equilibrium by the very existence of the new product version. This premium in turn incentivizes more upgraded products to be released and so has implications for profits and welfare.1

1Note this information-based account differs from the marketing literature which has generally explained consumer attraction to new products by a desire for uniqueness, stimulation, or novelty-seeking (Roehrich, 2004; Hirschman, 1980).
I present a model in which a monopolist chooses an R&D effort level that results in a stochastic outcome and must decide whether to adopt this innovation or not. The firm knows the true value of the innovation while consumers initially only observe the binary “innovation signal” of whether or not the product has been modified. Consumers form beliefs about product value and buy (or not) accordingly, and then learn from product trial and other sources such as product review websites so that they are more informed in the second period when they repeat their purchase decision. I find that consumers correctly expect the average quality of modified products to be higher. But because of this expectation, firms whose R&D has generated only a trivial improvement or no improvement have an incentive to present the product as new and improved. The result is a partial pooling equilibrium in which consumers are initially unsure whether modified products represent a genuine improvement. In making a trivial improvement firms face a trade-off between inducing an initial “new product” premium and the loss of future profits when the true quality is revealed.

That the firm might incur upgrade costs to sell a new product version only trivially different from the old may appear to unambiguously lower welfare. Indeed, I find that if the innovation signal is relatively strong and the firm’s upgrade costs are relatively low that socially inefficient upgrades will be made. However, a stronger incentive to signal may result in a net gain to welfare by offsetting previously existing distortions. It is well known that under full information a monopolist has less marginal incentive to make costly upgrades than does the social planner due to the firm’s inability to appropriate all of the benefits of the innovation (Arrow, 1959). In the present context, a firm may reject a product innovation whose upgrade expense is justified by the increase in welfare but not profits. By providing an additional incentive to make a product upgrade, the innovation signal alleviates Arrow’s underinvestment problem and so may increase welfare.

Though welfare may or may not rise, I show the equilibrium effect of consumers’
initial lack of information is to unambiguously lower expected profits. This arises from
the fact that significantly improved versions are not fully rewarded by consumers wary
of the trivially new products firms may occasionally introduce. By the same logic I
find that the more difficult it becomes for consumers to learn about product quality
from experience the lower will be expected profits.

Relation to prior literature

I model a situation close to that of Milgrom and Roberts (1986), who themselves
formalized ideas proposed by Nelson (1970 and 1974). In their model, a monopolist
has private information about its exogenously determined product quality and must
choose price and dissipative advertising expenditures that induce beliefs among con-
sumers who are uninformed in the first period but informed thereafter. As Milgrom
and Roberts describe it, theirs is a model “...in which the firm’s R&D effort has gen-
erated a product of some particular given quality that the firm must decide how to
introduce.” I instead consider the information content of the antecedent decision of
whether such a product should be introduced at all.

The Milgrom and Roberts result has many variants and extensions applied to
monopoly (Kihlstrom and Riordan, 1984; Wilson, 1985; Horstmann and MacDonald,
1994; Daughety and Reinganum, 1995), duopoly (Fluet and Garella, 2002; Yehezkel,
2008) and oligopoly (Janssen and Roy, 2010). Price signaling can even occur in a one-
period model when some proportion of consumers is informed about product quality
(Bagwell and Riordan, 1991; Linnemer, 2002). The common thread among each of
these models is a firm choosing marketing variables such as price or advertising to sig-
nal its exogenous quality. I abstract from these already well-understood mechanisms
to focus on a new one: the product adoption decision itself. Because every “new and
improved” product is the output of a random R&D process that has survived the
firm’s censoring rule, the existence of the new product version may serve as a signal
of improved quality.

To a limited extent then, my firm has some control over its product’s quality. However, my model differs from the endogenous quality literature that focuses on the moral hazard problem of the firm (see Spence, 1977; Wolinsky, 1983; and Miklós-Thal and Schumacher, 2013; among others). In that literature stream, it is assumed the firm derives a cost benefit from supplying a low-quality product while purporting it is of high-quality. Consequently, a firm with both high quality and cost may suffer from consumer wariness of being cheated and thus a corresponding low willingness to pay.² My model differs from such models in two main respects. First, the unobservable component of my firm’s quality “choice” is limited to accepting or rejecting a random R&D outcome, not a deterministic choice as in endogenous quality models. Second, I have no difference in production costs between types, a crucial component of endogenous quality models.

I focus on innovation signaling as a particular way to transmit private information from sellers to buyers, though other mechanisms exist. Crawford and Sobel (1982) showed that coarse information can be conveyed even when firms can costlessly make claims whose truth is unverifiable to consumers. The ensuing cheap talk literature contains numerous extensions of this result, including the ability of such communication to induce more consumer search (Mayzlin and Shin, 2011; Gardete, 2013) or make comparative claims about a product (Chakraborty and Harbaugh, 2014). The driving force of my model differs from these because it is the existence of the new product version that serves as the firm’s message, and product launch and upgrade expenses imply this message is costly to send.

²Various mechanisms have been proposed to ameliorate this problem, including reputation or offering a brand name as collateral (Spence, 1977; Klein and Leffler, 1981; Allen, 1984; Wernerfelt, 1988), price signaling (Wolinsky, 1983) as well as risk-sharing devices such as warranties (Grossman, 1981) and money-back guarantees (Mann and Wissink, 1988). Biglaiser (1993) models middlemen as quality guarantors while Miklos-Thal and Schumacher (2013) examine the role of third-party monitors.
The finding that “new” versions may differ only trivially from the old relates to prior work in which the firm degrades or otherwise denies the consumer the full value of its product. For example, Denekere and McAfee (1996) showed that firms can price discriminate by “crimping” a product—that is, degrading its performance and selling both the high and low quality versions. Oligopolists may engage in “planned obsolescence” whereby they produce goods with uneconomically short useful lives, forcing customers to make otherwise unnecessary repeat purchases (Bulow, 1986). And Moorman et al. (2012) found evidence some firms withhold innovative new products from the market for strategic reasons.

While I show the private benefit from signaling quality can induce more new product launches and raise welfare, other examples of seemingly excessive signaling actions increasing welfare can be found. Glazer and Konrad (1996) observe that individuals who make charitable donations to signal their status or wealth can raise total welfare if they fund public goods. In Denicolò (2008) a firm signals compliance costs are low by voluntarily over-complying with a regulation, thereby inducing stricter regulation that harms its competitors and possibly raises welfare. Leppämäki and Mustonen (2009) show that workers may contribute to open source software to signal their skill level, while it is well known that the signaling incentive to over-educate can raise welfare if education has positive externalities (Spence, 1973).

Finally, since I rule out disclosure or other forms of direct communication of information the model best applies to experiential or hedonic attributes of new products, which by their nature are difficult to value before use. For example, technical units of quality are often difficult to interpret, as Kamenica (2008) points out (“How much more...would you be willing to pay for a flashlight that delivers 40 rather than 35 lumens of light?”). However, it should be noted that I do not require consumers to be completely ignorant of the value of a new version since the model can be interpreted as applying to the residual uncertainty that remains after consumers have received
any information regarding new non-hedonic attributes.

The rest of the article proceeds as follows. Section 2 explains my assumptions and introduces the base model. In Section 3 I establish the existence of equilibria and discuss welfare results. Section 4 presents two extensions. In the first the firm exerts observable effort to improve its distribution of innovations while in the second consumer learning about product quality is imperfect. I then conclude and discuss areas for future research.

2 Model

A monopolist performs research and development (R&D) to improve the quality of its product, the outcome of which is a random variable $a$ with continuous log-concave density $f$ with full support on $[a, \bar{a}] \subseteq \mathbb{R}$ and distribution $F$. This distribution is common knowledge and exogenous though later in an extension I endogenize it. Once an innovation is realized, the firm privately observes its value and makes its new product adoption decision. It may sell a new version with the attribute developed in the R&D stage for a one-time fixed cost of $M > 0$ or avoid these costs by maintaining the existing version of the product. Captured in $M$ are new product launch costs such as the cost to alter production facilities or marketing expenditures that inform consumers of the existence of the new product version and for this reason I assume it is independent of the realization of $a$.

The firm’s innovation signal to consumers is binary: either it sells a new version or not. The firm might not adopt the innovation it developed if the improvement is deemed too marginal or too costly, or if the firm fears consumers will not make positive enough inferences about it. In this case new product launch costs can be avoided by rejecting the R&D outcome in favor of continuing to sell the old version of the product, the quality of which consumers are informed.
If the firm does introduce a new version by adopting its innovation potential consumers do not know its value but do know the distribution from which it was drawn. In addition, I assume there is no credible direct way the firm can provide information about the innovation. Consumers have unit demand in each of two periods and purchase in the first period if and only if

$$E[U_i] = v_i + E_{\mu} [a] - p_1 \geq 0,$$

where consumer $i$ has idiosyncratic valuation $v_i$ for the product which is i.i.d. across consumers on $[0, \tau]$ with continuous density $g$ and distribution $G$; $E_{\mu} [a]$ is consumers’ expectation of innovation $a$ given their beliefs $\mu$, where for simplicity $a$ is assumed to be commonly valued; and $p_1$ is the first period price charged. Thus the addition of an innovation does not change the dispersion of the distribution of consumers’ valuations and so results in a shift in demand.\(^3\) I initially assume all consumers learn the realization of $a$ at the end of the first period, through either their own perfectly informative consumption experience, word-of-mouth communications, or product review websites.\(^4,\,5\) Therefore second period consumers are fully informed and buy if and only if

$$U_i = v_i + a - p_2 \geq 0,$$

where $p_2$ is the second period price charged. I later conceptualize learning from consumption as a noisy process in which second period consumers Bayesian update their beliefs after observing the sum $a + \epsilon$, where $\epsilon$ is a mean zero error term.

\(^3\)Hence I do not analyze rotations in demand (Johnson and Myatt, 2006).

\(^4\)Thus there is no role for strategic buying to acquire information. See Grossman, Kihlstrom and Mirman (1977) for treatment of this subject.

\(^5\)Just as price and advertising may signal quality in a one-period setting in which some consumers are informed (Bagwell and Riordan, 1991; Linnemer, 2002), so too can the innovation signal, as shown in a previous version of this paper in which the role of second period informed consumers is replaced with a proportion of first period consumers informed of the firm’s R&D outcome. The current model allows innovation signaling to be analyzed separately from price signaling.
Utility maximization gives rise to demand function \( q(E_\mu[a], p) = 1 - G(p - E_\mu[a]) \), which represents the proportion of consumers that buy each period at price \( p \) given their posterior expectation \( E_\mu[a] \). Note that consumers know their valuation of the old version and in this case \( a = 0 \) can be used in the equations above. Thus all consumers begin the game informed and become uninformed only if the firm introduces a new version of its product. This feature of the model differs from many other signaling games and simplifies its structure. In addition, the firm has a constant marginal cost of production which is normalized to zero and no fixed costs, and these are not affected by the R&D innovation \( a \).

The equilibrium concept is perfect Bayesian equilibrium subject to a restriction on off-the-equilibrium-path beliefs explained below. In general consumers can form beliefs about the firm’s type given its decision to sell a new or old product and its price. However, by construction in my case there is no means by which price can serve as a signal: there are no cost differences across types nor do benefits vary by the assumption that all consumers learn each firm’s type at the end of the first period.\(^6\) I can therefore reasonably expect consumer beliefs to be invariant to price and thus assume that off the equilibrium path consumers do not update their beliefs about firm type based on price.

3 Equilibrium characterization and properties

I’m actually as proud of the things I haven’t done as the things I have done. Innovation is saying ‘no’ to 1,000 things. —Steve Jobs\(^7\)

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\(^6\)As discussed in Banks and Sobel (1987), without such type-dependent payoff differences standard forward-induction refinements do not apply. In Milgrom and Roberts (1986) price (and advertising) may signal exogenously determined product quality because of their requirement that consumers purchase the product in order to learn its type.

Like any other decision, introducing a new product or retaining the old version entails costs and benefits. When consumers are fully informed of quality, the tradeoff is a relatively straightforward one between the additional cost to launch a new version and the increased profit margins obtained by selling a more highly valued product. However, when consumers are unsure of quality their beliefs play a crucial role in determining profits and thus may result in different adoption decisions. A social planner that places weight on consumers’ well-being may reach a still different decision than a firm regarding which new products to launch.

In this section I show that when consumers are initially uninformed of quality, a firm’s binary decision to implement an R&D outcome or not involves a trade-off between inducing a “new product” premium in the first period by incurring new product launch costs and selling to consumers who will be fully informed of the product’s value in the second period. For a low enough type \( a \), the launch costs and potential decline in second period profits exceed the benefits conferred by the new product premium and thus the firm censors its R&D outcome. Though uninformed, first period consumers place a demand premium on new products because they know the firm needs to earn profits from informed consumers in the second period to recoup its product launch cost \( M \).

I show the firm’s adoption strategy consists of a simple threshold whereby a new version is launched when its quality is sufficiently high and the existing version is maintained otherwise. Note the use of a threshold strategy is not particular to the asymmetric information case but also appears when consumers have full information or if a social planner were in charge. In subsequent sections I will explore the effect these different threshold values have on profitability and welfare.

I now characterize the equilibrium. Let \( \pi(E_\mu[a]) \) be the maximum profits gross of \( M \) attainable in a period when consumers have expectation \( E_\mu[a] \), let \( p_{E_\mu[a]} = \arg\max_p \pi(E_\mu[a]) \) be the profit maximizing price, and let \( \delta \) be the discount factor.
between periods. Conjecture the firm’s adoption strategy consists of a threshold whereby a new version is launched when its quality is sufficiently high and the existing version is maintained otherwise. If so, the threshold type $a^*$ is indifferent to selling a new product and thus

$$\pi (E [a \mid a > a^*]) + \delta \pi (a^*) - M = (1 + \delta) \pi (0).$$

(3)

I assume away the case where the firm never introduces a new version by specifying $\pi$ high enough that this type always prefers to adopt its innovation. This leads to the following equilibrium characterization.

**Proposition 1** In the unique equilibrium there exists an adoption threshold $a^*$ such that the firm modifies its product whenever $a > a^*$ and otherwise does not. In period 1 all “new and improved” types sell for the high price $p^H = p_{E[a \mid a > a^*]}$ while all unmodified types sell for the low price $p_0$. In period 2 each type sells for its full information monopoly price. When $\pi(E[a]) + \delta \pi(a) - M \leq (1 + \delta) \pi(0)$ the equilibrium is partially separating with $a^*$ defined by line (3), and otherwise $a^* = a$ so that all types sell a new product.

**Proof** The firm uses a threshold strategy because higher types always have more incentive to sell a new version for any given beliefs. For the same reason an equilibrium involving mixed strategies cannot exist. Consider a candidate equilibrium in which there is a threshold type-$a^*$ firm which should be indifferent to selling a new version or not. By backward induction it receives a discounted payoff of $\delta \pi(a^*)$ in period 2. In period 1 any modifying type will price pool on $p^H = p_{E[a \mid a > a^*]}$ provided off-the-equilibrium-path beliefs following an unexpected price are not greater than $E[a \mid a > a^*]$, a condition satisfied because prices do not affect beliefs.

Let the gains and losses to the threshold type-$a^*$ firm from modifying its product be $G(a^*)$ and $L(a^*)$, respectively. A gain is derived from first period consumers who
are uninformed and is the increase in payoffs from selling the new version instead of the old: \( G(a^*) = \pi(E[a | a > a^*]) - \pi(0) \). The loss is the cost \( M \) to launch the product plus the discounted decline in profits, if any, from second period informed consumers: \( L(a^*) = M + \delta [\pi(0) - \pi(a^*)] \). \( G \) is continuous and monotonically increasing in \( a^* \) while \( L \) is continuous and monotonically decreasing in \( a^* \). By the assumption that type \( a \) innovation is always adopted I conclude that \( G(a^*) > L(a^*) \) for high enough \( a^* \) and thus \( G(a^*) \) and \( L(a^*) \) either cross at a unique point or not at all. Finally, \( \pi(E[a]) + \delta \pi(a) - M \leq (1 + \delta) \pi(0) \) is the necessary and sufficient condition for a unique crossing and is derived by setting \( G(a^*) < L(a^*) \) in the limit as \( a^* \to a \). ■

See Figure 1 for a visual depiction of the proof. Reading from right to left, as the proposed equilibrium threshold decreases the \( G \) curve decreases and the \( L \) curve increases towards their respective asymptotes. The \( L \) asymptote exceeds the \( G \) asymptote (i.e., the separating condition in the proposition is satisfied) so the two curves have a unique intersection \( a^* \) which is the equilibrium threshold since this type is indifferent to selling a new product. Note that off-the-equilibrium-path beliefs with respect to the product launch decision do not exist since both possible actions occur and so no action is off the equilibrium path.\(^8\) Off-the-equilibrium-path beliefs are invariant with respect to price, as discussed in footnote 6.

I have found there will be a threshold firm type indifferent to selling a new version, and for this type beliefs will be more favorable among first period uninformed consumers than second period informed consumers. This is because in the first period consumers form a pooled expectation over all types that would sell a new version while in the second period consumers can discern each type. The upshot is the firm uses a less stringent adoption threshold, since marginal types are enticed by the initial new product premium to introduce new versions that would otherwise be unprofitable.

\(^8\)One may hypothesize a “no new products” equilibrium in which any new product is believed to be the lowest type \( (E_\mu[a] = a) \) in the first period. However, this fails by the assumption that the highest type \( \bar{a} \) will always modify its product; i.e., \( \pi(a) + \delta \pi(\bar{a}) - M > (1 + \delta) \pi(0) \).
had all consumers been informed.

The extra incentive for types to incur the product launch cost \( M \) provided by first period uninformed consumers has implications for firm use of trivial product modifications. Here I define a product modification as trivial if the new product price premium exceeds the value \( a \) of the new attribute. Note that while this definition involves a trade-off between attribute value and price, the marketing literature has generally defined a trivial attribute purely in terms of its value without consideration of price (e.g., Carpenter, Glazer, and Nakamoto, 1994).

**Definition 1** The attribute \( a \) is trivial if \( \Delta p \equiv p^H - p_0 \geq a \) and useful if \( a > \Delta p \).

Thus all consumers that purchase a trivially new product are worse off on net than if the product were never improved. Pushing this idea further, it may even be possible a firm would adopt a *schlimmbesserung* attribute (“improvement for the worse” (Rheingold, 2000)) \( a < 0 \) in order to be initially pooled with more significant improvements. Such types would incur both adoption costs and a decline in second period profits relative to what the old version would have earned. Despite this, in
equilibrium the average new product will indeed be improved as noted below.

**Proposition 2** There exist thresholds $M_t > M_s > 0$ such that trivial attributes are adopted only if the product adoption cost $M \in [M_s, M_t]$ while schlimmbesserung attributes are adopted only if $M < M_s$. Notwithstanding this, in any equilibrium new products are on average improved (i.e., $E[a | a > a^*] > 0$) and thus command a new product pricing premium.

**Proof** See the appendix.

The new product pricing premium is strictly positive and so any type $a < \Delta p$ that adopts its innovation is trivial. Consider an example where $a \sim U[-0.5, 0.5]$, $M = \frac{1}{4}$, $\delta = \frac{9}{10}$ and $q = 1 - p + E[a]$. This implies $a^* \approx 0.136$ and $\Delta p \approx 0.159$ so that $a \in [0.136, 0.159]$ are trivial attributes while $a > 0.159$ are useful. Note marginal consumers who purchase a new product with a trivial attribute will ex-post regret doing so since equation (2) is violated. Intuitively, selling a trivially new version will be profitable provided first period uninformed consumers have a sufficiently high posterior over new products relative to the new product adoption cost $M$. Finally, I make one more definition here that will be useful in a subsequent subsection in which the welfare effects of selling trivially new versions are explored.

**Definition 2** Let $\tilde{a}$ be the welfare maximizing adoption threshold. Define attributes $a < \tilde{a}$ as inefficient and $a \geq \tilde{a}$ as efficient.

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<th>Symbol</th>
<th>Meaning</th>
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<td>$a^*$</td>
<td>Equilibrium adoption threshold</td>
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<tr>
<td>$\hat{a}$</td>
<td>Full information adoption threshold</td>
<td>$a^* &lt; \hat{a}$</td>
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<tr>
<td>$\tilde{a}$</td>
<td>Social planner’s adoption threshold</td>
<td>$a &lt; \tilde{a}$ are inefficient, $a \geq \tilde{a}$ efficient</td>
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<tr>
<td>$\Delta p$</td>
<td>New product price premium</td>
<td>$a \leq \Delta p$ are trivial, $a &gt; \Delta p$ useful</td>
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Thus I contemplate a social planner committing the firm to an adoption threshold, conditional on uninformed consumers’ inferences and the firm’s pricing decisions in the two-period model. As I will show, the threshold that maximizes the sum of expected profits and consumer surplus could be higher or lower than the firm’s threshold \( a^* \). Table 1 above summarizes the definitions from this section and indicates when useful or trivial attributes might exist. Figure 2 illustrates the existence ranges for these attribute types for the example under consideration. Improvements above the threshold \( a^* \) are adopted while those below are not. In the example \( \tilde{a} < \Delta p < \hat{a} \) though this need not hold in general. Notice there may exist attributes that are both efficient and trivial. This is because triviality of a product improvement is judged by consumer welfare, while efficiency is judged by total welfare, and monopoly pricing drives a wedge between the two.

### Profits and firm commitment

I now explore the profit implications of the more lax adoption threshold that is used when consumers are initially uninformed. The proposition below shows that a lower threshold reduces profits and thus the firm would prefer to commit itself against its ex-post incentive to more freely introduce new products. In addition, if consumers could observe product quality from the beginning then the game of symmetric information that follows would yield higher profits.
**Proposition 3** Let $G$ be concave. Compared to the game where consumers are initially uninformed of quality, the firm’s expected profits are higher when either:

(i) all consumers are fully informed, or

(ii) the firm commits to using the full information threshold $\hat{a}$.

**Proof** See the appendix.

The firm therefore faces a commitment problem in determining its adoption threshold: although using $\hat{a}$ is ex-ante profit maximizing the type $\hat{a}$ firm strictly prefers adoption, leading to a lower equilibrium threshold and therefore lower profits. Profits are lower in this case because the immediate gain from adopting an $a < \hat{a}$ is offset by two factors. First, consumers learn the product’s quality in the second period and so the initial benefit does not last. Second, such types that adopt impose a negative externality on all higher types: more significant improvements made to the product will not initially be rewarded enough because consumers anticipate the firm occasionally introduces marginal or even trivial new products.

The finding that the firm prefers to commit to the full information threshold also has useful implications for the disclosure of product quality. Although not modeled here, if the firm were capable of credibly disclosing its product’s quality at no cost one can see through an unraveling argument that all quality types would disclose. Then since consumers will always be informed the full information threshold $\hat{a}$ would be used, resulting in higher firm profits. However, the implications for welfare are less clear, as discussed below.

**The underinvestment problem**

It has been shown that the incentive to signal a “new and improved” product leads firms to adopt new attributes that in a full information environment do not justify the firm’s product launch costs, and that may even be harmful. Given this result it might seem welfare must be lower than if consumers could immediately learn the
exact quality of a new attribute. However, even with full information the firm’s adoption decision is already inefficient. As Arrow (1959) observed, a firm does not consider the gains to consumer surplus from adopting an innovation so it will tend to underinvest. Given this problem, I show that the signaling incentive to adopt a new innovation and receive a demand and pricing premium from uninformed consumers might on average lead to higher rather than lower efficiency.

To see how the innovation signaling effect can mitigate or even reverse the Arrow underinvestment problem, first consider the monopolist’s adoption decision when consumers are fully informed in both periods. The type-\(a\) monopolist adopts its innovation whenever its increase in profits, as seen in regions B, C and D in the left panel of Figure 3, exceed the product launch cost \(M\). Recall the type \(\hat{a}\) firm is indifferent to adoption whereas types \(a > \hat{a}\) strictly prefer to adopt the attribute. In contrast, a social planner that takes monopoly pricing to fully informed consumers as given is indifferent to adoption when the increase in total surplus from selling the new product, as seen in regions E, C and D in the left panel of Figure 3, equals \(M\). Because the social planner has regard for consumer surplus his threshold must be less than the monopolist’s and the planner strictly prefers adoption for the type \(\hat{a}\) firm.

The same incentives underlying the monopoly underinvestment problem arise in my model with asymmetric information but they are counteracted by the incentive to signal to first period uninformed consumers. First consider the equilibrium determination of the investment threshold \(a^*\) and hypothesize \(a^* = \hat{a}\). But then the type \(\hat{a}\) firm must strictly prefer investment because of the demand premium it receives from first period consumers who infer quality \(E[a | a > \hat{a}]\) and so buy more of the new product at a higher price than they would if informed. The resulting higher profits imply it must instead be \(a^* < \hat{a}\). The right panel of Figure 3 displays this argument graphically by drawing the informed demand curve for a new product of type \(\hat{a}\). Additional purchases by uninformed consumers reduce the deadweight loss of monopoly
in region $F_1$, whereas the price premium consumers pay results in a transfer of regions $E_2$ and $T$ to the firm. Therefore the type $\hat{a}$ firm must strictly prefer adoption of the new attribute.

The determination of the social planner’s preferred threshold is more complicated because it involves consideration of a type’s investment on the new product demand premium enjoyed by all other types that adopt. Thus, in the right panel of Figure 3 although the type $\hat{a}$ firm facing uninformed first period consumers prefers adoption in part due to the reduction in deadweight loss $F_1$, it does not consider the effect this adoption has on higher types that might have been realized. In fact, a lower type adopting implies a weaker innovation signal and therefore a lower demand premium for all other adopting types. Since the planner considers such externalities, is indifferent to transfers $E_2 + T$, but has regard for consumer surplus $E_1$, a general comparison of the planner’s threshold to the firm’s is difficult.

The proposition below characterizes when consumers’ lack of information actually increases welfare by incentivizing the right amount of additional new product
launches. I then present an explicit example in which this occurs. Intuitively, welfare will be higher when product launch costs are sufficiently large that: (1) the Arrow underinvestment problem to be solved is severe, and (2) the firm is deterred from adopting marginal, welfare reducing innovations.

**Proposition 4** Consumer uncertainty about new product quality mitigates the underinvestment problem and raises welfare if and only if the new product adoption cost exceeds a threshold value $\tilde{M}$.

**Proof** See the appendix.

**Example 1** In Figure 2 an example with $q = 1 - p + E[a]$, $a \sim U[-\frac{1}{2}, \frac{1}{2}]$, $M = \frac{1}{4}$, and $\delta = \frac{9}{10}$ was presented. In period 1 all types greater than $a^* \approx 0.136$ adopt their innovation and charge a price $p^H \approx 0.659$ and types $a \in [a^*, \Delta p] \approx [0.136, 0.159)$ are trivial. Each type charges its own full information monopoly price in period 2. Notwithstanding the addition of some trivial types the monopoly underinvestment problem is mitigated and expected welfare is higher under imperfect information (0.814) than full information (0.812).

Finally, the right panel of Figure 3 can help resolve a seeming oddity that first appeared in Figure 2: the existence of types that are both efficient and trivial. Let the informed demand curve for a type $a \in [\tilde{a}, \Delta p]$ be given in the right panel of Figure 3. Then marginal purchasing consumers make transfer payment $T$ to the firm and therefore ex-post regret buying such a product because of the trivial attribute. Nonetheless, because the type is efficient the increased output and resulting reduction in deadweight loss $F_1$ increase expected surplus.
4 Extensions

4.1 Ex-ante investment to develop quality-enhancing innovations

I now consider an extension of the main model in which the firm makes an ex-ante investment to improve the R&D distribution and ask how the innovation signal may affect the incentive for such an investment. Let $e \geq 0$ be a publicly observable effort level that incurs increasing cost $C(e)$ with $C(0) = C''(0) = 0$ and results in a stochastic R&D outcome $a(e) \equiv a + e$. Thus more effort incurs a greater cost but also increases the likelihood of more valuable innovations being developed. Note this extended model includes the original as a subgame and so the equilibrium characterization in Proposition 1 goes through but now the adoption threshold depends on $e$ and the firm chooses $e$ to maximize expected profits in the subgame, $\Pi(e)$, net of $C(e)$.

What is the firm’s incentive to improve its distribution of R&D outcomes? Although statically greater ex-ante investment provides a benefit in that it makes higher realizations more likely and increases consumers’ expectations of quality, the equilibrium effect of greater $e$ is to lower the adoption threshold in the subgame by inducing more marginal types to introduce a new product version. This tends to weaken the innovation signal. I first clarify the net effect of $e$ on $E[a | a > a^*]$ and profits gross of $C(e)$ in the lemma below.

**Lemma 1** A better distribution of R&D outcomes from higher $e$ induces a lower equilibrium threshold $a^*$ but a higher innovation signal, and provides the firm a positive expected benefit. Mathematically, $\frac{\partial a^*}{\partial e} < 0$, $\frac{\partial E[a | a > a^*]}{\partial e} > 0$, and $\frac{\partial \Pi(e)}{\partial e} > 0$.

---

9The public observability of R&D effort is a reasonable approximation of the fact that some firms are known for engaging in higher levels of R&D than others. If more R&D intensive firms draw their product improvements from a better distribution this will have implications for the innovation signal.
Proof See the appendix.

The lemma leads to several useful observations. First, while the adoption threshold \( a^* \) decreases in \( e \), the same is not true of \( \hat{a} \) as it only depends on the realized value of the innovation, not its distribution. This fact is recorded below.

Remark 1 The firm’s full information threshold \( \hat{a} \) is invariant to \( e \).

Next, the lemma shows that the firm receives a benefit from effort since it makes more valuable improvements more likely to be developed. However, the firm ignores surplus it creates but does not capture, leading to the following remark.

Remark 2 Conditional on any subgame threshold the Arrow underinvestment problem exists for the ex-ante effort decision.

I now consider the welfare effects of uninformed consumers in the extended model. It was shown that in the subgame the adoption threshold was too low, a fact that has implications for welfare in the subgame but also in the full game through its effect on the marginal benefit from ex-ante effort. To see this, note that the benefit from effort is only realized for those states for which an innovation is adopted, and thus a lower adoption threshold implies a greater benefit from effort. For this reason a threshold that is too low in an ex-post sense has the compensating feature that it incentives more ex-ante effort, which could improve welfare or even lower it if effort becomes too high. Further confounding matters, by Lemma 1 the subgame adoption threshold itself decreases in the amount of effort.

To untangle these effects I calculate the marginal private and social benefit from effort, which can be expressed in terms of the total derivative:

\[
\frac{d\Pi}{de} = \frac{\partial \Pi}{\partial a} \frac{\partial a}{\partial e} + \frac{\partial \Pi}{\partial e}, \quad (4)
\]

\[
\frac{dW}{de} = \frac{\partial W}{\partial a} \frac{\partial a}{\partial e} + \frac{\partial W}{\partial e}, \quad (5)
\]
where I denote a generic threshold by $a$. Notice the direct effect of effort on welfare (gross of the effort cost), $\frac{\partial W}{\partial e}$, is clearly positive while the sign of the indirect effect $\frac{\partial W \partial a}{\partial a \partial e}$ depends on $\frac{\partial W}{\partial a}$, which is negative only if the threshold exceeds $\tilde{a}$. Since $\frac{\partial a^*}{\partial e} < 0$ by Lemma 1, $\frac{\partial W(a^*)}{\partial a^*} > 0$ when $a^* > \tilde{a}$. In contrast, when consumers are informed $\frac{\partial \tilde{a}}{\partial e} = 0$ by Remark 1 and so the indirect effect is 0 in this case, implying $\frac{dW(a^*)}{de} > \frac{dW(\tilde{a})}{de}$ when $a^* > \tilde{a}$. Then provided the profit maximizing effort levels when consumers are uninformed and informed (denoted $e^*$ and $\tilde{e}$ respectively) are sufficiently close welfare will be higher when consumers are uninformed. These points are clarified in the last proposition.

**Proposition 5** Suppose $C'(e) = (e + 1)^n - 1$ for $n \geq 1$. There exist thresholds $\tilde{n}'$ and $\tilde{M}'$ such that when $n$ and $M$ exceed these respective thresholds $W(a^*, e^*) - C(e^*) > W(\tilde{a}, \tilde{e}) - C(\tilde{e})$. That is, when the adoption cost is sufficiently high and the marginal cost of effort increases sufficiently fast welfare in the full game is higher when consumers are initially uninformed than informed.

**Proof** See the appendix.

Although a specific functional form for $C'(e)$ was selected for the proposition, it can be seen from its proof that this is for convenience only; all that is required of $C'$ is that it increase “fast enough.” Below I present a final example that adds an ex-ante investment stage to Example 1 and uses a different functional form for $C$ and $C'$.

**Example 2** Let $q = 1 - p + E[a]$, $a \sim U[-\frac{1}{2} + e, \frac{1}{2} + e]$, $M = \frac{1}{4}$, $\delta = \frac{9}{10}$, and the cost of effort be given by $C(e) = \frac{16e - 1}{16 \ln 2}$. Then the ex-ante underinvestment problem is mitigated as the imperfect information firm chooses an investment level of $e^*_{II} \approx 0.665$ which is greater than that chosen under full information, $e^*_{FI} \approx 0.628$. Total welfare is higher under imperfect information (1.331) than full information (1.304).
4.2 Imperfect learning

In the previous sections I assumed a simple learning structure that enabled all consumers to determine the value of the new product attribute after the first period. Learning occurred from personal consumption experiences or secondary sources such as word-of-mouth communications or product review websites. Whereas such information was not available to consumers in the first period when the product was new, it is assumed by the second period it is widely available at no cost.

I now conceptualize learning as an imperfect process from which only some of consumers’ uncertainty about a product’s value is resolved. Such products are of practical interest and lay along a continuum between two theoretical extremes: Nelson’s (1970) experience good, for which all uncertainty is resolved after consumption, and Darby and Karni’s (1973) credence good, for which the consumer learns nothing from consumption. In this subsection I allow for imperfect, or noisy, learning from consumption and other sources and show that the main results from the model with perfect learning generalize to this context. In addition, I develop managerially relevant insights regarding the effect noise has on profits and the quality of inferences consumers make about new products.

I operationalize noisy learning by assuming each consumer receives a common signal $x = a + \epsilon$ after the first period that contains information about the realization of $a$ as well as an independent mean zero error term $\epsilon$ whose distribution is common knowledge. I assume $f$ and $h$, the densities of $a$ and $\epsilon$ respectively, are continuous with full support on $\mathbb{R}$ and $h$ is log-concave so that consumers’ posterior mean $E[a \mid x]$ is increasing in the noisy signal $x$.\textsuperscript{10,11} Including a common error term is justified on the grounds that consumers, whether or not they made a first period purchase, may

\textsuperscript{10}The assumption of full support simplifies the presentation of the results.

\textsuperscript{11}This ensures $L(a^*)$ is monotonic and thus a unique attribute adoption threshold strategy exists. Consumers’ posterior mean is increasing in $x$ when $x$ and $a$ are affiliated, a sufficient condition for which is the log-concavity of $h$ (Milgrom and Weber, 1982).
receive the same information from influential experts, product review websites, blogs, or word-of-mouth communication. For this reason I assume the firm observes the signal $x$ as well.

I now characterize the equilibrium with imperfect learning and compare its properties to that found in Section 3 under perfect learning.

**Proposition 6** Let consumers learn about product quality from noisy signal $x = a + \epsilon$. Then the equilibrium characterization from Proposition 1 applies except that in period 2 all modified types charge the optimal price given the consumption signal $x$ consumers have received. This results in a lower attribute adoption threshold $a^*$ and weaker innovation signal $E[a \mid a > a^*]$ when learning is noisy than when it is perfect. Finally, expected profits are lower provided $G$ is concave.

**Proof** See the appendix.

I thus conclude greater noise exacerbates the frictions caused by asymmetric information. A more noisy consumption experience renders learning more difficult and thus decreases the likelihood that consumers will detect the firm’s true type. When forming his posterior the consumer will place less weight on the signal and more on prior beliefs, which benefits low types. Knowing this, firms are emboldened on the margin to adopt relatively minor innovations so that the adoption threshold $a^*$ decreases, thus weakening the innovation signal. Ex-ante profits decline as the likelihood of introducing more marginal products increases.

Another interpretation is that an easier learning environment induces firms to apply a more stringent standard to releasing new products. While any new product will initially enjoy a demand premium, products of marginal quality will be quickly exposed in an easy learning environment, causing profits to be too low to justify product launch. Thus the firm will not introduce such products in the first place but will only do so when their quality is sufficiently high.$^{12}$

$^{12}$Metacritic.com co-founder Marc Doyle echoes this sentiment when he speculates that giving
The result that profits are lower in a more noisy learning environment has managerial relevance. Although a difficult learning environment may at least temporarily shroud a firm’s products of marginal quality from being recognized as such, it also makes proving the value of high quality products more difficult. Even if the consumer of a high quality product has a positive experience with it, he knows that in a difficult learning environment experiences are more volatile and is wary of the low type products that exploit this fact. The proposition above tells us that on net firms benefit the easier it is for consumers to learn from their consumption experiences. Thus I find if the firm is able to ease the learning environment through product design or marketing communications it has incentive to do so.

Example 3 I extend Example 1 in which \( q = 1 - p + E[a] \), \( a \sim U \left[ -\frac{1}{2}, \frac{1}{2} \right] \), \( M = \frac{1}{4} \), and \( \delta = \frac{9}{10} \), by allowing imperfect learning from the signal \( x = a + \epsilon \) where \( a \) is the firm’s true type and \( \epsilon \sim U [-0.1, 0.1] \). The adoption threshold of \( a^* \approx 0.106 \) is less than the perfect learning threshold of 0.136, leading to a weaker innovation signal. In period 1 all “new and improved” types set \( p^H \approx 0.651 \) whereas types below the adoption threshold sell their existing product and set price \( \frac{1}{2} \). In period 2 consumers form a posterior mean from signal \( x \) and the firm sets a price accordingly.

Like many, I used to be suckered into seeing movies or buying games based on glowing review quotations in magazines or newspapers (“One of the year’s best!”) from critics nobody has heard of or from skilled PR department writers. A site like ours helps people cut through that unobjective promotional language. By giving consumers information on the objective quality of a game, not only are they more educated about their choices, but it forces publishers to demand more from their developers, license owners to demand more from their licensees, and eventually, hopefully, the games get better.

5 Conclusion

This paper investigates firms’ incentive to introduce improved products and the welfare implications of these improvements when consumers are uncertain of the quality of the improvement. I show that information is revealed by the very existence of a new product that has survived a firm’s endogenous censoring rule so that product “newness” alone signals higher quality on average and hence confers a pricing and demand premium.

This premium induces the firm to adopt attributes that would prove unprofitable had all consumers been informed. This fact has two main consequences. First, firms may sell a “new” product only trivially different from their older version in the sense that its improvement in performance does not justify its higher price. However, the greater the likelihood of such products the weaker will be the inferred value of the innovation signal, and it is shown this lowers the firm’s ex-ante profits. The second consequence of a more lax attribute adoption policy is for welfare. By incentivizing the introduction of new versions, the innovation signaling effect can offset the existing monopoly underinvestment problem. For this reason it is possible that welfare can be lower when consumers are fully informed.

I show the robustness of these results by extending the model to a noisy learning environment in which consumers become better, though not perfectly, informed of the new product’s value after the first period. I find the more difficult it is for consumers to learn a product’s quality, the more incentive a firm has to introduce marginal improvements. Finally, an extension in which the firm takes effort to affect the distribution of R&D outcomes shows the qualitative results of the model continue to hold. Potential extensions of the model include allowing the monopolist to concurrently sell the old and new versions of its product, and generalizing the model to an oligopoly context.
6 Appendix: proofs

Proof of Proposition 2 \( E[a \mid a > a^*] \) follows from the equilibrium construction proof from Proposition 1. To establish the threshold values for \( M \), note that if \( M = 0 \) then \( a^* < 0 \) and thus schlimmbesserung improvements will occur. By line (3) \( \frac{\partial a^*}{\partial M} > 0 \), proving the existence of \( M_s > 0 \) such that schlimmbesserung occurs only if \( M < M_s \). Next, establishing \( 0 < \frac{\partial p^H}{\partial M} < \frac{\partial a^*}{\partial M} \) is sufficient to prove \( p^H - p_0 \geq a^* \) for all \( M \) below a threshold value \( M_t > M_s \). Since \( \frac{\partial a^*}{\partial M} > 0 \), \( E[a \mid a > a^*] \) and \( pE[a \mid a > a^*] = p^H \) also increase in \( M \). The log-concavity of \( f \) implies \( \frac{\partial E[a \mid a > a^*]}{\partial a^*} < 0 \) and thus \( \frac{\partial p^H}{\partial E[a \mid a > a^*]} < 1 \). Finally, since demand is downward sloping \( \frac{\partial p^H}{\partial M} < \frac{\partial a^*}{\partial M} \).

Proof of Proposition 3 I first prove a stronger claim than (ii): commitment to the threshold \( a_2^* \) results in higher profits than \( a_1^* \) whenever \( a_1^* < a_2^* \leq \hat{a} \). I restrict attention to \( a > a_1^* \) since otherwise profits are equal for either threshold and define \( t = \frac{F(a_2^*) - F(a_1^*)}{1 - F(a_1^*)} \) and \( 1 - t = \frac{1 - F(a_2^*)}{1 - F(a_1^*)} \). Since a lower threshold always results in lower second period profits, it suffices to consider the first period and show

\[
(1 - t) \left[ \pi(E[a \mid a > a_2^*]) - M \right] + t \pi(0) > \pi(E[a \mid a > a_1^*]) - M.
\]

Using the equilibrium substitution \((1 + \delta) \pi(\hat{a}) = (1 + \delta) \pi(0) + M\) it suffices to show

\[
(1 - t) \pi(E[a \mid a > a_2^*]) + t \pi(\hat{a}) > \pi(E[a \mid a > a_1^*]).
\] (6)

Profits are convex in \( a \) by the assumption that \( G \) is weakly concave\(^{13}\) so that the left

---

\(^{13}\)Profits \( \pi(a) = \max_p p(1 - G(p - a)) \) are convex in \( a \) by the envelope theorem when \( \frac{\partial^2}{\partial a^2} p(1 - G(p - a)) \geq 0 \iff -G''(p - a) \geq 0 \); i.e., when \( G \) is weakly concave.
hand side of (6) is
\[
\begin{align*}
> & \quad \pi (t \, \hat{a} + (1 - t) \, E [a \mid a > a^*_2]) \\
= & \quad \pi \left( \frac{\int_{a^*_1}^{\hat{a}} a \, f(a) da + \int_{a^*_2}^{\infty} a \, f(a) da}{1 - F(a^*_1)} \right) \\
> & \quad \pi \left( \frac{\int_{a^*_1}^{\infty} a \, f(a) da}{1 - F(a^*_1)} \right) \\
= & \quad \pi (E [a \mid a > a^*_1]), \quad (7)
\end{align*}
\]

Finally, that commitment to the full information threshold is preferred to no commitment is shown by letting \( a^*_2 = \hat{a} \) and \( a^*_1 = a^* \).

To prove (i) let \( a^*_2 = \hat{a} \) and \( a^*_1 = a^* \) above. By convexity \( \int \pi (a) \, f(a) \, da > \pi \left( \int a \, f(a) \, da \right) \) so that profits given fully informed consumers are higher than given uninformed consumers using threshold \( \hat{a} \), and thus in turn higher than uninformed consumers using threshold \( a^* \). ■

**Proof of Proposition 4** Consider the equilibrium thresholds \( \hat{a} \) and \( a^* \) as a function of the adoption cost \( M \). The equilibrium condition that the threshold type is indifferent to introducing a new version implies

\[(1 + \delta) \, \pi(\hat{a}) = (1 + \delta) \, \pi(0) + M = \pi (E [a \mid a > a^*]) + \delta \, \pi(a^*).\]

By the fact that \( a^* < \hat{a} \) it must also be \( a^* < \hat{a} < E [a \mid a > a^*] \). In addition, each of these terms is increasing in \( M \). Finally, the log-concavity of \( f \) implies \( E [a \mid a > a^*] - a^* \) is decreasing in \( a^* \) (Bagnoli and Bergstrom, 2005), and thus in \( M \), so that the difference between the thresholds \( \hat{a} \) and \( a^* \) decreases in \( M \). Since the planner’s threshold \( \tilde{a} \) contemplates consumer surplus, \( \frac{\partial \tilde{a}}{\partial M} < \frac{\partial a^*}{\partial M} \) so that the difference between \( \tilde{a} \) and \( \hat{a} \) increases in \( M \), and thus \( \tilde{a} < a^* < \hat{a} \) for sufficiently high \( M \). ■
Proof of Lemma 1  Let \( e_2 > e_1 \). I claim \( a^*(e_2) < a^*(e_1) \) but

\[
E [a(e_2) | a(e_2) > a^*(e_2)] > E [a(e_1) | a(e_1) > a^*(e_1)].
\] (8)

By construction the type \( a^*(e_1) \) is indifferent to selling a new version given the new product’s expected quality \( E [a(e_1) | a(e_1) > a^*(e_1)] \) and therefore this type must strictly prefer to sell a new version given a more favorable distribution and expectation, \( E [a(e_2) | a(e_2) > a^*(e_1)] \). Equilibrium is restored when the threshold decreases until indifference and thus \( a^*(e_2) < a^*(e_1) \).

Now note that threshold types for any distribution \( a(e) \) earn equal (two-period) payoffs since each is indifferent to selling a new version. Since \( a^*(e_2) < a^*(e_1) \), type \( a^*(e_2) \) earns a lower payoff in the second period than type \( a^*(e_1) \) and therefore must earn a higher payoff in the first period. Thus line 8 holds.

Finally, I show there is an expected benefit from effort by establishing that all types receive weakly higher payoffs under \( e_2 \) than \( e_1 \). Given \( e_2 \) all types \( a \geq a^*(e_1) \) are better off by line 8. Next, types \( a \in (a^*(e_2), a^*(e_1)) \) (which sell a new product given \( e_2 \) but not \( e_1 \)) receive a higher payoff than type \( a(e_2) \), who itself is indifferent to selling a new product. Types \( a \leq a^*(e_2) \) are indifferent between \( e_1 \) and \( e_2 \) because in either case they sell their old product version. \( \blacksquare \)

Proof of Proposition 5  By the proof of Proposition 3, for all \( M \) exceeding a threshold value we have \( \tilde{a} < a^* < \hat{a} \). I claim this implies that for a given level of effort \( e' \) for which this chain of inequalities holds, \( \frac{dW(a^*,e')}{de} > \frac{dW(\tilde{a},e')}{de} \). This follows since: (i) \( a^* < \hat{a} \) and the direct effect of \( e \) on welfare, \( \frac{\partial W}{\partial e} \), is greater the lower is the adoption threshold; and (ii) \( \tilde{a} < a^* \) and the indirect effect of \( e \) on welfare, \( \frac{\partial W}{\partial a} \frac{\partial a}{\partial e} \), is positive under asymmetric information but 0 when consumers are informed. The latter is true by Remark 1 (since \( \frac{\partial a^*}{\partial e} = 0 \)), while \( \frac{\partial a^*}{\partial e} < 0 \) by Lemma 1 and \( \frac{\partial W}{\partial a^*} < 0 \) when \( \tilde{a} < a^* \) since \( a^* \) is already too high for the planner.
Next, \( a^* < \hat{a} \) always, but by the proof of Proposition 3 the difference between \( \hat{a} \) and \( a^* \) decreases in \( M \), and so the marginal benefit from effort, \( \frac{d\Pi}{de} \), given in line (4) under asymmetric and full information can be made arbitrarily close to each other for sufficiently high \( M \). This implies that \( e^* \) can be made arbitrarily close to \( \hat{e} \) for high enough \( n \). Thus by \( \frac{dW(a^*,e^*)}{de} > \frac{dW(\hat{a},\hat{e})}{de} \) established above and continuity there exist threshold values for \( n \) and \( M \) such that \( W(a^*,e^*) - C(e^*) > W(\hat{a},\hat{e}) - C(\hat{e}) \).

The following lemma will be used to prove Proposition 6.

**Lemma 2** The threshold type’s expected profits approach zero as the threshold declines. Formally, \( \lim_{a^* \to -\infty} E_{x|a=a^*}[\pi(E[a | a > a^*, x])] = 0 \).

**Proof** Because \( f(x) = h(x - a^*) \), the expected gross profits to the threshold type-\( a^* \) firm \( E_{x|a=a^*}[\pi(E[a | a > a^*, x])] \) equal

\[
\int_{-\infty}^{t} \pi(E[a | a > a^*, x]) h(x - a^*)dx + \int_{t}^{\infty} \pi(E[a | a > a^*, x]) h(x - a^*)dx. \tag{9}
\]

Since \( \forall t, H(t - a^*) \to 1 \) as \( a^* \to -\infty \), \( t \) and \( a^* \) can be selected to assign arbitrarily large mass to arbitrarily negative values of the signal. This implies the integral on the right in (9) can be made to approach 0 since its mass approaches 0 while the integral on the left approaches 0 because \( E[a | a > a^*, x] \to -\infty \) and thus \( \pi(E[a | a > a^*, x]) \to 0 \) as \( a^* \) and \( x \) approach \( -\infty \).

**Proof of Proposition 6** I first characterize the equilibrium. Type \( a^* \)'s first period gain \( G(a^*) \) is identical to that in the perfect learning case as no learning has yet occurred in the first period, while the expected loss \( L \) is

\[
L(a^*) = M + \delta E_x[\pi(0) - \pi(E[a | a > a^*, x])], \tag{10}
\]

where the expectation is taken with respect to the distribution of possible signals the type-\( a^* \)'s consumers receive, with \( x = a + \epsilon \) and the firm knows \( a = a^* \) but \( \epsilon \) is
stochastic. Now, $a_1^* < a_2^* \implies \forall \epsilon, E [a | a > a_1^*, x = a_1^* + \epsilon] < E [a | a > a_2^*, x = a_2^* + \epsilon]$, which implies $E_x [\pi(E [a | a > a^*, x = a^* + \epsilon])]$ is monotonically increasing in $a^*$ so that the function in (10) is continuous and non-increasing in $a^*$ and thus the proof of Proposition 1 applies. By Lemma 2 the condition from line 3 remains the same.

I now prove the claims about the equilibrium threshold and innovation signal. The gain $G(a^*)$ is realized in the first period and therefore is not directly affected by the learning structure but the second period loss is. For any non-degenerate prior and any realization of the signal $x$, the posterior mean $E[a | a > a^*, x] > a^*$ and thus a type-$a^*$ firm induces more favorable beliefs when learning is noisy than when it is perfect. Integrating the firm’s profits over $x$ preserves the inequality:

$$E_x [\pi (E[a | a > a^*, x])] > E_x [\pi (a^*)] = \pi (a^*) ,$$

and implies the second period cost $L(a^*)$ in line (10) is weakly lower with noisy learning and therefore so too is the adoption threshold and innovation signal.

Finally, I show profits are lower under noisy learning. Let $a_p^*$ and $a_n^*$ be the equilibrium thresholds under perfect and noisy learning, respectively. Since $a_p^* > a_n^*$, by Proposition 3(ii) it suffices to establish that expected profits are higher under perfect learning when the firm is committed to the threshold $a_n^*$ than when learning is noisy. Since first period profits are equal in this case and second period profits are equal when $a < a_n^*$, I am left to compare second period profits when $a \geq a_n^*$. I wish to show

$$\int_{a_n^*}^{\infty} \pi (a) f (a) da > \int_{a_n^*}^{\infty} E_x [\pi (E[a | a > a_n^*, x])] f (a) da$$

where the integrand on the right hand side is the profit a type-$a$ firm expects to earn, where this expectation is taken over the distribution of consumption signals such a type’s customers receive, $x = a + \epsilon$, and where the realization of each such signal induces a posterior mean $E[a | a > a_n^*, x]$. 30
To proceed, rewrite the right hand side as profits integrated with respect to the distribution of induced posterior means. With \( f_{a > a^*_n} \) as the prior distribution, define 
\[
a = E \left[ a \mid a > a^*_n, x = \bar{x} \right]
\]
as the posterior mean induced by signal \( \bar{x} \). The likelihood of consumers holding expectation \( \bar{a} \) is given by 
\[
k(\bar{a}) \equiv \int_{a^*_n}^{\infty} f(a) \ h(\bar{x} - a) \ da,
\]
which integrates over all possible types \( a \) and noise realizations \( \epsilon \) that result in \( a + \epsilon = \bar{x} \). I now state two useful properties of \( k(\bar{a}) \) given in Gelman et al., 2013. First, the prior mean is the average over all possible posterior means; that is, 
\[
\int a f(a) \ da = \int \bar{a} k(\bar{a}) \ da.
\]
Second, the expected posterior variance is less than the prior variance. Now represent line 12 equivalently as 
\[
\int_{a^*_n}^{\infty} \pi(a) \ f(a) \ da > \int_{a^*_n}^{\infty} \pi(\bar{a}) \ k(\bar{a}) \ da,
\]
which holds as \( \pi \) is convex (per footnote 13) and \( f(a) \) is a mean-preserving spread of \( k(\bar{a}) \).

References


