Multi-period competitive cheap talk with completely biased experts*

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Abstract

Each of $n$ experts communicates to a principle about the privately observed quality of his own project via cheap talk, and new independently drawn projects are available each period until the principle has adopted $k < n$ projects. Even when experts are completely biased in that they only receive a positive payoff if their own project is selected, we show that informative equilibria may exist, characterize the set of symmetric equilibria, and find the Pareto dominant equilibrium. Experts face a tradeoff between inducing acceptance now versus waiting for a better project should the game continue. When the future is more highly valued experts send more informative messages, increasing the average quality of an adopted project and resulting in a Pareto improvement, while the opposite is true when there is more competition between experts from either higher $n$ or lower $k$.

JEL Classification: D23, D74, D82

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1 Introduction

A decision maker often consults many experts over time before taking an action. For example, consider the division managers of a company who report to the CEO about the profitability of projects available to them. The CEO has enough resources to fund some, but not all, projects and cannot directly observe their quality while each division’s manager is privately informed about his own best project. The CEO wishes to select only the best projects while each manager is only concerned with his own division’s profits and so statically wants his project adopted even if it has low profitability. However, better projects may arrive over time which will influence the desirability of adopting projects currently available.

In this paper we ask whether the decision maker (the CEO) in such a setting can rely on the unverifiable reports of completely biased experts (each division manager) to make an adoption decision where the projects are independent across experts and time. The defining characteristics of this motivating example are as follows. Each of $n$ experts simultaneously report their project’s type using cheap talk to a decision maker (DM) who will adopt only $k < n$ of them. Each expert observes only his own type, not that of the other experts, and receives a payoff equal to his project’s type if it is adopted but obtains no benefit when a competing expert’s project is adopted. The DM makes a binary choice to adopt an expert’s project or not and has a payoff equal to the sum of the adopted projects’ types. At the end of any period if less than $k$ projects have been adopted so far the players proceed to the next period where independent draws are available. Thus each expert competes with the others for the adoption of his project over an indefinite time horizon.

Examples of experts competing in this manner can be found in many settings. Consider lobbyists who wish to convince the chairman of a government budget committee to spend on their own favored programs. One lobbyist proposes an educational intervention while the other an environmental one, and each is informed of his own policy’s effectiveness but not that of the other. The chairman has money sufficient to fund only one policy but may also adopt neither, deferring the decision to next year when additional proposals will be available. Another example is given in Li, Rantakari and Yang (2016) in which an economics
department has one open position to be filled by either a micro- or macroeconomist. The search committees can determine the quality of the candidate in their own field but not the other, and each prefers a hire in its own field. The department chair is a labor economist who prefers to hire the best candidate irrespective of the field but cannot observe either candidate’s quality. In our setting the chair may also refrain from hiring anyone now and wait for next year’s applicant pool.

This paper explores how the existence of future periods and competition between experts affects communication in the current period, and we show the two are intimately related. For example, it can easily be seen that when experts vie for their project to be adopted in a one period model only a babbling equilibrium will exist: each expert wants his project adopted regardless of the state and has only this period to convince the DM to do so. One way to avoid this outcome is to change the experts’ utility functions and in fact Li, Rantakari and Yang (2016) show that in a static model with two experts informative equilibria exist if the experts have a low enough Crawford and Sobel (1982) style additive or multiplicative bias. When the stage game is repeated with new projects realized each period it is no longer clear that babbling must ensue in our model since each expert has a continuation value and so might not attempt to induce acceptance of low types. However, future periods are valued only if they are reached and so more competing experts tend to make informative communication harder to support.

In order to disentangle the effects of future periods from competition between experts on the current period’s outcomes, we first consider a game between a single expert and a decision maker where incentives are aligned except for an outside option that provides a benefit to the DM but not the expert. This setting closely resembles Che, Dessein and Kartik’s (2013) static model in which the expert observes the value of finitely many projects and recommends one by use of comparative cheap talk. Our expert’s recommendation can similarly be viewed as a comparison between the value of the single project currently available with the value of projects that might be realized in the future, the crucial difference being that in our model the realization of future projects is not yet known to the expert. We show that even when an
informative equilibrium does not exist in a one period model the addition of future periods may allow for meaningful first period communication. This is because both the expert and decision maker benefit from rejecting states below a threshold value since better outcomes are likely next period, and thus the expert may be willing to separate these and other states. A higher discount factor increases the continuation value from the game and so expands the parameter values over which informative communication can occur.

The intuition that having future periods improves communication remains when there are two or more experts but now an additional factor is at play. Each expert is concerned that if he divulges information leading to rejection of his own project a competitor’s project may be selected now and the game will terminate. For this reason competition makes experts act as if they are “present biased” in that they put more weight on getting a project approved now than waiting for a better choice. This bias is not behavioral, but rather reflects the strategic incentives of competitive cheap talk across time. Since competing experts recommend adoption more often the DM infers a lower average quality for recommended projects and so rejects for a larger range of his outside option, making an informative equilibrium harder to sustain. Nonetheless, the basic structure of the equilibrium remains unchanged when there is competition: there is a threshold below which an expert prefers to induce rejection due to the continuation value of the game. Loosely speaking, an equilibrium exists when projects have a high chance of low outcomes so that an expert does not fear losing out to competitors, and yet a high expected value so that arriving at the next period is enticing enough. For states above the threshold, each expert prefers immediate acceptance and therefore wishes to induce as high a posterior as possible in order to be selected over competing experts. This implies credible distinctions between states above the threshold cannot be made and thus these states must be pooled.

Having shown that equilibria will entail the use of a threshold with a pooled message above and separation possible below we establish that any equilibrium in which no expert babbles is symmetric. However, this symmetric threshold is too low in that there is a higher threshold that would constitute a Pareto improvement. We use this result to select a
symmetric equilibrium and interpret comparative statics on the intensity of competition and value of future periods. We show that as the future becomes more important (i.e., a higher discount factor) each expert’s equilibrium threshold increases, which allows for informative communication for a larger range of parameter values and implies a conditionally higher project value, improving payoffs for all.

Increasing the intensity of competition has the opposite effect of lowering the equilibrium threshold and thus harming communication and payoffs, a result that contrasts with much of the multiple-sender literature (e.g., Battaglini, 2002) and owes to our experts’ extreme bias and knowledge of just one dimension of the state. In our model competition can manifest itself as an increase in the number of experts $n$ or a decrease in the number of projects $k$ the DM will adopt, and in either case our result obtains. This suggests that the DM would prefer to consult fewer experts, and indeed for any symmetric equilibrium with $n$ experts there is an asymmetric equilibrium in which $i < n$ experts babble and are ignored while the remaining $n - i$ experts use the equilibrium threshold from the symmetric game with $n - i$ experts. In the extreme, all but one expert babble, and any such equilibrium is best for the DM. However, we show these equilibria do not survive a refinement in which each expert believes there is a small chance he will not be ignored and for this reason we focus on equilibria in which no expert babbles, which we show must be symmetric.

This paper builds on a growing project selection literature. Li (2015) considers a cheap talk model with a Crawford and Sobel (1982) style additive bias in which two experts take turns reporting their project types over time, where the DM commits to a probabilistic decision rule that determines which player is active each period. Bonatti and Rantakari (2016) allow agents to exert costly effort that effects project completion time when each expert can veto the adoption of the other’s project. Rantakari (2016) explores the consequences of allowing the principle himself to exert effort in order to probabilistically obtain a better project while Rantakari (2013) allows the principle to ex-ante publicly commit to a decision mechanism when experts have an unknown bias. More generally this paper relates to the literature on competition between experts, as for example in Gilligan and Krehbiel (1989),
Krishna and Morgan (2001a; 2001b) and Battaglini (2002). However, in each of these models both experts observe the same state of the world whereas in the present paper each expert has private information about just one dimension of the state.

In Quint and Hendricks (2013) bidders in a two-stage auction first report their valuations by cheap talk and then enter the second stage to submit binding bids only if their first stage report was among the highest two. In this way the bidders are competing in the first period. However, in this model the seller commits to a message space and decision rule by which the first stage winners are determined and together with other assumptions it is shown this induces a Crawford and Sobel (1982) style bias for each bidder in the cheap talk stage. Our analysis differs in that we do not assume the receiver can commit to a mechanism and our senders do not have Crawford and Sobel style preferences.

Other models allow for influential communication with just a single agent through other means such as reputation. For example, Kim (1996) explores how reputation can effect cheap talk over an infinite horizon though unlike his paper we do not require infinitely many periods nor do we have ex-post verifiability. Along similar lines Sobel (1985) explores reputation in a cheap talk model in which the state is fixed across periods. Finally, in a static model Chakraborty and Harbaugh (2007) demonstrate that a single expert who observes all dimensions of the state space can make credible comparative statements even when it would not be credible on a single dimension.

The paper proceeds as follows. We first present the model and then show future periods improve communication and payoffs when there is one expert. Next we consider more than one expert competing in each period and across time and establish that all symmetric equilibria will involve a stationary threshold strategy. We then give conditions guaranteeing existence of symmetric equilibria in a general setting, identify the Pareto optimal such equilibrium, perform comparative statics, explore a specific family of distributions, and briefly discuss asymmetric equilibria. We then conclude.
2 Model

Let $\theta_i \sim F$ be the independent and identically distributed profits generated from the project available to expert $i$, for $i = 1, 2, ..., n$ and $n \geq 1$, where $F$ is a $C^1$ function, has finite expectation, and full support on a closed connected set $A \subseteq \mathbb{R}_+$. $F$ is common knowledge while the realization of $\theta_i$ is private information to expert $i$. In contrast to other multiple-sender models, here expert $i$ does not observe the outcome of $\theta_j$, $j \neq i$. The experts simultaneously send cheap talk messages to the decision maker (DM) who must decide which single project to accept. Though we initially assume the DM adopts only one project we extend this to $k < n$ projects in a subsequent section. We denote the DM’s choice by $d \in \{0, 1, 2, ..., n+1\}$, where $d = 0$ means all projects are rejected and $d = n + 1$ is an outside option always available to the DM. The outside option has commonly known value $\theta_{n+1} = r$, and we assume $r < \max A$ so the problem is non-trivial.

Payoffs in the stage game are as follows. For each project accepted the DM receives payoff $U_{DM}(d) = \theta_d$. Each expert is completely biased in that he only receives positive utility if his own project is adopted:

$$U_i(d) = \begin{cases} 
0 & \text{if } d \neq i \\
\theta_i & \text{if } d = i
\end{cases}. $$

If the DM rejects all projects the game enters the next period with new independent draws for each $\theta_i$ such that $i \neq n + 1$, and continues indefinitely until a project is adopted. Any project that has been rejected is lost and cannot be brought back, though we will show this is without loss of generality with i.i.d. projects and an indefinite number of periods. Finally, future payoffs are discounted by $\delta \in (0, 1)$ and all players are expected utility maximizers.

A strategy for an expert is a function that for each period maps the history of the game to a probability distribution over the message space $M = \mathbb{R}_+$. The DM updates beliefs about each $\theta_i$ given messages $m_i \in M$ using Bayes rule. A strategy for the DM is a function that for each period maps the history of the game to a probability distribution over the set of possible actions $\{0, 1, 2, ..., n + 1\}$. We look for a perfect Bayesian equilibrium that is non-babbling.
3 One expert

Initially we focus on the case of one expert and show that the existence of future periods can improve communication and payoffs. Since the DM takes a binary action of adopting the current project or not we will refer to any message that induces acceptance or rejection as a recommendation to accept or reject, respectively. As we shall see below, such recommendations may arise from messages pooled over many states or from separating messages.

When the game has just one period the expert prefers all projects be accepted while the DM wishes to only accept those projects above the outside option \( r \). Accordingly, the expert can induce acceptance by pooling his recommendation over all states provided that \( r \leq E[\theta] \). For \( r \) strictly less than \( E[\theta] \) the recommendation need not be pooled over all states; any subset containing \([0, r]\) with conditional expectation exceeding \( r \) will suffice, with states outside this subset separated. In fact, in the extreme case of \( r = 0 \), incentives are fully aligned and full separation is possible. We record these observations below.

Remark 1 With one expert and one period, in any equilibrium the expert recommends all projects be accepted, the DM follows all recommendations when \( r \leq E[\theta] \), and otherwise the DM rejects. The state can be truthfully revealed when \( r = 0 \) but otherwise some states must be pooled.

Now suppose there are two periods and discount factor \( \delta \). Using backwards induction, in the second period if \( 0 \leq r \leq E[\theta] \) the DM can be induced to accept any project, the expected present value of which is \( \delta E[\theta] \). This continuation value is enjoyed by the DM and the expert since both can benefit from a better second period project. Then in the first period the expert prefers adoption of only those projects exceeding \( \delta E[\theta] \) while the DM wishes to only accept projects above \( \max\{\delta E[\theta], r\} \). If \( r \leq \delta E[\theta] \) incentives are fully aligned in the first period and full separation is possible. If instead \( r \in (\delta E[\theta], E[\theta]) \), the DM prefers rejection of \( \theta \in (\delta E[\theta], r) \) while the expert prefers acceptance and can induce it, for example by pooling all states above \( \delta E[\theta] \), which would be accepted since \( E[\theta|\theta > \delta E[\theta]] > E[\theta] \geq r \). In fact this chain of inequalities shows there is slack so that some states above \( r \) can be separated. See
the first and third rows of Figure 1. Finally, since the expert always recommends acceptance in the last period the DM rejects if \( r > E[\theta] \), and so by an unraveling argument the expert will also always recommend acceptance and be rejected in any prior period. We summarize these observations below.

**Remark 2** With one expert and two periods, in any equilibrium the expert recommends acceptance in the first period if and only if the project exceeds \( E[\theta] \), the DM follows all recommendations when \( r \leq E[\theta] \), and otherwise the DM rejects. The state can be truthfully revealed when \( r \leq \delta E[\theta] \) but otherwise some states must be pooled.

When \( r \leq E[\theta] \) the existence of a second period improves outcomes. By having a future to look forward to, the expert is willing to recommend rejection of low quality projects in the first period since it is likely a better one can be obtained next period. This leads to higher expected profits, since \( \int_{0}^{\infty} \max\{\delta E[\theta], \theta\} \, dF(\theta) > \int_{0}^{\infty} \theta \, dF(\theta) \). Furthermore, the expert can fully separate states whenever \( r \in [0, \delta E[\theta]] \), in contrast to the one period case in which the same result obtains only when \( r = 0 \).

Finally we show that allowing the game to repeat indefinitely can further improve communication and payoffs. Initially consider \( r = 0 \) so that there is no divergence in incentives. There is no last period so we conjecture projects are adopted only if they exceed a threshold \( t \). This is a familiar optimal stopping problem and the threshold must satisfy

\[
t = \delta \int_{t}^{\infty} \theta \, dF(\theta) \sum_{i=0}^{\infty} (\delta F(t))^i \quad \Leftrightarrow \quad t = \frac{\delta \int_{t}^{\infty} \theta \, dF(\theta)}{1 - \delta F(t)}.
\]

**Lemma 1** Equation 1 has a unique solution \( \hat{t} \) which increases in \( \delta \), and the DM’s payoff is single-peaked at \( \hat{t} \).

**Proof** See Appendix A. \( \blacksquare \)
Values of the DM’s outside option that support communication

|                      | 0       | $\delta \cdot E[\theta]$ | $E[\theta]$ | $\hat{t}$ | $E[\theta|\theta > \hat{t}]$ | $\infty$ |
|----------------------|---------|--------------------------|-------------|---------|--------------------------|---------|
| **Persuasive**       |         |                          |             |         |                          |         |
| communication        |         |                          |             |         |                          |         |
| Finitely repeated    | Yes     | Yes                      | No          | No      | No                       | No      |
| Indefinitely         |         |                          |             |         |                          |         |
| repeated             |         |                          |             |         |                          |         |
| **Full**             |         |                          |             |         |                          |         |
| separation           |         |                          |             |         |                          |         |
| Repeated twice       | Yes     | No                       | No          | No      | No                       | No      |
| Indefinitely         |         |                          |             |         |                          |         |
| repeated             |         |                          |             |         |                          |         |

Figure 1: Persuasive communication and full separation are supported for a larger range of DM’s outside option when the game is indefinitely rather than finitely repeated. Note: the ordering of $E[\theta]$ and $\hat{t}$ is ambiguous in general.

It is a straightforward exercise to show that the adoption threshold is more stringent with infinitely many periods than two ($\hat{t} > \delta E[\theta]$). Now suppose $r > 0$, and we note the logic proceeds much the same as in the two period case. In each period the expert prefers adoption only if the project exceeds $\hat{t}$ while the DM’s preferred threshold is $\max\{\hat{t}, r\}$. When $r \leq \hat{t}$ incentives are fully aligned and full separation is possible. If $r > \hat{t}$ the DM prefers rejection of $\theta \in (\hat{t}, r)$ while the expert prefers acceptance and can induce it by pooling with higher states provided $r \leq E[\theta|\theta > \hat{t}]$. Finally, if $r > E[\theta|\theta > \hat{t}]$ future periods have no effect since the DM takes his outside option immediately. We summarize these observations in the following remark.

**Remark 3** With one expert and infinitely many periods, in any equilibrium the expert recommends acceptance in each period if and only if the project exceeds $\hat{t}$ from Lemma 1, the DM follows all recommendations when $r \leq E[\theta|\theta > \hat{t}]$, and otherwise the DM rejects. The state can be truthfully revealed when $r \leq \hat{t}$ but otherwise some states must be pooled.

Repeating the game an indefinite number of times further improves outcomes (see Figure 1). A higher threshold is used since the expert is more willing to let relatively low states be rejected, and this further increases payoffs since $\int_0^\infty \max\{\hat{t}, \theta\} dF(\theta) > \int_0^\infty \max\{\delta E[\theta], \theta\} dF(\theta)$. In addition, the DM can be induced away from his outside provided that $r \leq E[\theta|\theta > \hat{t}]$ in
an indefinitely repeated game, while the condition is \( r \leq E[\theta] \) if the game is finite. Thus persuasive communication can be supported for a larger range of parameter values, a general property that is demonstrated in the example below.

**Example 1** Let \( \theta \sim U[0,1] \) be i.i.d. across periods and \( \delta = 0.8 \). Table I below records first period outcomes for a game with one, two, or infinitely many periods.

**Table I**

<table>
<thead>
<tr>
<th>Total number of periods</th>
<th>All recommendations are followed and...</th>
<th>DM always rejects</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Full separation</td>
<td>Some pooling</td>
</tr>
<tr>
<td>1</td>
<td>( r = 0 )</td>
<td>( 0 &lt; r \leq 0.5 )</td>
</tr>
<tr>
<td>2</td>
<td>( 0 \leq r \leq 0.4 )</td>
<td>( 0.4 &lt; r \leq 0.5 )</td>
</tr>
<tr>
<td>( \infty )</td>
<td>( 0 \leq r \leq 0.5 )</td>
<td>( 0.5 &lt; r \leq 0.75 )</td>
</tr>
</tbody>
</table>

We conclude this section by commenting on the shared features of our one sender model with the comparative cheap talk literature (e.g., Chakraborty and Harbaugh, 2007 and 2010; Che, Dessein, and Kartik, 2013). To see this, suppose the game has reached period \( n \) and the expert must decide whether or not to recommend the adoption of the project whose value is \( \theta_n \).\(^1\) A recommendation to adopt indicates the current project is better than what will likely arise in the future, while a recommendation to reject indicates the converse. That is, the expert makes a comparative cheap talk statement that ranks the two relevant dimensions of the state space: this period’s project versus next period and beyond. However, in our model the expert does not have private information about future states, in contrast to static comparative cheap talk models in which the expert observes all dimensions of the state. It is for this reason, for example, that the pandering effect found in Che, Dessein, and Kartik (2013) is not present here.

\(^1\)Pursuant to the strategies in Remark 3, having reached period \( n \) implies the prior \( n - 1 \) projects were below the threshold \( \ell \). Since \( \theta_i \) are i.i.d. any previously rejected project would never be returned to and so the decision reduces to recommending this period’s project or not. This observation demonstrates that within our model the no recall assumption is without loss of generality.
In our model comparative statements can be informative due to an endogenous opportunity cost of lying. If the expert induces acceptance by falsely ranking the current project ahead of what can be expected in the future then he himself suffers from the forgone benefit of continuing in the game. The continuation value depends on the discount factor $\delta$ and the number of periods the game might have. Lower $\delta$ increases the relative value of the present and so causes the expert to recommend adoption of the current project more often; that is, $\hat{t}$ decreases per Lemma 1. Thus when the expert becomes more present-biased the value of a recommended project $E[\theta|\theta > \hat{t}]$ declines, which leads to rejection for a larger set of values of the DM’s outside option $r$. In the next section we explore another cause of present-bias, namely competition with other experts.

4 Competition among experts

Having seen that future periods help improve communication between a single expert and the DM, we now consider the effect of competition between experts to have their own favored projects adopted. Since expert $i$ only observes $\theta_i$ and receives payoff 0 when project $j \neq i$ is adopted, if there is only one period all experts always recommend adoption of their project. However, if there is a second period an expert with a low realization may recommend rejection of his own project in the hopes that both next period will be reached and a better project will be realized and accepted then. We proceed by showing any equilibrium entails use of a threshold strategy and then consider equilibrium existence, selection, and comparative statics. We initially assume the DM selects at most one project and extend to $k < n$ projects in a subsequent subsection.

**Proposition 1 (Equilibrium characterization)** In any symmetric equilibrium there is a stationary threshold $t^*$ such that projects $\theta \leq t^*$ can be separated and are rejected while $\theta > t^*$ are pooled and recommended for acceptance. If one or more project is recommended the DM chooses one of these to adopt with equal probability.
Proof We first prove a threshold is used. Given any strategy, the continuation value from inducing rejection is invariant to type while the expected payoff from inducing acceptance is increasing in type. Thus any message must be sent by a connected set of types. Next, project adoption can only be recommended for one set of pooled states. If not, then any type assigned to a message accepted with lower probability would defect to a message accepted with a higher probability.

Next, if a single expert prefers his project be rejected then the DM does a fortiori since the latter benefits when any expert’s future project is above the threshold; thus rejected states can be separated. Also, since the problem is stationary the threshold does not change between periods nor depend on previous realizations. Finally, given the experts employ a stationary threshold strategy it is a best response for the DM to accept a recommended project with probability \( \frac{1}{j} \) when \( j \geq 1 \) projects are recommended, and reject when rejection is recommended by all experts.

Due to the continuation value of the game an expert may prefer his relatively low quality project be rejected in the hope that he obtains a better project should next period be reached. This implies the DM would also prefer such a project be rejected since he is concerned with the more likely possibility that any expert receives a better project, and for this reason the expert is willing to separate when the state is below \( t^* \). In contrast, competition with other experts precludes the expert from making credible recommendations to accept that are separated since an expert who wants his project accepted will always send the most favorable message possible to avoid losing out to a competitor. This concern was not present in the single expert case and so there separation for some accepted states was possible. In addition, waiting for a possibly better project next period only incurs the lone expert a time delay cost while competing experts are also cognizant that the game may terminate this period, which we will show causes competing experts to use a lower threshold.

We now define \( G(t) \equiv \frac{\delta F(t)^{n-1}}{1-\delta F(t)^n} \int_t^\infty \theta \, dF(\theta) \). As will be demonstrated in Proposition 2, the existence of an equilibrium will correspond to fixed points of \( G \). To this end, the following properties will be useful.
Lemma 2

(i). \( G(t) \) is continuous, \( G(0) = 0 \), and \( \lim_{t \to \infty} G(t) = 0 \).

(ii). \( G'(t) \) is continuous and \( G'(0) = \begin{cases} \delta f(0) E[\theta] & \text{if } n = 2 \\ 0 & \text{if } n > 2 \end{cases} \).

(iii). Let \( t_0 > 0 \) be the largest fixed point of \( G \). Then \( G'(t_0) \leq 1 \), and if \( \theta \) is bounded above by \( \bar{\theta} \) then \( t_0 < \frac{\bar{\theta}}{n} \).

Proof For part (i), the continuity of \( G(t) \) follows from the continuity of \( F(t) \) and \( G(0) = 0 \) by direct computation. Next, \( \lim_{t \to \infty} G(t) = \frac{\delta}{1-\delta} \times 0 = 0 \). For part (ii), \( G'(t) \) calculates to

\[
\frac{\partial}{\partial t} \left( \frac{\delta F(t)^{n-1}}{1-\delta F(t)^{n}} \right) \int_t^\infty \theta dF(\theta) + \frac{\delta F(t)^{n-1}}{1-\delta F(t)^{n}} \frac{\partial}{\partial t} \left( \int_t^\infty \theta dF(\theta) \right). \tag{2}
\]

Next, \( \frac{\partial}{\partial t} \left( \int_t^\infty \theta dF(\theta) \right) = -t f(t) \) and thus when \( t = 0 \) the second term in line 2 evaluates to 0 for \( n \geq 2 \). Next,

\[
\frac{\partial}{\partial t} \left( \frac{\delta F(t)^{n-1}}{1-\delta F(t)^{n}} \right) = \frac{(1-\delta F(t)^n) (n-1) \delta F(t)^{n-2} f(t) + \delta F(t)^{n-1} n \delta F(t)^{n-1} f(t)}{(1-\delta F(t)^n)^2} \tag{3}
\]

and this evaluates to 0 when \( t = 0 \) and \( n > 2 \) while if \( n = 2 \) it becomes

\[
\frac{(1-\delta F(t)^n) (n-1) \delta F(t)^{n-2} f(t)}{(1-\delta F(t)^n)^2} = \frac{\delta f(0)}{1-\delta F(0)^2} = \delta f(0)
\]

and the result follows by substituting this into line 2. Finally, \( G'(t) \) is continuous by the continuity of \( f \) and \( F \) and lines 2 and 3.

Part (iii). Let \( t_0 \) be the largest \( t \) such that \( G(t) = t \). Then the graph of \( G \) cannot cross the graph of \( t \) from below at \( t_0 \), since this implies \( G(t_0 + \epsilon) > t_0 + \epsilon \) for sufficiently small \( \epsilon > 0 \) by the continuity of \( G \). But then by part (i) of this lemma there must be another fixed point \( t_1 > t_0 \) since \( G \to 0 \) in the limit, a contradiction. Thus at \( t_0 \) the graph of \( G(t) \) crosses \( t \) from above, or is just tangent to it. In the former case \( G'(t_0) < 1 \) while in the latter \( G'(t_0) = 1 \).
We now prove $t_0 < \frac{\bar{n}}{n}$ if $\theta$ is bounded above by $\bar{\theta}$. First, denote the upper bound of $G$ by $x$ and note $t_0 \leq x$. Next, by inspection $\frac{\partial G}{\partial \theta} > 0$ for all $t > 0$, and so it suffices to bound $G$ by $\frac{\bar{n}}{n}$ for $\delta = 1$:

$$G(t; \delta = 1) = \frac{F(t)^{n-1}}{1 - F(t)^n} \int_t^{\infty} \theta dF(\theta) = \left( \frac{F(t)^{n-1}}{\sum_{i=0}^{n-1} F(t)^i} \right) E[\theta | \theta \geq t]. \quad (4)$$

We claim line 4 increases in $t$. Since $E[\theta | \theta \geq t]$ obviously increases in $t$ it suffices to show

$$\frac{\partial}{\partial t} \left( \frac{F(t)^{n-1}}{\sum_{i=0}^{n-1} F(t)^i} \right) = \frac{(n-1) F(t)^{n-2} f(t) \sum_{i=0}^{n-1} F(t)^i - F(t)^{n-1} \frac{\partial}{\partial n} \left( \sum_{i=0}^{n-1} F(t)^i \right)}{\left( \sum_{i=0}^{n-1} F(t)^i \right)^2} > 0.$$

This is true since the numerator is positive, which follows from

$$(n-1) F(t)^{n-2} f(t) \sum_{i=0}^{n-1} F(t)^i > F(t)^{n-1} \frac{\partial}{\partial n} \left( \sum_{i=0}^{n-1} F(t)^i \right) \iff \sum_{i=0}^{n-1} (n-1) F(t)^i > F(t) \left( \sum_{i=0}^{n-2} (1+i) F(t)^i \right),$$

where the equivalence is established by evaluating the derivative and dividing by $F(t)^{n-2}$ and the last inequality holds since the left hand side is larger than even the term in the parentheses on the right hand side. Finally, since $G(t; \delta = 1)$ is increasing in $t$, taking the limit of line 4 as $t \to \bar{\theta}$ gives the result. 

We now define $A(t) \equiv \sum_{i=0}^{n-1} \frac{(n-1) F(t)^{n-1-i}(1-F(t))^i}{1+i}$, the probability an expert’s project is chosen given he recommends it when all experts use threshold $t$. This probability depends on how likely no other experts recommend acceptance, exactly one does, exactly two do, and so on, where each project is accepted with equal probability if more than one is recommended. Next let $B(t) \equiv \int_t^{\infty} \theta \ dF(\theta)$, which integrates over the possible types that might be realized and accepted should future periods be reached. Finally, unless otherwise specified all references to fixed points of $G$ will refer to non-zero fixed points.
Proposition 2 (Existence of equilibria) A symmetric threshold \( t^* \) satisfies each expert’s incentive compatibility constraint if and only if \( G(t^*) = t^* \). At least one fixed point of \( G \) exists if \( n = 2 \) and \( E[\theta] > 1/\delta f(0) \); otherwise, at least two fixed points exist if there is a \( t \) such that \( G(t) > t \). The DM’s participation constraint is satisfied and thus an equilibrium exists if \( E[\theta|\theta > t^*] \geq r \).

Proof The DM will accept the project only if its expected value is not less than his outside option; i.e., \( E[\theta|\theta > t^*] \geq r \). Next, to check each expert’s constraint we conjecture a threshold \( t \) above which each recommends adoption, as indicated in Proposition 1, and show type \( t \) is indifferent between recommending acceptance or rejection. The payoff to the threshold type from attempting to induce acceptance is his type \( t \) times the probability of winning while inducing rejection gives a continuation payoff:

\[
\begin{align*}
t A(t) &= \delta F(t)^{n-1} \left( A(t) B(t) + \delta F(t)^n \left( A(t) B(t) + \delta F(t)^n \left( \ldots \right) \right) \right) \\
t A(t) &= \sum_{l=0}^{\infty} A(t) B(t) \delta F(t)^{n-1} (\delta F(t)^n)^l \\
t &= \frac{\delta F(t)^{n-1}}{1-\delta F(t)^n} B(t)
\end{align*}
\]

(5)

In the first line, the ellipses in the last parentheses repeats the expressions given in the big parentheses. The second line uses summation notation for this infinite sum, and the third line reduces the sum and cancels the common \( A(t) \) term. Thus satisfying the sender constraints reduces to finding a fixed point of \( G(t) \).

By Lemma 2, \( G(0) = 0 \) and if \( n > 2 \) then \( G'(0) = 0 \). Since \( \lim_{t \to \infty} G(t) = 0 \), if there is a \( \hat{t} \) such that \( G(\hat{t}) > \hat{t} \) then there are at least two fixed points since \( G \) is continuous. In fact this argument only requires \( G'(0) < 1 \) and thus it applies when \( n = 2 \) and \( E[\theta] < 1/\delta f(0) \), since by Lemma 2(ii) \( G'(0) < 1 \) in this case. If \( n = 2 \) and \( E[\theta] > 1/\delta f(0) \) then \( G'(0) > 1 \) and thus \( \lim_{t \to \infty} G(t) = 0 \) implies at least one fixed point exists. ■

By inspection of \( G(t) = \frac{\delta F(t)^{n-1}}{1-\delta F(t)^n} \int_t^\infty \theta dF(\theta) \) the condition that \( G(t) > t \) occur is satisfied when, for example, \( F(t) \) is large for small \( t \) but \( E[\theta|\theta > t] \) is nonetheless high. Intuitively,
the probability of low realizations must be high enough that an expert with a low outcome doesn’t fear being scooped by the competing expert, but at the same time the expectation of a new project must be sufficiently high that the continuation value from proceeding to the next period is enticing enough. When these conditions fail only a babbling equilibrium in which experts recommend all project types be adopted \((t = 0)\) exists,\(^2\) as depicted in Figure 2(a). This is in stark contrast to the single expert case, in which the existence of a non-trivial threshold was guaranteed for \textit{any} distribution. In fact, if \(\delta = 1\) a single expert would never recommend acceptance \((t = \infty)\) while two competing experts may instead \textit{always} do so \((t = 0)\), or if \(E[\theta] > 1/f(0)\) use a non-zero but finite threshold.

Though informative communication is harder to sustain when experts compete, repetition of the stage game may help in a nuanced way. The DM does not directly observe the state and therefore does not benefit from a typical order statistic effect from, say, adding another period or increasing the discount factor in a decision problem. Instead the DM only infers the average quality of a recommended project given the experts’ threshold, so that the communication strategy used in the future effects how valuable the future will be. For example, a finitely repeated game with babbling by all experts in each period leads to

\(^2\)A babbling equilibrium always exists since \(G(0) = 0\) by Lemma 2(i).
the same expected actions and payoffs as an indefinitely repeated game with similar strategies, since potentially more periods of babbling has no effect. However, if the equilibrium threshold is positive there will in general be an equilibrium effect of $\delta$ on the threshold, and thus on payoffs and communication. We explore such comparative statics further in a later subsection, but we first select an equilibrium.

**Equilibrium selection**

By Proposition 2 there can be multiple non-trivial equilibria, an example of which is depicted in Figure 2(c). In addition, since $G(0) = 0$ there is always a trivial equilibrium in which $t = 0$ so that all projects are recommended regardless of their type. In Section 5 below we show there are also equilibria in which an individual player or players babble and are ignored by the DM while other players use non-zero thresholds. For the time being we restrict away from these cases, show that any equilibrium must therefore be symmetric, and select an equilibrium which is Pareto dominant.

**Definition 1** An expert is influential in an equilibrium if in each period there is a positive probability his recommended project is accepted.

**Proposition 3** Any equilibrium in which each expert is influential is symmetric.

**Proof** We first show an expert’s best-response threshold is increasing in the other experts’ thresholds. Let $t_i$ denote the threshold that expert $i$ uses. Without loss of generality we establish the result by proving $\frac{\partial t_i}{\partial t_i} > 0$ for all $i \neq 1$. The indifference condition for threshold $t_1$, analogous to that in line 5, is

$$t_1 = \frac{\delta \sum_{j=2}^{n} F(t_j)}{1 - \delta \sum_{j=1}^{n} F(t_j)} B(t_1). \quad (6)$$

Let $t^* \equiv (t_1, ..., t_n)$ be a solution to line 6 and define $H(t_1, ...t_n) = \frac{\delta \sum_{j=2}^{n} F(t_j)}{1 - \delta \sum_{j=1}^{n} F(t_j)} B(t_1) - t_1$. 

17
By the implicit function theorem $\frac{\partial H}{\partial t_i} (t^*) = -\frac{\partial H}{\partial t_1} (t^*)$. We now show $\frac{\partial H}{\partial t_i} (t^*) > 0$ for $i \neq 1$.

$$\frac{\partial H}{\partial t_i} (t^*) = \delta B (t_1) \left( \frac{(1 - \delta \Pi_{j=1}^n F(t_j)) f(t_i) \Pi_{j=1, j \neq i}^n F(t_j) + (\Pi_{j=2}^n F(t_j)) f(t_i) \delta \Pi_{j=2, j \neq i}^n F(t_j)}{(1 - \delta \Pi_{j=1}^n F(t_j))^2} \right)$$

and thus $\frac{\partial H}{\partial t_i} (t^*) > 0$ since both the numerator and denominator are positive. Finally, we show $\frac{\partial H}{\partial t_1} (t^*) < 0$ and thus conclude $\frac{\partial H}{\partial t_i} (t^*) > 0$.

$$\frac{\partial H}{\partial t_1} (t^*) = -t_1 f(t_1) \left( \frac{\delta \Pi_{j=2}^n F(t_j)}{1 - \delta \Pi_{j=1}^n F(t_j)} \right) + B(t_1) \left( \frac{-(\delta \Pi_{j=2}^n F(t_j)) (-f(t_1) \delta \Pi_{j=2}^n F(t_j))}{(1 - \delta \Pi_{j=1}^n F(t_j))^2} \right) - 1$$

and rearranging terms and substituting from line 6 gives $\frac{\partial H}{\partial t_1} (t^*) = -1 < 0$. Thus each expert's threshold is increasing each other expert's threshold.

Finally, toward a contradiction suppose there exists an asymmetric equilibrium in which each expert is influential. Then there are experts $i$ and $j$ such that $t_i > t_j$, for $i \neq j$. If these thresholds are part of an equilibrium then the symmetry of the players' type distributions implies there is also an otherwise identical equilibrium in which expert $i$ uses threshold $t_j$ and expert $j$ uses threshold $t_i$, contradicting the fact that each best response threshold is increasing in other experts' thresholds. See Figure 3.

Proposition 3 together with Proposition 1 go a long way towards characterizing the equilibrium set: any equilibrium in which each expert is influential must be symmetric, and any symmetric equilibrium must conform to Proposition 1. We complete the characterization of the full set of stationary equilibria in Section 5 by discussing equilibria in which some players are not influential. We now proceed by selecting a symmetric equilibrium and to this end we establish the following lemma.

**Lemma 3** The DM’s expected payoff is $\frac{1-F(t^*)}{1-\delta F(t^*)} E[\theta | \theta > t]$ while each of the $n$ experts’ expected payoff is $\frac{1}{n}$ of this.
Figure 3: If point $a$ represents mutual best response thresholds then so too does point $b$, contradicting that a best response threshold $t_j$ is increasing in $t_i$.

**Proof** The DM’s expected payoff with threshold $t$ and discount factor $\delta$ is

$$\sum_{i=0}^{\infty} (\delta F(t)^n)^i (1 - F(t)^n) E[\theta|\theta > t] = \frac{1 - F(t)^n}{1 - \delta F(t)^n} E[\theta|\theta > t]$$

while each expert’s expected payoff is

$$\sum_{i=0}^{\infty} (\delta F(t)^n)^i (1 - F(t)) A(t) E[\theta|\theta > t] = \frac{(1 - F(t)) A(t) E[\theta|\theta > t]}{1 - \delta F(t)^n}.$$ 

Eliminating common terms the result then follows since

$$\frac{1 - F(t)^n}{1 - F(t)} = \sum_{i=0}^{n-1} F(t)^i = n A(t).$$

Intuitively, since experts are ex-ante identical and use symmetric strategies the probability any particular expert’s project is selected is $\frac{1}{n}$. In addition, as soon as the DM accepts an expert’s project the game ends and that expert and the DM receive the same payoff, the expectation of which is $E[\theta|\theta > t^*]$. Thus among the set of equilibrium thresholds the DM and each expert share the same maximizer, a result we utilize below.
Proposition 4 The equilibrium with the highest threshold is Pareto optimal.

Proof A Pareto ranking follows from Lemma 3. To show the highest equilibrium threshold is Pareto optimal we note that (i) it suffices to track the DM’s payoff; (ii) by Lemma 1 the DM’s payoff is single-peaked at $\hat{t}$ (which solves line 1); and (iii) in any symmetric equilibrium with $n > 1$ the threshold is less than $\hat{t}$ (by Proposition 6 below).

Unless otherwise specified we henceforth focus on the Pareto optimal symmetric equilibrium. We next present a stylized example in which multiple equilibria exist.

Example 2 Let $\delta = 1$ and suppose the DM will adopt one project from among three experts whose projects are i.i.d. with pdf parameterized by $b \geq 1$ as follows:

$$f(\theta; b) = \begin{cases} 
1 - \theta & \text{if } \theta \in [0, 1] \\
1 & \text{if } \theta \in [b, b + 0.5] \\
0 & \text{otherwise} 
\end{cases}$$

and let $F(t; b)$ be the corresponding CDF.\(^3\) Thus $F(t; b)$ is invariant to $b$ for $t \leq 1$ but

\(^3\)Although $f$ is not continuous and differentiable at every point of its domain it is for $\theta < 1$ and thus the properties of $G(t)$ previously established hold when $t < 1$. 

Figure 4: Determination of equilibrium thresholds for Example 2
$E[\theta|\theta > t]$ is increasing in $b$ and so an equilibrium threshold exists for high enough $b$. Figure 4 graphs $G(t)$ for $b = 6.5$ and shows there are two thresholds that satisfy the sender constraints, $t_1^* \approx 0.470$ and $t_2^* \approx 0.961$. If the DM’s outside option $r < E[\theta|\theta > t_1^*] \approx 5.412$ both thresholds constitute an equilibrium though $t_2^*$ is Pareto dominant. If $r > E[\theta|\theta > t_2^*] \approx 6.741$ the DM will always reject while for intermediate $r$ only $t_2^*$ is an equilibrium threshold.

**Comparative statics**

Since each expert’s recommendation is a comparison of his current project to the future, anything that increases the relative value of the present will induce experts to recommend adoption of their current project more often; i.e., each expert will use a lower threshold. In turn the DM will infer a lower expected type for those projects that are recommended, leading to rejection for a larger range of values of the outside option $r$. In this subsection we explore the qualitative effect of competition between experts and the role of future periods by showing how each affects the “present bias” of experts. A decline in the discount factor $\delta$ places less weight on future periods and so relatively more on the present, a case we consider in the proposition immediately below. Increased competition between experts also increases the relative value of the present since it implies a lower likelihood that future periods will be reached, which we explore in Propositions 6 and 7 below.

**Proposition 5** The equilibrium threshold increases in the discount factor ($\frac{\partial t^*}{\partial \delta} > 0$), resulting in a Pareto improvement.

**Proof** By inspection, $\frac{\partial G}{\partial \delta} > 0$. Next, by Lemma 2(iii) either $G'(t^*) < 1$ and so $\frac{\partial G}{\partial \delta} > 0$ and the implicit function theorem imply $\frac{\partial r^*}{\partial \delta} > 0$, or $G'(t^*) = 1$ and so $G(t^*)$ is tangent to $t$ and thus by $\frac{\partial G}{\partial \delta} > 0$ higher $\delta$ gives rise to two fixed points, the greater of which exceeds $t^*$. Thus $\frac{\partial r^*}{\partial \delta} > 0$. By Lemma 3 to establish the Pareto improvement it suffices to calculate the change in the DM’s payoff with respect to $\delta$, which can be decomposed into a direct and indirect effect:

$$\frac{\partial}{\partial \delta} \left( \frac{1 - F(t^*)}{1 - \delta F(t^*)} E[\theta|\theta > t^*] \right) + \frac{\partial}{\partial t^*} \left( \frac{1 - F(t^*)}{1 - \delta F(t^*)} E[\theta|\theta > t^*] \right) \frac{\partial t^*}{\partial \delta}.$$
The first term is positive by inspection while $\frac{\partial \xi}{\partial t} > 0$ was just proved. Finally, the DM’s payoff increases in $t$ by arguments (ii) and (iii) in the proof of Proposition 4.

We find that future periods improve communication and increase expected payoffs, confirming our intuition from the single expert case. When the future is discounted less (higher $\delta$) there is a direct effect that improves payoffs but also an indirect effect through the equilibrium threshold. With higher $\delta$ the value of waiting for a better project increases for the DM and each expert. This makes experts more willing to divulge information about states that are relatively low and improves payoffs since the DM also prefers to reject these low states. While an increased weight on the future induces experts to be more selective in recommending their own projects, the presence of more rival experts has the opposite effect, as shown in the next proposition.

**Proposition 6** The equilibrium threshold decreases in the number of experts, resulting in a Pareto loss.

**Proof** By Lemma 2(iii), $G'(t^*) \leq 1$ and so it suffices to show that $G(t)$ is decreasing in $n$ for all $t$. Using the $B(t)$ notation, $G(t; n) = \frac{B(t)}{F(t)^n}$ and the statement follows since $F(t)^{n-1}$ and $F(t)^n$ are decreasing in $n$ for all $t > 0$. Finally, regarding each player’s payoff, by Lemma 3 it suffices to track the DM’s payoff and by Lemma 1 this payoff declines as the equilibrium threshold declines further below $\hat{t}$.

We find that competition among experts causes each to value the future less since it is less likely future periods will be reached. For this reason each expert is less selective when recommending projects, even though this lowers the average quality of adopted projects and results in a Pareto loss. In fact, the proof of the proposition establishes that the sum of all players’ payoffs, including the new expert, is lower than the corresponding sum without this new expert.

To conclude this subsection we explore one more notion by which the degree of competition between experts can vary. Suppose the DM chooses precisely $k < n$ projects from among the $n$ experts where the projects can be chosen in any manner across time. If an
expert’s project is chosen that expert is removed from the game in all subsequent periods, should they arise, and we assume the number of remaining projects to be selected is common knowledge. Thus for a fixed $n$ higher $k$ implies less competition between experts.\footnote{If $k = n$ there is no competition at all since each expert will have his project selected eventually, and so the results from Section 3 regarding one expert and one project would apply.}

Equilibria are qualitatively similar to the $k = 1$ case: every equilibrium consists of a threshold value below which experts can separate and will be rejected but above which they must pool. Appendix B iteratively constructs an equilibrium for the $k = i$ case for $i = 2, 3, ..., n - 1$ by referring to the $k = i - 1$ case. For example, the threshold determination for $k = 2$ contemplates the possibility that next period there may again be two projects left to be selected, one project left in which case the expected profits to an expert from continuing can be calculated from Lemma 3, or next period may not be reached since two or more experts recommended acceptance this period. Determining the threshold for $k = 3$ similarly refers back to the profits earned in the $k = 1$ and 2 cases, and so on.

We leave the details of the equilibrium characterization to Appendix B but explore existence and comparative statics here. Let $t_k$ be the highest equilibrium threshold given there are $k$ projects left to be selected.

\textbf{Proposition 7} \textit{If there is an equilibrium threshold $t_k > 0$ then there exists an equilibrium threshold $t_{k+1} > t_k$. Additionally, higher $k$ results in a Pareto improvement.}

\textbf{Proof} See Appendix B. \hfill \blacksquare

The comparative static on $k$ confirms our intuition that less competition results in better outcomes. Here, if the DM is able to accept more projects ($k$ increases) there is a greater possibility of reaching subsequent periods and so each expert is willing to use a higher threshold, implying a conditionally higher project quality and thus higher payoffs for all players.
An example: the symmetric beta distribution

Let \( k = 1 \) and \( \theta_i \sim B[\alpha, \beta] \) where we specify \( \alpha = \beta > 0 \) so that the distribution is symmetric. The Beta distribution has full support on \((0, 1)\), and with our assumptions \( E[\theta_i] = \frac{1}{2} \) while \( Var[\theta_i] = \frac{1}{4(2\alpha + 1)} \), so lowering \( \alpha \) constitutes a mean-preserving spread.

**Proposition 8** An equilibrium exists if and only if \( \alpha < \hat{\alpha}_n \) for some \( \hat{\alpha}_n > 0 \). Additionally, the equilibrium threshold is decreasing in \( \alpha \).

**Proof** If an equilibrium threshold \( t^* \) exists it is implicitly defined by \( H(t) = 0 \), where \( H(t) = G(t) - t \). By the implicit function theorem

\[
\frac{\partial t}{\partial \alpha}(t^*) = -\frac{\partial H}{\partial \alpha}(t^*),
\]

and Lemma 2(iii) implies \( \frac{\partial H}{\partial \alpha}(t^*) < 0 \) (except for the knife-edge case \( G'(t^*) = 1 \), where higher \( \alpha \) implies the fixed point no longer exists). Thus it suffices to show \( \frac{\partial H}{\partial \alpha}(t^*) < 0 \), which is established if \( \frac{\partial G}{\partial \alpha}(t^*) < 0 \). Now, any fixed point \( t^* < \frac{1}{2} \) by Lemma 2(iii). But then by the properties of the symmetric Beta distribution \( \frac{\partial F}{\partial \alpha} < 0 \) and \( \frac{\partial E[\theta|\theta > t]}{\partial \alpha} < 0 \). Writing \( G \) as

\[
\left( \frac{\delta F(t)^{n-1} (1 - F(t))}{1 - \delta F(t)^n} \right) E[\theta|\theta > t],
\]

it suffices to show the term in the large parentheses decreases in \( \alpha \). Since the denominator is increasing in \( \alpha \) it suffices to show that

\[
\frac{\partial}{\partial \alpha} \left( \delta F(t)^{n-1} (1 - F(t)) \right) < 0 \quad \iff \quad \delta \frac{\partial F}{\partial \alpha} F(t)^{n-2} \left( (1 - F(t)) (n - 1) - F(t) \right) < 0,
\]

which holds since \( t^* < \frac{1}{2} \) implies \( F(t^*) < \frac{1}{2} \) and \( \frac{\partial F}{\partial \alpha} < 0 \), and thus \( (1 - F(t)) (n - 1) - F(t) > 0 \) since \( n \geq 2 \).

By Proposition 2, to establish existence of \( t^* > 0 \) it suffices to show that \( G(t) > t \) is
possible. For $t \in (0, \frac{1}{2})$, $\lim_{\alpha \to 0} E[\theta|\theta > t] = 1$, $\lim_{\alpha \to 0} F(t) = \frac{1}{2}$, and thus using line 7 gives

$$\lim_{\alpha \to 0} G(t) = \frac{\delta (\frac{1}{2})^{n-1} (\frac{1}{2})}{1 - \delta (\frac{1}{2})^n} \times 1 = \frac{\delta}{\frac{1}{2^n} - \delta} > 0.$$  

Thus for any $n$ there is a $t$ and an $\alpha_n$ depending on $n$ such that $G(t) > t$, and furthermore since $\frac{\partial t}{\partial \alpha} < 0$ this remains true for $\alpha < \alpha_n$. ■

Notice that when $\alpha = 1$ the distribution is uniform, and letting $n = 2$ we find $G(t) = \frac{t}{1 + t} (\frac{1 + t}{2}) = \frac{t}{2} < t$ except for the trivial solution $t = 0$. Given Propositions 5 and 6 we can conclude that for any $n > 1$ and $\delta$ no equilibrium exists when $\alpha \geq 1$.

Example 3 Suppose $\theta_i \sim B[\frac{1}{10}, \frac{1}{10}]$ and the DM’s outside option is $r = 0$. Table 2 shows the equilibrium thresholds for 1, 2, and 3 experts for two different discount factors when $k = 1$ and 2. Consistent with the comparative statics propositions above, increased competition results in a lower threshold while a greater importance assigned to the future increases the threshold.

Table II

<table>
<thead>
<tr>
<th>Number of experts</th>
<th>$k = 1$</th>
<th>$k = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\delta = 0.399$</td>
<td>$\delta = 0.399$</td>
</tr>
<tr>
<td>1</td>
<td>0.238</td>
<td>0.238</td>
</tr>
<tr>
<td>2</td>
<td>0.084</td>
<td>0.238</td>
</tr>
<tr>
<td>3</td>
<td>0.025</td>
<td>0.120</td>
</tr>
</tbody>
</table>

5 Asymmetric equilibria

Proposition 3 established that when all experts are influential an equilibrium must be symmetric. Here we complete the characterization of stationary equilibria by exploring those in which not every expert is influential. To this end, note that one can change any symmetric equilibrium with $n$ experts into an asymmetric equilibrium as follows. Specify $i < n$ experts
who will babble while the remaining $n - i$ experts play the equilibrium strategy that ensues in the symmetric case with $n - i$ experts. Given these strategies the DM will ignore the babbling experts and accept projects from the other experts as in the symmetric case. It is plain to see these strategies constitute mutual best responses.

This observation together with Proposition 3 then provides a full characterization of all stationary equilibria. In any stationary equilibrium the players either use a symmetric threshold strategy as described in Proposition 1, or some subset of them do while the others babble (use threshold 0). Thus all the equilibrium properties derived in the prior section still apply in this setting. In particular, Proposition 2 tells us when an asymmetric equilibrium in which $n - i$ experts use a positive threshold exists, Proposition 3 informs us there are no other such equilibria, Proposition 4 selects a Pareto optimal equilibrium in which $i$ players babble, while the comparative statics established in Propositions 5, 6, and 7 also carry over.

The number of such equilibria can be large. There are \( \binom{n}{i} \) asymmetric equilibria in which the same $i$ experts babble each period, making the total number of these equilibria $\sum_{j=1}^{n} \binom{n}{j} = 2^n - 1$. This naturally leads to the question of equilibrium selection. One idea is to look for an equilibrium that results in the highest payoff for the DM.

**Proposition 9** Among all equilibria, the DM’s expected payoff is highest in a stationary equilibrium in which only one expert is influential.

**Proof** The result follows immediately when the DM’s outside option is less than \( \hat{t} \) (which solves line 1) since in this case the first-best outcome is achievable (see Lemma 1 and Remark 3). Otherwise, we note that the sole influential expert’s continuation value is greater in the stationary equilibrium than in any non-stationary one and thus his threshold is higher, which leads to a higher payoff for the DM by Lemma 1.

We now briefly discuss asymmetric equilibria that are non-stationary. For example, conjecture an equilibrium in which expert 1 is ignored only if the period is odd while expert 2 is ignored only if the period is even, and all other experts are always ignored. Given this, it is a best response for each expert to babble during the periods he is ignored and use an
optimal threshold strategy otherwise. This equilibrium closely resembles an outcome in Li’s (2015) model of sequential cheap talk in which the DM receives alternating offers from two experts over time, both of whom have an additive Crawford and Sobel (1982) style bias. There the DM commits to a Markov decision rule that probabilistically determines which single expert will be consulted in each period, and this process continues until one project is adopted. Li finds that the best such decision rule for the DM involves no alternation between experts at all but instead consulting the same expert each period. In Proposition 9 we find the same is true when experts compete across time and within each period.

We conclude this section by showing any equilibrium in which an expert babbles and is ignored fails to survive a refinement that permutes the indices of the players. Let $e_{i,j}$ denote any asymmetric equilibrium in which expert $i$ babbles but $j$ does not, and suppose there is an $\epsilon > 0$ probability that the DM mistakes expert $i$ for expert $j$. Then expert $i$ will defect from his babbling strategy in $e_{i,j}$ since there is a chance he will be influential, and the equilibrium $e_{i,j}$ fails. Thus every asymmetric equilibrium fails the refinement while symmetric equilibria survive since in these all experts use the same threshold.

6 Conclusion

When a decision maker consults multiple experts about which projects to adopt, the information revealed by each expert will depend on both the amount of competition between experts and whether new projects arrive over time. In this paper we develop a model of competitive cheap talk in which $n$ experts receive i.i.d. projects each period and vie to have their own project adopted by the decision maker through simultaneously reporting their type by cheap talk. Each expert is only informed of the quality of his own current project and only $k < n$ projects will ultimately be adopted due to resource constraints.

When experts compete we show that any equilibrium is characterized by a threshold above which states are pooled and recommended for adoption and below which states can be separated and will be rejected. Since the DM has the option to reject all experts’ projects
and proceed to the next period, recommendations can be viewed as comparative cheap talk about today’s project versus what is expected in the future. Our main finding is that in any symmetric equilibrium each expert’s threshold is decreasing in the amount of competition and increasing in the value of future periods. The logic behind each result is the same: an expert is willing to divulge that his type is low and thus be rejected in the current period if the value from continuing in the game is high enough. More experts imply a higher probability the game will terminate this period and thus a lower continuation value, while a higher discount factor has the opposite effect.

Multiple symmetric equilibria may exist though we show the equilibrium with the highest threshold is Pareto dominant. This result also implies that a lower threshold arising from more competition between experts results in a lower payoff for the decision maker. That is, competition harms the decision maker. Asymmetric equilibria in which some experts babble and are ignored lead to a higher payoff for the decision maker but do not survive a refinement in which each expert thinks there is some small chance he will not be ignored.

Appendix A: the optimal stopping problem

When there is only one expert and \( r = 0 \) the game reduces to an optimal stopping problem. Here we prove Lemma 1 by establishing there is a unique solution in the form of a threshold \( \hat{t} \) that increases in \( \delta \), and that the DM’s payoff is single-peaked at \( \hat{t} \).

**Proof of Lemma 1** Letting \( n = 1 \) in Lemma 3, the DM’s expected payoff using threshold \( t \) is

\[
\frac{(1-F(t))E[\theta]_{\theta > t}}{1-\delta F(t)} = \frac{\int_t^{\infty} \theta \, dF(\theta)}{1-\delta F(t)}.
\]

Differentiating with respect to \( t \) gives

\[
\frac{-t \, f(t)}{1-\delta F(t)} + \frac{\delta \, f(t) \int_t^{\infty} \theta \, dF(\theta)}{(1-\delta F(t))^2}.
\]

Setting line 8 equal to 0 and rearranging terms yields

\[
\frac{\int_t^{\infty} \theta \, dF(\theta)}{1-\delta F(t)} = \frac{t}{\delta}.
\]
Differentiating line 8 with respect to $t$ gives

\[
\frac{- (f(t) + t f'(t))}{1 - \delta F(t)} - \frac{-t f(t) (-\delta f(t))}{(1 - \delta F(t))^2}
\]

\[+ \frac{-t f(t) \delta f(t) + \delta f'(t) \int_t^\infty \theta \, dF(\theta)}{(1 - \delta F(t))^2} - \frac{2 \int_t^\infty \theta \, dF(\theta) \, \delta f(t) (-\delta f(t))}{(1 - \delta F(t))^3}. \tag{10}
\]

Substituting the right hand side of line 9 into the terms in line 10 gives the value of the second derivative when the first order condition is satisfied:

\[
\frac{- (f(t) + t f'(t))}{1 - \delta F(t)} - \frac{t f(t)^2 \delta}{(1 - \delta F(t))^2} - \frac{t f(t)^2 \delta}{(1 - \delta F(t))^2} + \frac{2 t f(t)^2 \delta}{(1 - \delta F(t))^2}
\]

\[= \frac{-f(t)}{1 - \delta F(t)} < 0.
\]

Thus if a critical point exists it is a maximum. Furthermore, since all critical points are maxima there is a unique maximum, and so the function is single-peaked at the maximizer.

To prove existence of the maximizer, we use line 9 and observe \( \frac{\delta \int_t^\infty \theta \, dF(\theta)}{1 - \delta F(t)} \) indeed has a fixed point: the function is continuous, is positive for $t = 0$, and approaches 0 as $t \to \infty$. Finally, by inspection the right hand side of line 1 increases in $\delta$ and thus by the implicit function theorem \( \frac{\partial t}{\partial \delta} > 0 \).

Appendix B: model with $k$ projects selected from among $n$ experts

Let $A_{j,n-1}(t)$ be the probability an expert’s recommended project is selected when all $n - 1$ other experts use threshold $t$ and there are $j$ projects left to be selected by the DM. We previously used $A_{1,n-1}(t)$ while in general

\[
A_{j,n-1}(t) \equiv \sum_{i=0}^{j-1} \binom{n - 1}{i} F(t)^{n-1-i} (1 - F(t))^i + \sum_{i=j}^{n-1} \binom{n - 1}{i} F(t)^{n-1-i} (1 - F(t))^i \frac{j}{1+i}.
\]
Let $\Pi_{j,n-1}$ be an expert’s ex-ante equilibrium payoff in a game with $j$ projects to be selected and $n-1$ other experts. We have already calculated $\Pi_{1,n-1}$ for arbitrary $n$, and $\Pi_{0,n-1} = 0$, and we define $\Pi_{-s,n-1} = 0$ for any $s > 0$. Let $M_{j,n-1}(t) \equiv \binom{n-1}{j} F(t)^{n-1-j} (1 - F(t))^j$ be the probability that exactly $j$ of the $n-1$ other experts recommend acceptance given threshold $t$. Note the invariance with $k$. Finally, let $t_k$ be the highest equilibrium threshold given $k$ projects will ultimately be selected.

**Proof of Proposition 7** The equilibrium characterization of Proposition 1 still applies in that there is a threshold below which separating rejection messages can be sent and above which there is a pooled recommendation for acceptance. We continue to look for an equilibrium in which all players are influential and thus each expert will use the same threshold by the same arguments made in Proposition 3. However, as will be shown below, the threshold used will depend on the number of projects left to be adopted.

We proceed iteratively by showing $t_1$ implies the existence of $t_2$, $t_2$ implies the existence of $t_3$, and so on. When $k = 2$ the payoff to the threshold type from recommending acceptance is $tA_{2,n-1}(t)$ while the payoff from recommending rejection is non-zero when exactly one other expert recommends acceptance or none do:

$$= \delta M_{1,n-1} \times \Pi_{1,n-2} + \delta F(t)^{n-1} \left( A_{2,n-1}(t) B(t) + F(t) \left( M_{1,n-1} \times \Pi_{1,n-2} + \delta F(t)^{n-1} \right) \right)$$

$$= \sum_{i=0}^{\infty} \delta \left( M_{1,n-1}(t) \Pi_{1,n-2} + A_{2,n-1}(t) B(t) F(t)^{n-1} \right) (\delta F(t)^n)^i$$

$$= \frac{\delta M_{1,n-1}(t) \Pi_{1,n-2}}{1 - \delta F(t)^n} + \frac{\delta A_{2,n-1}(t) B(t) F(t)^{n-1}}{1 - \delta F(t)^n}$$

The ellipses in the first line indicate the expression in the large parentheses is repeated. The second line uses summation notation for this infinite sum while the third reduces it and rearranges terms. Equating the payoffs from inducing acceptance and rejection and dividing
each by $A_{2,n-1}$ gives

$$t = \frac{\delta M_{1,n-1}(t)}{A_{2,n-1}(t)} \frac{\Pi_{1,n-2}}{(1 - \delta F(t)^n)} + \frac{\delta B(t) F(t)^{n-1}}{1 - \delta F(t)^n},$$

$$t = \frac{\delta M_{1,n-1}(t)}{1 - \delta F(t)^n} \frac{\Pi_{1,n-2}}{A_{2,n-1}(t)} + G(t). \quad (11)$$

We claim the existence of $t_1 > 0$ that solves the $k = 1$ case (i.e., $G(t_1) = t_1$) implies the existence of $t_2 > t_1$ that solves line 11. First, $\frac{\delta M_{1,n-1}(t) \Pi_{1,n-2}}{A_{2,n-1}(t)} > 0$ for all $t > 0$ and so $t_1$ is not a solution of line 11, and by inspection this term is continuous in $t$. We now show that this term converges to 0. Recalling $\Pi_{1,n-2}$ is a constant, $\delta < 1$ implies

$$\lim_{t \to \infty} \frac{\delta M_{1,n-1}(t) \Pi_{1,n-2}}{1 - \delta F(t)^n} = \lim_{t \to \infty} \frac{\delta (n-1) F(t)^{n-2} (1 - F(t)) \Pi_{1,n-2}}{1 - \delta F(t)^n} \frac{1}{A_{2,n-1}(t)} \quad (12)$$

$$= \frac{0}{1 - \delta \Pi_{1,n-2}} = 0$$

since $F(t)^i \to 1$ for all $i$, $1 - F(t) \to 0$, and $A_{2,n-1}(t) \to 1$ and $(n-1)$ is a constant. By Lemma 2(i) $\lim_{t \to \infty} G(t) = 0$ and thus the left hand side of line 11 must exceed the right hand side for sufficiently high $t$. Thus there exists a fixed point $t_2 > t_1$.

Having solved the $k = 1$ and $k = 2$ cases we now proceed by induction. By the induction hypothesis suppose there is a $t_i$ which is a fixed point of

$$G(t) + \sum_{j=1}^{i-1} \frac{\delta M_{i-j,n-1}(t) \Pi_{j,n-1-i+j}}{1 - \delta F(t)^n} \frac{1}{A_{i,n-1}(t)}. \quad (13)$$

We now solve the $k = i + 1$ case by showing there exists a fixed point of

$$\frac{\delta M_{i,n-1}(t) \Pi_{1,n-1-i}}{1 - \delta F(t)^n} \frac{1}{A_{i+1,n-1}(t)} + G(t) + \sum_{j=1}^{i-1} \frac{\delta M_{i-j,n-1}(t) \Pi_{j+1,n-1-i+j}}{1 - \delta F(t)^n} \frac{1}{A_{i+1,n-1}(t)}. \quad (14)$$
We explicitly write out the \( j = 0 \) term of the summation in line 14 to emphasize that it has no corresponding term in line 13. We proceed by arguing that

**Claim 1** Line 14 is greater than line 13 since the \( j = 0 \) term of line 14 is positive for \( t > 0 \) and each term in the sum in line 14 exceeds its counterpart in line 13; i.e.,

\[
\frac{\delta M_{i-j,n-1}(t)}{1-\delta F(t)} \frac{\Pi_{j+1,n-1-i+j}}{A_{i,n-1}(t)} \geq \frac{\delta M_{i-j,n-1}(t)}{1-\delta F(t)} \frac{\Pi_{j+1,n-1-i+j}}{A_{i,n-1}(t)} \quad \text{for all } j \in \{1, 2, ..., i - 1\}.
\]

**Claim 2** The terms \( j \in \{0, 1, ..., i - 1\} \) in the sum in line 14 approach 0 as \( t \to \infty \).

These claims together with the continuity of the expression in line 14 and Lemma 2(i) then imply there exists a \( t_{i+1} > t_i \) that solves line 14, and we are able to calculate the value of \( \Pi_{i+1,n} \) for all \( n \).

**Proof of Claim 1** By inspection the \( j = 0 \) term of line 14 is positive for \( t > 0 \). Next, we wish to establish

\[
\frac{A_{i+1,n-1}(t)}{A_{i,n-1}(t)} < \frac{j+1}{j} < \frac{\Pi_{j+1,n-1-i+j}}{\Pi_{j,n-1-i+j}}.
\]

To establish the first inequality in line 15 it suffices to show the \( j = i - 1 \) case:

\[
\frac{A_{i+1,n-1}(t)}{A_{i,n-1}(t)} < \frac{i}{i-1} \iff A_{i+1,n-1}(t) < \frac{i}{i-1} A_{i,n-1}(t) \iff
\]

\[
\sum_{l=0}^{i} \binom{n-1}{l} F(t)^{n-1-l}(1-F(t))^l + \sum_{l=i+1}^{n-1} \binom{n-1}{l} F(t)^{n-1-l}(1-F(t))^l \left(\frac{i+1}{l+1}\right)
\]

\[
< \sum_{l=0}^{i-1} \binom{n-1}{l} \frac{i}{i-1} F(t)^{n-1-l}(1-F(t))^l + \sum_{l=i}^{n-1} \binom{n-1}{l} F(t)^{n-1-l}(1-F(t))^l \left(\frac{i}{l+1}\right),
\]

which follows since \( i \left(\frac{i}{i-1}\right) \) is greater than \( i+1 \).
We now establish the second inequality in line 15. For any symmetric threshold each expert has the same ex-ante probability \( \frac{k}{n} \) of having his project selected, and thus comparing any game with \( j + 1 \) projects to be selected to any game with \( j \) projects to be selected, each expert is \( \frac{i+1}{n} / \frac{i}{j} = \frac{i+1}{j} \) times more likely to have his project selected in the former than the latter. Additionally, the ex-ante expected value of a project that is accepted is greater in the former case than the latter since the adoption threshold is higher, and thus the inequality \( \frac{i+1}{j} < \frac{\Pi_{j+1,n-1-i+i}}{\Pi_{j,n-1-i+j}} \) holds for all \( n \). ■

**Proof of Claim 2** Substituting for \( M_{i-j,n-1} \), a generic term of the sum in line 14 is

\[
\frac{\left( \delta (n-1) F(t)^{n-1-i+j} (1 - F(t))^{i-j} \right) \Pi_{j+1,n-1-i+j}}{1 - \delta F(t)^{n}} \frac{\Pi_{j+1,n-1-i+j}}{A_{i+1,n-1}(t)}
\]

and the term in large parentheses converges to \( \frac{0}{1-\delta} \) and thus the entire expression converges to 0. Also, note that the first term outside the summation in line 14 would be the \( j = 0 \) term of the summation and so it too converges to 0. ■

Finally, we prove that higher \( k \) constitutes a Pareto improvement. By Lemma 3 it suffices to track the DM’s payoff. First, the direct effect is clearly positive as the DM enjoys payoffs from more projects. The indirect effect is also positive since \( t^* \) increases in \( k \) and payoffs increase in \( t^* \). The former statement was just been proved while the latter follows from parts (ii) and (iii) in the proof of Proposition 4. ■
References


