New and Improved?*

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Abstract

Are new versions of products necessarily better? We analyze product innovation by a firm that engages in research and development designed to improve an existing product, the outcome of which is uncertain. If the firm adopts the innovation its modified product appears to consumers as “new and improved,” but consumers do not immediately know whether or how much the product is better. We find that new products are on average improved and therefore command a pricing premium. This induces some types to exploit the innovation signal by selling new versions that are only trivially different from their older version or that require inefficiently high upgrade costs. Nevertheless, the incentive to “show off” by introducing a new product may improve total welfare by inducing more innovation adoption and thereby mitigating the standard monopoly underinvestment problem. Firms benefit ex-ante from better consumer information about quality or from committing to not exploit their informational advantage.

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JEL Classification: D82, O31


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1 Introduction

How do consumers update their beliefs about a “new” or “improved” version of a product before purchase? For example, how much value will the latest edition of a college textbook provide over its predecessor? Should a consumer who is told “roads change by as much as 15% every year” purchase an updated map for her GPS device? Or suppose a familiar household cleanser’s packaging states “WOW! Powerful New Formula,” but its price has increased by 10%. Is the touted improvement in performance worth the higher price?

In each of these examples consumers are likely unaware of the exact value offered by the “new” or “improved” version of the product. Facing perhaps thousands of such new products each year, consumers must discern major breakthroughs from the more common incremental improvements before making their purchase decision. This often proves difficult due to “a plethora of choice, a surge of marketing communications, decreasing inter-brand differences, [and the] increasing complexity of information” (Walsh et al., 2007). For their part, although firms may devote significant resources to research and development the outcome of such efforts is highly volatile and often results in failure (Stevens and Burley, 1997). Firms must decide which research outcomes to implement and which to censor from the market, knowing that some consumers may not be willing or able to become immediately informed of the new product’s value.

This paper uses a signaling model to investigate the incentive of firms to introduce improved products and the welfare consequences of these introductions when consumers are uncertain of the quality of the improvement. Consumers form beliefs about quality knowing the product exceeds the firm’s minimal threshold for new product launch. We find this leads to a “newness premium” resulting from the information conveyed in equilibrium by the very existence of the new product version. This premium in turn incentivizes more upgraded products to be released and so has
implications for profits and welfare.\(^1\)

In our model a monopolist chooses an R&D effort level that results in a stochastic outcome and must decide whether to adopt this innovation or not. The firm knows the true value of the innovation while consumers initially only observe the binary “innovation signal” of whether or not the product has been modified. Consumers form beliefs about product value and buy (or not) accordingly, and then learn from product trial and other sources such as product review websites so that they are more informed in the second period when they repeat their purchase decision. We find that consumers correctly expect the average quality of modified products to be higher. But because of this expectation, firms whose R&D has generated only a trivial improvement or no improvement have an incentive to present the product as new and improved. The result is a partial pooling equilibrium in which consumers are initially unsure whether modified products represent a genuine improvement. In making a trivial improvement firms face a trade-off between inducing an initial “new product” premium and the loss of future profits when the true quality is revealed.

That the firm might incur upgrade costs to sell a new product version only trivially different from the old may appear to unambiguously lower welfare. Indeed, we find that if the innovation signal is relatively strong and the firm’s upgrade costs are relatively low that socially inefficient upgrades will be made. However, a stronger incentive to signal may result in a net gain to welfare by offsetting previously existing distortions. It is well known that under full information a monopolist has less marginal incentive to make costly upgrades than does the social planner due to the firm’s inability to appropriate all of the benefits of the innovation (Arrow, 1959). In the present context, a firm may reject a product innovation whose upgrade expense is justified by the increase in welfare but not profits. By providing an additional

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\(^1\)Note our information-based account differs from the marketing literature which has generally explained consumer attraction to new products by a desire for uniqueness, stimulation, or novelty-seeking (Roehrich, 2004; Hirschman, 1980).
incentive to make a product upgrade, the innovation signal alleviates Arrow’s under-investment problem and so may increase welfare.

Though welfare may or may not rise, we show the equilibrium effect of consumers’ initial lack of information is to unambiguously lower expected profits. This arises from the fact that significantly improved versions are not fully rewarded by consumers wary of the trivially new products firms may occasionally introduce. By the same logic we find that the more difficult it becomes for consumers to learn about product quality from experience the lower will be expected profits.

Relation to prior literature

We model a situation close to that of Milgrom and Roberts (1986), who themselves formalized ideas proposed by Nelson (1970 and 1974). In their model, a monopolist has private information about its exogenously determined product quality and must choose price and dissipative advertising expenditures that induce beliefs among consumers who are uninformed in the first period but informed thereafter. As Milgrom and Roberts describe it, theirs is a model “...in which the firm’s R&D effort has generated a product of some particular given quality that the firm must decide how to introduce.” We instead consider the information content of the antecedent decision of whether such a product should be introduced at all.

The Milgrom and Roberts result has many variants and extensions applied to monopoly (Kihlstrom and Riordan, 1984; Wilson, 1985; Horstmann and MacDonald, 1994; Daughety and Reinganum, 1995), duopoly (Fluet and Garella, 2002; Yehezkel, 2008) and oligopoly (Janssen and Roy, 2010). Price signaling can even occur in a one-period model when some proportion of consumers is informed about product quality (Bagwell and Riordan, 1991; Linnemer, 2002). The common thread among each of these models is a firm choosing marketing variables such as price or advertising to signal its exogenous quality. We abstract from these already well-understood mech-
anisms to focus on a new one: the product adoption decision itself. Because every “new and improved” product is the output of a random R&D process that has survived the firm’s censoring rule, the existence of the new product version may serve as a signal of improved quality.

To a limited extent then, our firm has some control over its product’s quality. However, our model differs from the endogenous quality literature that focuses on the moral hazard problem of the firm. In that literature stream, it is assumed the firm derives a cost benefit from supplying a low-quality product while purporting it is of high-quality. Consequently, a firm with both high quality and cost may suffer from consumer wariness of being cheated and thus a corresponding low willingness to pay.

Our model differs from such models in two main respects. First, the unobservable component of our firm’s quality “choice” is limited to accepting or rejecting a random R&D outcome, not a deterministic choice as in endogenous quality models. Second, we have no difference in production costs between types, a crucial component of endogenous quality models.

We focus on innovation signaling as a particular way to transmit private information from sellers to buyers, though other mechanisms exist. Crawford and Sobel (1982) showed that coarse information can be conveyed even when firms can costlessly make claims whose truth is unverifiable to consumers. The ensuing cheap talk literature contains numerous extensions of this result, including the ability of such communication to induce more consumer search (Mayzlin and Shin, 2011; Gardete, 2013) or make comparative claims about a product (Chakraborty and Harbaugh, 2014). Our model differs from these because it is the existence of the new product 

\footnote{Various mechanisms have been proposed to ameliorate this problem, including reputation or offering a brand name as collateral (Spence, 1977; Klein and Leffler, 1981; Allen, 1984; Wernerfelt, 1988), price signaling (Wolinsky, 1983) as well as risk-sharing devices such as warranties (Grossman, 1981) and money-back guarantees (Mann and Wissink, 1988). Biglaiser (1993) models middlemen as quality guarantors while Miklos-Thal and Schumacher (2013) examine the role of third-party monitors.
version that serves as the firm’s message, and product launch and upgrade expenses imply this message is costly to send.

Finally, our finding that “new” versions may differ only trivially from the old relates to prior work in which the firm degrades or otherwise denies the consumer the full value of its product. For example, Denekere and McAfee (1996) showed that firms can price discriminate by “crimping” a product—that is, degrading its performance and selling both the high and low quality versions. Oligopolists may engage in “planned obsolescence” whereby they produce goods with uneconomically short useful lives, forcing customers to make otherwise unnecessary repeat purchases (Bulow, 1986). And Moorman et al. (2012) found evidence some firms withhold innovative new products from the market for strategic reasons.

The rest of the article proceeds as follows. Section 2 explains our assumptions and introduces our base model. In Section 3 we establish the existence of equilibria while in Section 4 we discuss welfare results. Section 5 considers profitability and extensions including imperfect consumer learning about product quality and firm commitment to only introduce new versions exceeding some minimal standard. We then conclude and discuss areas for future research.

2 Model

A monopolist performs research and development (R&D) to improve the quality of its product, the outcome of which is a random variable $a$ with continuous log-concave density $f$ with full support on $[a, \bar{a}] \subseteq \mathbb{R}$ and distribution $F$. This distribution is common knowledge and exogenous though later in an extension we endogenize it. Once an innovation is realized, the firm privately observes its value and makes its new product adoption decision. It may sell a new version with the attribute developed in the R&D stage for a one-time fixed cost of $M > 0$ or avoid these costs by maintaining
the existing version of the product. Captured in $M$ are new product launch costs such as the cost to alter production facilities or marketing expenditures that inform consumers of the existence of the new product version and for this reason we assume it is independent of the realization of $a$.

The firm’s innovation signal to consumers is binary: either it sells a new version or not. The firm might not adopt the innovation it developed if the improvement is deemed too marginal or too costly, or if the firm fears consumers will not make positive enough inferences about it. In this case new product launch costs can be avoided by rejecting the R&D outcome in favor of continuing to sell the old version of the product, the quality of which consumers are informed.

If the firm does introduce a new version by adopting its innovation potential consumers do not know its value but do know the distribution $a$ from which it was drawn. In addition, we assume there is no credible direct way the firm can provide information about the innovation. Consumers have unit demand in each of two periods and purchase in the first period if and only if

$$E[U_i] = v_i + E_{\mu}[a] - p_1 \geq 0,$$

where consumer $i$ has idiosyncratic valuation $v_i$ for the product which is i.i.d. across consumers on $[0, \pi)$ with continuous density $g$ and distribution $G$ and $a \leq -\pi$ in order to simplify presentation of the results; $E_{\mu}[a]$ is consumers’ expectation of innovation $a$ given their beliefs $\mu$, assumed for simplicity to be commonly valued; and $p_1$ is the first period price charged. Thus the addition of an innovation does not change the dispersion of the distribution of consumers’ valuations and so results in a shift in demand.\(^3\) We initially assume all consumers learn the realization of $a$ at the end of the first period, through either their own perfectly informative consumption

\(^3\)Hence we do not analyze rotations in demand (Johnson and Myatt, 2006).
experience, word-of-mouth communications, or product review websites.\textsuperscript{4,5} Therefore second period consumers are fully informed and buy if and only if

\begin{equation}
U_i = v_i + a - p_2 \geq 0,
\end{equation}

where $p_2$ is the second period price charged. We later conceptualize learning from consumption as a noisy process in which second period consumers Bayesian update their beliefs after observing the sum $a + \epsilon$, where $\epsilon$ is a mean zero error term.

Utility maximization gives rise to demand function $q(E_{\mu}[a], p) = 1 - G(p - E_{\mu}[a])$, which represents the proportion of consumers that buy each period at price $p$ given their posterior expectation $E_{\mu}[a]$. Note that consumers know their valuation of the old version and in this case we use $a = 0$ in the equations above. Thus all consumers begin the game informed and become uninformed only if the firm introduces a new version of its product. This feature of the model differs from many other signaling games and simplifies its structure. In addition, the firm has a constant marginal cost of production which is normalized to zero and no fixed costs, and these are not affected by the R&D innovation $a$.

Our equilibrium concept is perfect Bayesian equilibrium subject to a restriction on off-the-equilibrium-path beliefs explained below. In general consumers can form beliefs about the firm’s type given its decision to sell a new or old product and its price. However, by construction in our case there is no means by which price can serve as a signal: there are no cost differences across types nor do benefits vary by the assumption that \textit{all} consumers learn each firm’s type at the end of the first

\textsuperscript{4}Thus there is no role for strategic buying to acquire information. See Grossman, Kihlstrom and Mirman (1977) for treatment of this subject.

\textsuperscript{5}Just as price and advertising may signal quality in a one-period setting in which some consumers are informed (Bagwell and Riordan, 1991; Linnemer, 2002), so too can the innovation signal, as shown in a previous version of this paper in which the role of second period informed consumers is replaced with a proportion of first period consumers informed of the firm’s R&D outcome. The current model allows innovation signaling to be analyzed separately from price signaling.
period.\textsuperscript{6} We can therefore reasonably expect consumer beliefs to be invariant to price and thus assume that off the equilibrium path consumers do not update their beliefs about firm type based on price.

3 Equilibrium characterization and properties

I’m actually as proud of the things we haven’t done as the things I have done. Innovation is saying ‘no’ to 1,000 things. —Steve Jobs\textsuperscript{7}

Like any other decision, introducing a new product or retaining the old version entails costs and benefits. When consumers are fully informed of quality, the tradeoff is a relatively straightforward one between the additional cost to launch a new version and the increased profit margins obtained by selling a more highly valued product. However, when consumers are unsure of quality their beliefs play a crucial role in determining profits and thus may result in different adoption decisions. A social planner that places weight on consumers’ well-being may reach a still different decision than a firm regarding which new products to launch.

In this section we show that when consumers are initially uninformed of quality, a firm’s binary decision to implement an R&D outcome or not involves a trade-off between inducing a “new product” premium in the first period by incurring new product launch costs and selling to consumers who will be fully informed of the product’s value in the second period. For a low enough type $a$, the launch costs and potential decline in second period profits exceed the benefits conferred by the new product premium and thus the firm censors its R&D outcome. Though uninformed,

\textsuperscript{6}As discussed in Banks and Sobel (1987), without such type-dependent payoff differences standard forward-induction refinements do not apply. In Milgrom and Roberts (1986) price (and advertising) may signal exogenously determined product quality because of their requirement that consumers purchase the product in order to learn its type.

first period consumers place a demand premium on new products because they know
the firm needs to earn profits from informed consumers in the second period to recoup
its product launch cost $M$.

We show the adoption strategy consists of a simple threshold whereby a new
version is launched when its quality is sufficiently high and the existing version is
maintained otherwise. Note the use of a threshold strategy is not particular to the
asymmetric information case but also appears when consumers have full information
or if a social planner were in charge. In subsequent sections we will explore the effect
these different threshold values have on profitability and welfare.

We now characterize the equilibrium. When consumers have expectation $E_\mu[a]$, let
$\pi(E_\mu[a]) = \max_p \{p \cdot q(E_\mu[a], p)\}$ and $p_{E_\mu[a]} = \arg \max_p \pi(E_\mu[a])$ be the maximum
profits (gross of $M$) attainable and the profit maximizing price, respectively, and let
$\delta$ be the discount factor between periods. We assume away the case where the firm
never introduces a new version by specifying $\bar{a}$ high enough that it always prefers to
adopt its innovation. Additionally, we assume

$$\pi(E[a]) - M \leq (1 + \delta) \pi(0),$$

(3)

to rule out the unlikely case in which the firm always introduces a new version. This
requires the lowest type’s payoff from introducing a new product to be less than the
profits that could be earned from the existing version.

**Proposition 1** In the unique equilibrium there exists an adoption threshold $a^*$ such
that the firm modifies its product whenever $a > a^*$ and otherwise does not. In period 1
all “new and improved” types charge the high price $p^H = p_{E[a|a>a^*]}$ while all unmodified
types charge the low price $p_0$. In period 2 each type charges its full information
monopoly price.

**Proof** The firm uses a threshold strategy because higher types always have more
incentive to sell a new version for any given beliefs. For the same reason an equilibrium involving mixed strategies cannot exist. Consider a candidate equilibrium in which there is a threshold type-$a^*$ firm which should be indifferent to selling a new version or not. By backward induction it receives a discounted payoff of $\delta \pi(a^*)$ in period 2. In period 1 any modifying type will price pool on $p^H = p_{E[a|a>a^*]}$ provided off-the-equilibrium-path beliefs following an unexpected price are not greater than $E[a|a>a^*]$, a condition satisfied because prices do not affect beliefs.

Let the gains and losses to the threshold type-$a^*$ firm from modifying its product be $G(a^*)$ and $L(a^*)$, respectively. A gain is derived from first period consumers who are uninformed and is the increase in payoffs from selling the new version instead of the old: $G(a^*) = \pi(E[a|a>a^*]) - \pi(0)$. The loss is the cost $M$ to launch the product plus the discounted decline in profits, if any, from second period informed consumers: $L(a^*) = M + \delta [\pi(0) - \pi(a^*)]$.

$G$ is continuous and monotonically increasing in $a^*$ while $L$ is continuous and monotonically decreasing in $a^*$. By the assumption that type $\tilde{a}$ innovation is always adopted we conclude that $G(a^*) > L(a^*)$ for high enough $a^*$ and thus $G(a^*)$ and $L(a^*)$ either cross at a unique point or not at all. Line 3 is the necessary and sufficient condition for a unique crossing and is derived by setting $G(a^*) < L(a^*)$ in the limit as $a^* \to a^*$.

See Figure 1 for a visual depiction of the proof. Reading from right to left, as the proposed equilibrium threshold decreases the $G$ curve decreases and the $L$ curve increases towards their respective asymptotes. The $L$ asymptote exceeds the $G$ asymptote (i.e., line 3 is satisfied) so the two curves have a unique intersection $a^*$ which is the equilibrium threshold since this type is indifferent to selling a new product. Note that off-the-equilibrium-path beliefs with respect to the product launch decision do not exist since both possible actions occur and so no action is off the
equilibrium path.\footnote{One may hypothesize a “no new products” equilibrium in which any new product is believed to be the lowest type ($E_\mu[a] = \underline{a}$) in the first period. However, this fails by our assumption that the highest type $\bar{a}$ will always modify its product; i.e., $\delta \pi(\bar{a}) - M > (1 + \delta) \pi(0)$.} Off-the-equilibrium-path beliefs are invariant with respect to price, as discussed in footnote 6.

We have found there will be a threshold firm type indifferent to selling a new version, and for this type beliefs will be more favorable among first period uninformed consumers than second period informed consumers. This is because in the first period consumers form a pooled expectation over all types that would sell a new version while in the second period consumers can discern each type. The upshot is the firm uses a less stringent adoption threshold, since marginal types are enticed by the initial new product premium to introduce new versions that would otherwise be unprofitable had all consumers been informed.

The extra incentive for types to incur the product launch cost $M$ provided by first period uninformed consumers has implications for firm use of trivial product modifications. Here we define a product modification as trivial if the new product price premium exceeds the value $a$ of the new attribute. Note that while our definition

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Determination of equilibrium threshold strategy from Proposition 1.}
\end{figure}
involves a trade-off between attribute value and price, the marketing literature has generally defined a trivial attribute purely in terms of its value without consideration of price (e.g., Carpenter, Glazer, and Nakamoto, 1994).

**Definition 1** The attribute \( a \) is trivial if \( \Delta p \equiv p^H - p_0 \geq a \) and useful if \( a > \Delta p \).

Thus all consumers that purchase a trivially new product are worse off on net than if the product were never improved. Pushing this idea further, it may even be possible a firm would adopt a *schlimmbesserung* attribute (“improvement for the worse” (Rheingold, 2000)) \( a < 0 \) in order to be initially pooled with more significant improvements. However, such types would incur both adoption costs and a decline in second period profits relative to what the old version would have earned. When also considering reputational concerns the use of such negative attributes would likely be rare. Regardless, in equilibrium the average new product will indeed be improved as established below.

**Proposition 2**

(i) New products are on average improved (i.e., \( E[a \mid a > a^*] > 0 \)) and thus command a new product demand and pricing premium.

(ii) The new product premium is large enough that the firm would make a trivial improvement whenever the product adoption cost \( M \) is sufficiently low.

**Proof** See the appendix.

The new product pricing premium is strictly positive and so any type \( a < \Delta p \) that adopts its innovation is trivial. Consider an example where \( a \sim U[-0.5, 0.5] \), \( M = \frac{1}{4} \), \( \delta = \frac{9}{10} \) and \( q = 1 - p + E[a] \). This implies \( a^* \approx 0.136 \) and \( \Delta p \approx 0.159 \) so that \( a \in [0.136, 0.159] \) are trivial attributes while \( a > 0.159 \) are useful. Note marginal consumers who purchase a new product with a trivial attribute will ex-post regret
doing so since equation (2) is violated. Intuitively, selling a trivially new version will be profitable provided first period uninformed consumers have a sufficiently high posterior over new products relative to the new product adoption cost $M$.

In the next section we consider the welfare effects of selling trivially new versions and will use the following definition: the type threshold a social planner would employ when weighing the welfare effects of a new product against the ex-post investment $M$ required to adopt a new attribute.

**Definition 2** Let $\tilde{a}$ be the welfare maximizing adoption threshold. We define attributes $a < \tilde{a}$ as inefficient and $a \geq \tilde{a}$ as efficient.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Notes</th>
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<tbody>
<tr>
<td>$a^*$</td>
<td>Equilibrium adoption threshold</td>
<td>—</td>
</tr>
<tr>
<td>$\hat{a}$</td>
<td>Full information adoption threshold</td>
<td>$a^* &lt; \hat{a}$</td>
</tr>
<tr>
<td>$\tilde{a}$</td>
<td>Social planner’s adoption threshold</td>
<td>$a &lt; \tilde{a}$ are inefficient, $a \geq \tilde{a}$ efficient</td>
</tr>
<tr>
<td>$\Delta p$</td>
<td>New product price premium</td>
<td>$a \leq \Delta p$ are trivial, $a &gt; \Delta p$ useful</td>
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Thus we contemplate a social planner committing the firm to an adoption threshold, conditional on uninformed consumers’ inferences and the firm’s pricing decisions in our two-period model. As we will show, the threshold that maximizes the sum of expected profits and consumer surplus could be higher or lower than the firm’s threshold $a^*$. Table 1 above summarizes the definitions from this section and indicates when useful or trivial attributes might exist. Figure 2 illustrates the existence ranges for these attribute types for the example under consideration. Improvements above the threshold $a^*$ are adopted while those below are not. In the example $\tilde{a} < \Delta p < \hat{a}$ though this need not hold in general. Notice there may exist attributes that are both efficient and trivial. This is because triviality of a product improvement is judged by
consumer welfare, while efficiency is judged by total welfare, and monopoly pricing drives a wedge between the two.

4 Firm investment in quality and welfare

We have found that the incentive to signal a “new and improved” product leads firms to adopt new attributes that in a full information environment do not justify the firm’s product launch costs, and that may even be trivial. Given this result it might seem welfare must be lower than if consumers could immediately learn the exact quality of a new attribute. However, even with full information the firm’s adoption decision is already inefficient. As Arrow (1959) observed, a firm does not consider the gains to consumer surplus from adopting an innovation so it will tend to underinvest.

In this section we consider two such types of underinvestment. The first is discussed in Section 4.1 and corresponds to the firm not adopting an unprofitable innovation even when doing so would increase the sum of profits and consumer surplus. The second type is discussed in Section 4.2 and involves an extension of the main model in which the firm makes a costly ex-ante investment to improve the distribution of its R&D outcomes. Given the underinvestment that may occur in each case even with informed consumers, we show that the signaling incentive to adopt a new innovation and receive a demand and pricing premium from uninformed consumers
might on average lead to higher rather than lower efficiency.\footnote{This second-best problem can readily arise with signaling games. For instance, if education has positive externalities then the signaling incentive to overeducate can raise rather than lower total welfare.}

4.1 The adoption of quality-enhancing innovations

To see how the innovation signaling effect can mitigate or even reverse the Arrow underinvestment problem, first consider the monopolist’s adoption decision when consumers are fully informed in both periods. The type-$a$ monopolist adopts its innovation whenever its increase in profits, as seen in regions B, C and D in the left panel of Figure 3, exceed the product launch cost $M$. Recall the type $\hat{a}$ firm is indifferent to adoption whereas types $a > \hat{a}$ strictly prefer to adopt the attribute. In contrast, a social planner that takes monopoly pricing to fully informed consumers as given is indifferent to adoption when the increase in total surplus from selling the new product, as seen in regions E, D and C in the left panel of Figure 3, equals $M$. Because the social planner has regard for consumer surplus his threshold must be less than the monopolist’s and the planner strictly prefers adoption for the type $\hat{a}$ firm.

The same incentives underlying the monopoly underinvestment problem arise in our model with asymmetric information but they are counteracted by the incentive to signal to first period uninformed consumers. First consider the equilibrium determination of the investment threshold $a^*$ and hypothesize $a^* = \hat{a}$. But then the type $\hat{a}$ firm must strictly prefer investment because of the demand premium it receives from first period consumers who infer quality $E[a | a > \hat{a}]$ and so buy more of the new product at a higher price than they would if informed. The resulting higher profits imply we must instead have $a^* < \hat{a}$. The right panel of Figure 3 displays this argument graphically by drawing the informed demand curve for a new product of type $\hat{a}$. Additional purchases by uninformed consumers reduce the deadweight loss of monopoly in region $F_1$, whereas the price premium consumers pay results in a trans-
Demand for new product
Demand for old product

Full information: the monopoly underinvestment problem

Uninformed consumers provide additional incentive to invest

Figure 3: The signaling effect induces adoption by marginal types below \( \hat{a} \) and decreases deadweight loss by region \( F_1 \). Note: \( E = E_1 + E_2 \) and \( F = F_1 + F_2 \).

The determination of the social planner’s preferred threshold is more complicated because it involves consideration of a type’s investment on the new product demand premium enjoyed by all other types that adopt. Thus, in the right panel of Figure 3 although the type \( \hat{a} \) firm facing uninformed first period consumers prefers adoption in part due to the reduction in deadweight loss \( F_1 \), it does not consider the effect this adoption has on higher types that might have been realized. In fact, a lower type adopting implies a weaker innovation signal and therefore a lower demand premium for all other adopting types. Since the planner considers such externalities, is indifferent to transfers \( E_2 + T \), but has regard for consumer surplus \( E_1 \), a general comparison of the planner’s threshold to the firm’s is difficult.

The proposition below characterizes when consumers’ lack of information actually increases welfare by incentivizing the right amount of additional new product launches. We then present an explicit example in which this occurs. Intuitively,
welfare will be higher when product launch costs are sufficiently large that: (1) the Arrow underinvestment problem to be solved is severe, and (2) the firm is deterred from adopting marginal, welfare reducing innovations.

**Proposition 3**

(i) The Arrow underinvestment problem exists for the new product adoption decision with complete information.

(ii) Consumer uncertainty about new product quality mitigates this underinvestment and raises welfare when the new product adoption cost $M$ is sufficiently large.

**Proof** See the appendix.

**Example 1** In Figure 2 an example with $q = 1-p+E[a], a \sim U \left[-\frac{1}{2}, \frac{1}{2}\right], M = \frac{1}{4}$, and $\delta = \frac{9}{10}$ is presented. In period 1 all types greater than $a^* \approx 0.136$ adopt their innovation and charge a price $p^H \approx 0.659$ and types $a \in [a^*, \Delta p) \approx [0.136, 0.159)$ are trivial. Each type charges its own full information monopoly price in period 2. Notwithstanding the addition of some trivial types the monopoly underinvestment problem is mitigated and expected welfare is higher under imperfect information (0.814) than full information (0.812).

Finally, the right panel of Figure 3 can help resolve a seeming oddity that first appeared in Figure 2: the existence of types that are both efficient and trivial. Let the informed demand curve for a type $a \in [\bar{a}, \Delta p]$ be given in the right panel of 3. Then marginal purchasing consumers make transfer payment $T$ to the firm and therefore ex-post regret buying such a product because of the trivial attribute. Nonetheless, because the type is efficient the increased output and resulting reduction in deadweight loss $F_1$ increase expected surplus.
4.2 Ex-ante investment to develop quality-enhancing innovations

We now consider an extension of the main model in which the firm makes an ex-ante investment to improve the R&D distribution and ask how the innovation signal may affect the incentive for such an investment. Let $e \geq 0$ be a publicly observable effort level that incurs increasing cost $C(e)$ with $C(0) = 0$ and results in a stochastic R&D outcome $a(e) \equiv a + e$.\footnote{The public observability of R&D effort is a reasonable approximation of the fact that some firms are known for engaging in higher levels of R&D than others. If more R&D intensive firms draw their product improvements from a better distribution this will have implications for the innovation signal.} Thus more effort incurs a greater cost but also increases the likelihood of more valuable innovations being developed. Note this extended model includes the original as a subgame and so the equilibrium characterization in Proposition 1 goes through but now the adoption threshold depends on $e$ and the firm chooses $e$ to maximize expected profits in the subgame, $\Pi(e)$, net of $C(e)$.

What is the firm’s incentive to improve its distribution of R&D outcomes? Although statically greater ex-ante investment provides a benefit in that it makes higher realizations more likely and increases consumers’ expectations of quality, the equilibrium effect of greater $e$ is to lower the adoption threshold in the subgame by inducing more marginal types to introduce a new product version. This tends to weaken the innovation signal. We first clarify the net effect of $e$ on $E[a \mid a > a^\star]$ and profits gross of $C(e)$ in the lemma below.

**Lemma 1** A better distribution of R&D outcomes from higher $e$ induces a lower equilibrium threshold $a^\star$ but a higher innovation signal, and provides the firm a positive expected benefit. Mathematically, $\frac{\partial a^\star}{\partial e} < 0$, $\frac{\partial E[a \mid a > a^\star]}{\partial e} > 0$, and $\frac{\partial \Pi(e)}{\partial e} > 0$.

**Proof** See the appendix.

Given there is a benefit from ex-ante investment, we might expect the firm to exert a positive effort level. The question remains whether this selection is too high
or low with respect to some benchmark. If consumers were fully informed the firm would clearly derive a private benefit from investment in R&D since more valuable improvements are more likely and this value would be observable by consumers. Given a uniform price monopolist consumer surplus would also increase in R&D effort and thus the marginal social benefit from ex-ante investment exceeds the private benefit, implying a social planner would choose a higher investment level than the firm so that Arrow’s underinvestment problem also exists in the ex-ante sense.

We now consider the welfare effects of this ex-ante problem together with the under-adoption problem in the full game. Most obviously, the cost of effort affects the optimal selection of effort and so the extent of ex-ante underinvestment. In addition, the benefit of ex-ante effort depends on the curvature of the profit function and thus the information structure, since effort increases the quality of each type under full information but only the average inferred quality of the types under imperfect information. Finally, because the choice of effort affects the distribution of R&D outcomes it has implications for innovation adoption as well. For these reasons a general welfare characterization is intractable though we establish the following existence results.

**Proposition 4**

(i) The Arrow underinvestment problem exists for the ex-ante investment decision with complete information.

(ii) Consumer uncertainty about new product quality can mitigate this underinvestment problem.

(iii) Welfare in the full game can be higher when consumers are uncertain of new product quality.

**Proof** Part (i) follows from the discussion above while parts (ii) and (iii) are established by Example 2 below.
Next we add the ex-ante investment stage to Example 1 in order to establish parts (ii) and (iii) of the proposition.

**Example 2** Let $q = 1 - p + E[a]$, $a \sim U \left[-\frac{1}{2} + e, \frac{1}{2} + e \right]$, $M = \frac{1}{4}$, $\delta = \frac{9}{10}$, and the cost of effort be given by $C(e) = \frac{16e - 1}{16 \ln 2}$. Then the ex-ante underinvestment problem is mitigated as the imperfect information firm chooses an investment level of $e^*_{II} \approx 0.665$ which is greater than that chosen under full information, $e^*_{FI} \approx 0.628$. Total welfare is higher under imperfect information (1.331) than full information (1.304).

5 Information and learning

5.1 Consumer information and firm commitment

We have found that the firm uses a lower adoption threshold when its consumers are initially uninformed but subsequently learn product quality than if they are fully informed from the beginning. In this subsection we find that a lower threshold reduces profits and thus the firm would prefer to commit itself against its ex-post incentive to more freely introduce new products. In addition, if consumers could observe product quality from the beginning then the game of symmetric information that follows would yield higher profits. These results, together with similar findings on noisy learning in Subsection 5.2, highlight the losses firms incur due to their inability to credibly communicate information about product quality and suggest a role for developing such means.

**Proposition 5** Let $G$ be concave. Compared to the game where consumers are initially uninformed of quality, the firm’s expected profits are higher when either:

(i) all consumers are fully informed, or

(ii) the firm commits to using the full information threshold $\hat{a}$. 


Proof See the appendix. ■

The proposition demonstrates the net effect of innovation signaling on the firm’s profits. When facing imperfectly informed consumers, the firm is tempted by the immediate gains from accepting an R&D outcome that would otherwise be unprofitable had consumers been informed. However, the modest benefit accrued from doing so does not last since consumers eventually learn the product’s quality. In addition, such marginal types impose a negative externality on higher types: real and significant improvements made to the product are not initially rewarded enough because consumers anticipate that the firm may occasionally introduce trivial new products. The net effect is the firm’s ex-ante profits are higher if it either it could commit not to introduce such marginal products or if consumers were initially informed.

We find that a firm has incentive to educate its customers about a new product’s features, for example by submitting products to an independent reviewer, certifier or granting advance access to critics. Such third-party reviewers have been found to be an important component and influencer of a firm’s marketing strategy (see, for example, Chen and Xie, 2005). While a firm cannot be assured that any particular review will prove favorable, a policy of submitting products in advance to third-party reviewers will prove beneficial on net by quickly rewarding the significantly improved new versions a firm does occasionally develop.

5.2 Imperfect learning

In the previous sections we assumed a simple learning structure that enabled all consumers to determine the value of the new product attribute after the first period. Learning occurred from personal consumption experiences or secondary sources such as word-of-mouth communications or product review websites. Whereas such information was not available to consumers in the first period when the product was new, it is assumed by the second period it is widely available at no cost.
We now conceptualize learning as an imperfect process from which only some of consumers’ uncertainty about a product’s value is resolved. Such products are of practical interest and lay along a continuum between two theoretical extremes: Nelson's (1970) experience good, for which all uncertainty is resolved after consumption, and Darby and Karni's (1973) credence good, for which the consumer learns nothing from consumption. In this subsection we allow for imperfect, or noisy, learning from consumption and other sources and show that the main results from the model with perfect learning generalize to this context. In addition, we develop managerially relevant insights regarding the effect noise has on profits and the quality inferences consumers make about new products.

We operationalize noisy learning by assuming each consumer receives a common signal $x \equiv a + \epsilon$ after the first period that contains information about the realization of $a$ as well as an independent mean zero error term $\epsilon$ whose distribution is common knowledge. We assume $f$ and $h$, the densities of $a$ and $\epsilon$ respectively, are continuous with full support on $\mathbb{R}$ and $h$ is log-concave so that consumers’ posterior mean $E[a \mid x]$ is increasing in the noisy signal $x$.\footnote{The assumption of full support simplifies the presentation of the results.} \footnote{This ensures $L(a^*)$ is monotonic and thus a unique attribute adoption threshold strategy exists. Consumers’ posterior mean is increasing in $x$ when $x$ and $a$ are affiliated, a sufficient condition for which is the log-concavity of $h$ (Milgrom and Weber, 1982).} Including a common error term is justified on the grounds that consumers, whether or not they made a first period purchase, may receive the same information from influential experts, product review websites, blogs, or word-of-mouth communication. For this reason we assume the firm observes the signal $x$ as well.

We now characterize the equilibrium with imperfect learning and compare its properties to that found under perfect learning.

**Proposition 6** Let consumers learn about product quality from noisy signal $x = a + \epsilon$. Then the equilibrium characterization from Proposition 1 applies except that in period
all modified types charge the optimal price given the consumption signal $x$ consumers have received. This results in a lower attribute adoption threshold $a^*$ and weaker innovation signal $E[a | a > a^*]$. Finally, expected profits are lower when learning is noisy than when it is perfect when $G$ is concave.

**Proof** See the appendix.

We thus conclude greater noise exacerbates the frictions caused by asymmetric information. A more noisy consumption experience renders learning more difficult and thus decreases the likelihood that consumers will detect the firm's true type. When forming his posterior the consumer will place less weight on the signal and more on prior beliefs, which benefits low types. Knowing this, firms are emboldened on the margin to adopt relatively minor innovations so that the adoption threshold $a^*$ decreases, thus weakening the innovation signal. Ex-ante profits decline as the likelihood of introducing more marginal products increases.

Another interpretation is that an easier learning environment induces firms to apply a more stringent standard to releasing new products. While any new product will initially enjoy a demand premium, products of marginal quality will be quickly exposed in an easy learning environment, causing profits to be too low to justify product launch. Thus the firm will not introduce such products in the first place but will only do so when their quality is sufficiently high.\(^{13}\)

\(^{13}\)Metacritic.com co-founder Marc Doyle echoes this sentiment when he speculates that giving consumers better information will tend to encourage the release of higher quality movies and videogames:

> Like many, I used to be suckered into seeing movies or buying games based on glowing review quotations in magazines or newspapers (“One of the year’s best!”) from critics nobody has heard of or from skilled PR department writers. A site like ours helps people cut through that unobjective promotional language. By giving consumers...information on the objective quality of a game, not only are they more educated about their choices, but it forces publishers to demand more from their developers, license owners to demand more from their licensees, and eventually, hopefully, the games get better.


http://www.guardian.co.uk/technology/gamesblog/2008/jan/17/interviewtheartofmetacriti
The result that profits are lower in a more noisy learning environment has managerial relevance. Although a difficult learning environment may at least temporarily shroud a firm’s products of marginal quality from being recognized as such, it also makes proving the value of high quality products more difficult. Even if the consumer of a high quality product has a positive experience with it, he knows that in a difficult learning environment experiences are more volatile and is wary of the low type products that exploit this fact. The proposition above tells us that on net firms benefit the easier it is for consumers to learn from their consumption experiences. Thus we find if the firm is able to ease the learning environment through product design or marketing communications it has incentive to do so.

**Example 3** We extend Example 1 in which \( q = 1 - p + E[a] \), \( a \sim U \left[ -\frac{1}{2}, \frac{1}{2} \right] \), \( M = \frac{1}{4} \), and \( \delta = \frac{9}{10} \), by allowing imperfect learning from the signal \( x = a + \epsilon \) where \( a \) is the firm’s true type and \( \epsilon \sim U \left[ -0.1, 0.1 \right] \). The adoption threshold of \( a^* \approx 0.106 \) is less than the perfect learning threshold of 0.136, leading to a weaker innovation signal. In period 1 all “new and improved” types set \( p^H \approx 0.651 \) whereas types below the adoption threshold sell their existing product and set price \( \frac{1}{2} \). In period 2 consumers form a posterior mean from signal \( x \) and the firm sets a price accordingly.

6 Conclusion

This paper investigates firms’ incentive to introduce improved products and the welfare implications of these improvements when consumers are uncertain of the quality of the improvement. We show that information is revealed by the very existence of a new product that has survived a firm’s endogenous censoring rule so that product “newness” alone signals higher quality on average and hence confers a pricing and demand premium.

This premium induces the firm to adopt attributes that would prove unprofitable
had all consumers been informed. This fact has two main consequences. First, firms may sell a “new” product only trivially different from their older version in the sense that its improvement in performance does not justify its higher price. However, the greater the likelihood of such products the weaker will be the inferred value of the innovation signal. The second consequence of a more lax attribute adoption policy is for welfare. By incentivizing the introduction of new versions, the innovation signaling effect can offset the existing monopoly underinvestment problem. For this reason it is possible that welfare can be lower when consumers are fully informed.

We show the robustness of these results by extending the model to a noisy learning environment in which consumers become better, though not perfectly, informed of the new product’s value after the first period. We find the more difficult it is for consumers to learn a product’s quality, the more incentive a firm has to introduce marginal improvements. Finally, consumers’ initial lack of information and subsequent imperfect learning both lower the firm’s profits by reducing their initial willingness to pay for significantly improved products. Potential extensions of the model include allowing the monopolist to concurrently sell the old and new versions of its product, and generalizing the model to an oligopoly context.

7 Appendix: proofs

Proof of Proposition 2 Part (i) follows from the equilibrium construction proof from Proposition 1. For Part (ii), note that trivial improvements occur in equilibrium whenever the threshold type is less than the price difference: \( p^H - p_0 \geq a^* \). If \( M = 0 \) then \( a^* < 0 \) and \( p^H - p_0 > 0 \), thus trivial improvements will occur. For \( M > 0 \), since \( p_0 \) is constant establishing \( 0 < \frac{\partial p^H}{\partial M} < \frac{\partial a^*}{\partial M} \) is sufficient to prove \( p^H - p_0 \geq a^* \) for all \( M \) below a threshold.

Since \( a^* \) solves \( \pi(E[a \mid a > a^*]) + \delta \pi(a^*) - M = (1 + \delta) \pi(0) \) it increases in \( M \).
and thus so too do \( E[a \mid a > a^*] \) and \( p_{E[a \mid a > a^*]} = p^H \). The log-concavity of \( f \) implies \( E[a \mid a > a^*] - a^* \) is decreasing in \( a^* \) (Bagnoli and Bergstrom, 2005) and thus \( \frac{\partial E[a \mid a > a^*]}{\partial a^*} < 1 \). Finally, since demand is downward sloping \( \frac{\partial p^H}{\partial E[a \mid a > a^*]} < 1 \) and thus \( \frac{\partial p^H}{\partial M} < \frac{\partial a^*}{\partial M} \).

**Proof of Proposition 3** Part (i) follows from the discussion above the proposition. For Part (ii) we consider equilibrium thresholds \( \hat{a} \) and \( a^* \) as a function of the adoption cost \( M \). The equilibrium condition that the threshold type is indifferent to introducing a new version implies

\[
(1 + \delta) \pi(\hat{a}) = (1 + \delta) \pi(0) + M = \pi(E[a \mid a > a^*]) + \delta \pi(a^*).
\]

By the fact that \( a^* < \hat{a} \) we must also have \( a^* < \hat{a} < E[a \mid a > a^*] \). In addition, each of these terms is increasing in \( M \). Finally, the log-concavity of \( f \) implies \( E[a \mid a > a^*] - a^* \) is decreasing in \( a^* \) (Bagnoli and Bergstrom, 2005), and thus in \( M \), so that the difference between the thresholds \( \hat{a} \) and \( a^* \) decreases in \( M \). Since the planner’s threshold \( \tilde{a} \) contemplates consumer surplus, \( \frac{\partial \tilde{a}}{\partial M} < \frac{\partial \hat{a}}{\partial M} \) so that the difference between \( \tilde{a} \) and \( \hat{a} \) increases in \( M \), and thus \( \tilde{a} < a^* < \hat{a} \) for sufficiently high \( M \).

**Proof of Lemma 1** Let \( e_2 > e_1 \). We claim \( a^*(e_2) < a^*(e_1) \) but

\[
E[a(e_2) \mid a(e_2) > a^*(e_2)] > E[a(e_1) \mid a(e_1) > a^*(e_1)].
\]  

(4)

By construction the type \( a^*(e_1) \) is indifferent to selling a new version given the new product’s expected quality \( E[a(e_1) \mid a(e_1) > a^*(e_1)] \) and therefore this type must strictly prefer to sell a new version given a more favorable distribution and expectation, \( E[a(e_2) \mid a(e_2) > a^*(e_1)] \). Equilibrium is restored when the threshold decreases until indifference and thus \( a^*(e_2) < a^*(e_1) \).

We now note that threshold types for any distribution \( a(e) \) earn equal (two-period) payoffs since each is indifferent to selling a new version. Since \( a^*(e_2) < a^*(e_1) \), type
$a^\ast (e_2)$ earns a lower payoff in the second period than type $a^\ast (e_1)$ and therefore must earn a higher payoff in the first period. Thus line 4 holds.

Finally, we show there is an expected benefit from effort by establishing that all types receive weakly higher payoffs under $e_2$ than $e_1$. Given $e_2$ all types $a \geq a^\ast (e_1)$ are better off by line 4. Next, types $a \in (a^\ast (e_2), a^\ast (e_1))$ (which sell a new product given $e_2$ but not $e_1$) receive a higher payoff than type $a (e_2)$, who itself is indifferent to selling a new product. Types $a \leq a^\ast (e_2)$ are indifferent between $e_1$ and $e_2$ because in either case they sell their old product version.

**Proof of Proposition 5** We first prove a stronger claim than (ii): commitment to the threshold $a^\ast_2$ results in higher profits than $a^\ast_1$ whenever $a^\ast_1 < a^\ast_2 \leq \hat{a}$. We restrict attention to $a > a^\ast_1$ since otherwise profits are equal for either threshold and define $t = \frac{F(a^\ast_2) - F(a^\ast_1)}{1 - F(a^\ast_1)}$ and $1 - t = \frac{1 - F(a^\ast_2)}{1 - F(a^\ast_1)}$. Since a lower threshold always results in lower second period profits, it suffices to consider the first period and show

$$(1-t) \left[ \pi(E[a | a > a^\ast_2]) - M \right] + t \pi(0) > \pi(E[a | a > a^\ast_1]) - M.$$  

Using the equilibrium substitution $(1 + \delta) \pi(\hat{a}) = (1 + \delta) \pi(0) + M$ it suffices to show

$$(1-t) \pi(E[a | a > a^\ast_2]) + t \pi(\hat{a}) > \pi(E[a | a > a^\ast_1]).$$  

(5)

Profits are convex in $a$ by the assumption that $G$ is weakly concave\(^{14}\) so that the left

---

\(^{14}\)Profits $\pi(a) = \max_p p(1 - G(p - a))$ are convex in $a$ by the envelope theorem when $\frac{\partial^2}{\partial a^2} p(1 - G(p - a)) \geq 0 \iff -G''(p - a) \geq 0$; i.e., when $G$ is weakly concave.
hand side of (5) is

\[
> \pi (t \hat{a} + (1 - t) E [a \mid a > a_2^*]) \\
= \pi \left( \frac{\int_{a_1^*}^{a_2^*} \hat{a} f(a) da + \int_{a_2^*}^{\infty} a f(a) da}{1 - F(a_1^*)} \right) \\
> \pi \left( \frac{\int_{a_1^*}^{\infty} a f(a) da}{1 - F(a_1^*)} \right) \\
= \pi (E [a \mid a > a_1^*]),
\]

(6)

Finally, that commitment to the full information threshold is preferred to no commitment is shown by letting $a_2^* = \hat{a}$ and $a_1^* = a^*$.

To prove (i) let $a_2^* = \hat{a}$ and $a_1^* = a^*$ above. By convexity $\int \pi (a) f(a) da > \pi \left( \int a f(a) da \right)$ so that profits given fully informed consumers are higher than given uninformed consumers using threshold $\hat{a}$, and thus in turn higher than uninformed consumers using threshold $a^*$. ■

To prove Proposition 6 we first establish the following lemma.

**Lemma 2** The threshold type’s expected profits approach zero as the threshold declines. Formally, $\lim_{a^* \to +\infty} E_{x \mid a = a^*}[\pi(E[a \mid a > a^*, x])] = 0$.

**Proof** Because $f(x) = h(x - a^*)$, the expected gross profits to the threshold type-$a^*$ firm $E_{x \mid a = a^*}[\pi(E[a \mid a > a^*, x])]$ equal

\[
\int_{-\infty}^{t} \pi(E[a \mid a > a^*, x]) h(x - a^*) dx + \int_{t}^{\infty} \pi(E[a \mid a > a^*, x]) h(x - a^*) dx.
\]

(7)

Since $\forall t$, $H(t - a^*) \to 1$ as $a^* \to -\infty$, $t$ and $a^*$ can be selected to assign arbitrarily large mass to arbitrarily negative values of the signal. This implies the integral on the right in (7) can be made to approach 0 since its mass approaches 0 while the integral on
the left approaches 0 because $E[a | a > a^*, x] \to -\infty$ and thus $\pi(E[a | a > a^*, x]) \to 0$ as $a^*$ and $x$ approach $-\infty$. ■

**Proof of Proposition 6** We first characterize the equilibrium. Type $a^*$’s first period gain $G(a^*)$ is identical to that in the perfect learning case as no learning has yet occurred in the first period, while the expected loss $L$ is

$$L(a^*) = M + \delta \cdot E_x[\pi(0) - \pi(E[a | a > a^*, x])],$$

(8)

where the expectation is taken with respect to the distribution of possible signals the type-$a^*$’s consumers receive, with $x = a + \epsilon$ and the firm knows $a = a^*$ but $\epsilon$ is stochastic. Now, $a_1^* < a_2^* \to \forall \epsilon, E[a | a > a_1^*, x = a_1^* + \epsilon] < E[a | a > a_2^*, x = a_2^* + \epsilon]$, which implies $E_x[\pi(E[a | a > a^*, x = a^* + \epsilon])]$ is monotonically increasing in $a^*$ so that the function in (8) is continuous and non-increasing in $a^*$ and thus the proof of Proposition 1 applies. By Lemma 2 the condition from line 3 remains the same.

We now prove the claims about the equilibrium threshold and innovation signal. The gain $G(a^*)$ is realized in the first period and therefore is not directly affected by the learning structure but the second period loss is. For any non-degenerate prior and any realization of the signal $x$, the posterior mean $E[a | a > a^*, x] > a^*$ and thus a type-$a^*$ firm induces more favorable beliefs when learning is noisy than when it is perfect. Integrating the firm’s profits over $x$ preserves the inequality:

$$E_x[\pi(E[a | a > a^*, x]) > E_x[\pi(a^*)] = \pi(a^*),$$

(9)

and implies the second period cost $L(a^*)$ in line (8) is weakly lower with noisy learning and therefore so too is the adoption threshold and innovation signal.

Finally, we show profits are lower under noisy learning. Let $a_p^*$ and $a_n^*$ be the equilibrium thresholds under perfect and noisy learning, respectively. Since $a_p^* > a_n^*$, by Proposition 5(ii) it suffices to establish that expected profits are higher under
perfect learning when the firm is committed to the threshold \( a_n^* \) than when learning is noisy. Since first period profits are equal in this case and second period profits are equal when \( a < a_n^* \), we are left to compare second period profits when \( a \geq a_n^* \). We wish to show

\[
\int_{a_n^*}^{\infty} \pi(a) f(a) \, da > \int_{a_n^*}^{\infty} E_x[\pi(E[a \mid a > a_n^*, x])] f(a) \, da
\]

where the integrand on the right hand side is the profit a type-\( a \) firm expects to earn, where this expectation is taken over the distribution of consumption signals such a type’s customers receive, \( x = a + \epsilon \), and where the realization of each such signal induces a posterior mean \( E[a \mid a > a_n^*, x] \).

To proceed, we rewrite the right hand side as profits integrated with respect to the distribution of induced posterior means. With \( f \mid_{a>a_n^*} \) as our prior distribution, we define \( \underline{a} \equiv E[a \mid a > a_n^*, x = \underline{x}] \) as the posterior mean induced by signal \( \underline{x} \). The likelihood of consumers holding expectation \( \underline{a} \) is given by \( k(\underline{a}) \equiv \int_{a_n^*}^{\infty} f(a) h(\underline{x} - a) \, da \), which integrates over all possible types \( a \) and noise realizations \( \epsilon \) that result in \( a + \epsilon = \underline{x} \). We now state two useful properties of \( k(\underline{a}) \) given in Gelman et al., 2013. First, the prior mean is the average over all possible posterior means; that is, \( \int a f(a) \, da = \int \underline{a} k(\underline{a}) \, d\underline{a} \). Second, the expected posterior variance is less than the prior variance. We now represent line 10 equivalently as

\[
\int_{a_n^*}^{\infty} \pi(a) f(a) \, da > \int_{a_n^*}^{\infty} \pi(\underline{a}) k(\underline{a}) \, d\underline{a}
\]

which holds as \( \pi \) is convex (per footnote 14) and \( f(a) \) is a mean-preserving spread of \( k(\underline{a}) \).

References


