Multi-period competitive cheap talk with highly biased experts*

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Abstract

Each of n experts communicates with a principal about the privately observed quality of the expert's own project via cheap talk, with new independently drawn projects available each period until the principal adopts one. Even when experts are highly biased in that they only receive a positive payoff if their own project is selected, we show that informative equilibria may exist, characterize a large class of stationary equilibria, and find the Pareto dominant symmetric equilibrium. Experts face a tradeoff between inducing acceptance now versus waiting for a better project should the game continue. When the future is more highly valued experts send more informative messages, increasing the average quality of an adopted project and resulting in a Pareto improvement, while communication is harmed and payoffs can decline when there is more competition between experts.

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1 Introduction

A decision maker often consults many experts over time before taking an action. For example, consider the division managers of a company who report to the CEO about the profitability of projects available to them. The CEO has enough resources to fund some, but not all, projects and cannot directly observe their quality, while each division's manager is privately informed about his own best project. The CEO seeks to select only the best projects, whereas each manager is only concerned with his own division's profits and so statically wants his project adopted even if it has low profitability. However, better projects may arrive over time, which will influence the desirability of adopting projects currently available.

In this paper we ask whether the decision maker (the CEO) in such a setting can benefit from the unverifiable reports of highly biased experts (each division manager) when making an adoption decision, where the projects are independent across experts and time. The defining characteristics of this motivating example are as follows. Each of n experts simultaneously report their project's type using cheap talk to a decision maker (DM). The DM then either adopts one of the projects, terminating the game, or chooses against adopting any of them, in which case the players proceed to the next period where past projects are lost but new independent draws are available. In each period an expert observes only his own type, not that of the other experts, and receives a payoff equal to his project's type if it is adopted, but obtains no benefit when a competing expert's project is adopted. The DM's payoff equals the adopted project's type. Thus each expert competes with the others for the adoption of his project over an indefinite time horizon.

Examples of experts competing in this manner can be found in many settings. Consider lobbyists who seek to convince the chairman of a government budget committee to spend on their own favored programs. One lobbyist proposes an educational intervention while the other an environmental one, and each is informed of his own policy's effectiveness but not that of the other. The chairman only has resources sufficient to fund one policy but may also adopt neither, deferring the decision to next year when additional proposals will be available. Another example is given in Li, Rantakari and Yang (2016) in which an economics department has one open position to be filled by either a micro- or macroeconomist. The search committees can determine the quality of the candidate in their own field but not the other, and each prefers a hire in its own field. The department chair is a labor economist who prefers to hire the best candidate irrespective of the field but cannot observe either candidate's quality. In our setting the chair may also refrain from hiring anyone now and wait for next year's applicant pool.

This paper explores how the existence of future periods and competition between experts affects communication in the current period, and we show the two are intimately related. For example, it can easily be seen that when experts vie for their project to be adopted in a one-period model only a babbling equilibrium will exist: each expert wants his project adopted regardless of the state and has only this period to convince the DM to do so. One way to avoid this outcome is to change the experts' utility functions and in fact Li, Rantakari and Yang (2016) show that in a static model with two experts informative equilibria exist if the experts have a low enough Crawford and Sobel (1982) style additive or multiplicative bias. When the stage game is repeated with new projects realized each period it is no longer clear that babbling must ensue in our model since each expert has a continuation value and so might not attempt to induce acceptance of low types. However, future periods are valued only if they are reached and so more competing experts tend to make informative communication harder to support.

In order to disentangle the effects of future periods from competition between experts on the current period's outcomes, we first consider a game between a *single* expert and a DM where incentives are aligned except for an outside option that provides a benefit to the DM but not the expert. This setting closely resembles Che, Dessein and Kartik's (2013) static model in which the expert observes the value of finitely many projects and recommends one by use of comparative cheap talk. Our expert's recommendation can similarly be viewed as a comparison between the value of the single project currently available with the value of projects that might be realized in the future, the crucial difference being that in our model the realization of future projects is not yet known to the expert. We show that even when an informative equilibrium does not exist in a one-period model, the addition of future periods can allow for meaningful first-period communication because both the expert and DM benefit from rejecting states below a threshold value since better outcomes are likely next period. When the future is discounted less, the continuation value from the game increases, expanding the parameter values over which informative communication can occur.

The intuition that having future periods improves communication in that only higher quality projects are recommended remains when there are two or more experts but now an additional factor is at play. Each expert is concerned that if he divulges information leading to rejection of his own project a competitor's project may be selected now and the game will terminate. For this reason competing experts put more weight on getting a project approved now than waiting for a better choice, and thus they recommend adoption more often. In turn, the DM infers a lower average quality for recommended projects and so rejects for a larger range of his outside option, making an informative equilibrium harder to sustain. Nonetheless, the basic structure of the equilibrium remains unchanged under competition: a threshold exists below which an expert prefers to induce rejection because continuation of the game has greater value. Loosely speaking, an equilibrium exists when projects have a high chance of low outcomes so that an expert does not fear being preempted by competitors, and yet a high expected value so that arriving at the next period is enticing enough. For states above the threshold, each expert prefers immediate acceptance and therefore wishes to induce as high a posterior as possible in order to be selected over competing experts. This implies credible distinctions between states above the threshold cannot be made and thus these states must be pooled.

Having shown that equilibria will entail the use of a threshold we establish that any equilibrium in which each expert's message is not ignored is symmetric. However, this symmetric threshold is too low in that there is a higher threshold that would constitute a Pareto improvement. We use this result to select a symmetric equilibrium and interpret comparative statics on the intensity of competition and value of future periods. We show that as the future is discounted less each expert's equilibrium threshold increases, which allows for informative communication for a larger range of parameter values and implies a conditionally higher project quality, improving payoffs for all. Increasing the intensity of competition has the opposite effect of lowering the equilibrium threshold and thus harming communication, which tends to lower the DM's payoff by causing a lower quality project to be adopted. However, consulting an additional expert has the offsetting effect of more quickly generating a successful project since it is more likely that at least one expert's project exceeds the threshold. We establish when the former communication effect dominates the latter time effect, in which case the DM prefers to consult a single expert, and identify asymmetric equilibria in which this can occur.

That consulting a single expert may be best for the DM contrasts with much of the multiple-sender literature (e.g., Battaglini, 2002) and owes to the experts' knowledge of just one dimension of the state. One other such exception is found in Li (2016), where a tradeoff between time and project quality is also present. In Li's model two experts compete over time to have their own project implemented, though unlike in the present paper they do internalize some benefit if the other's project is selected. Only one expert receives a project and makes a recommendation in each period, and the DM commits to consulting the same expert he consulted last period with probability p. In addition, an expert only receives a draw from the distribution with some probability, which is increasing in the expert's search effort. Li finds that when search is free only one expert should be consulted, but otherwise some competition ($p < 1$) between the experts over time may be best by inducing greater search effort and therefore a lower waiting time for a success. Our results are complementary to Li's in that we find the same basic tradeoff in a setting with experts competing both across time and within each period, and where the DM lacks commitment power.

This paper builds on a growing project selection literature. Bonatti and Rantakari (2016) allow agents to exert costly effort that affects project completion time when each expert can veto the adoption of the other's project. Rantakari (2016) explores the consequences of allowing the principal himself to exert effort in order to probabilistically obtain a better project while Rantakari (2013) allows the principal to ex-ante publicly commit to a decision mechanism when experts have an unknown bias. In Moldovanu and Shi (2013) a committee of experts receives new projects each period until one is adopted by unanimous consent. More generally this paper relates to the literature on competition between experts; see for example Gilligan and Krehbiel (1989), Krishna and Morgan (2001a; 2001b) as well as Battaglini (2002). In each of these models, however, both experts observe the same state of the world, whereas in the present paper each expert has private information about just one dimension of the state.

In Quint and Hendricks (2013) bidders in a two-stage auction first report their valuations by cheap talk and then enter the second stage to submit binding bids only if their first stage report was among the highest two. In this way the bidders are competing in the first period. However, in this model the seller commits to a message space and decision rule by which the first stage winners are determined and together with other assumptions it is shown this leads to partially aligned incentives for each bidder in the cheap talk stage. Our analysis differs in that we do not assume the receiver can commit to a mechanism and our senders do not have Crawford and Sobel (1982) style preferences.

The present paper is also related to preemption games in which competitors decide when to end the game, where there is a first-mover advantage in doing so. Applications have included the decision of when to patent an invention or introduce a new product (Bobtcheff and Mariotti, 2012; Hopenhayn and Squintani, 2011; Hopenhayn and Squintani, 2015), where waiting can help a firm by allowing it time to improve its product but risks being preempted by a competitor. These models differ from the present paper in that they lack a decision-making principal and termination of the game by an informed player is an assumed feature of the model rather than an endogenous outcome.

Other models allow for influential communication with just a single agent through other means such as reputation. For example, Kim (1996) explores how reputation can affect cheap talk over an infinite horizon though unlike his paper we do not require infinitely many periods nor do we have ex-post verifiability. Along similar lines Sobel (1985) explores reputation in a cheap talk model in which the state is fixed across periods. Finally, in a static model Chakraborty and Harbaugh (2007) demonstrate that a single expert who observes all dimensions of the state space can make credible comparative statements even when it would not be credible on a single dimension.

In the next section, we present the model and then demonstrate that future periods improve communication and payoffs when there is only one expert. Next we consider more than one expert competing in each period and across time and establish that all symmetric equilibria will involve a threshold strategy, provide conditions guaranteeing existence of symmetric equilibria in a general setting, identify the Pareto dominant such equilibrium, perform comparative statics, and discuss asymmetric equilibria. We then conclude.

2 Model

Let $\theta_i \sim F$ be the independent and identically distributed profits generated from the project available to expert i, for $i = 1, 2, ..., n$ and $n \geq 1$, where F is a C^1 function, has finite expectation, and full support on a closed connected set $A \subseteq \mathbb{R}_+$, with $\min A = 0$. Assume that F is common knowledge while the realization of θ_i is private information to expert i. In contrast to other multiple-sender models, here expert i does not observe the outcome of θ_j, $j \neq i$. The experts simultaneously send cheap talk messages to the decision maker (DM) who can choose at most one project to accept. Multiple project cannot be adopted, perhaps due to resource constraints. We denote the DM's choice by $d \in \{0, 1, 2, ..., n+1\}$, where $d = 0$ means all projects are rejected and $d = n + 1$ is an outside option always available to the DM, with commonly known value $\theta_{n+1} = r$.

Payoffs in the stage game are as follows. For the project accepted the DM receives payoff $U_{DM}(d) = \theta_d$. Each expert is highly biased in that he only receives positive utility if his own project is adopted:

$$U_i(d) = \begin{cases} 0 & \text{if } d \neq i \\ \theta_i & \text{if } d = i \end{cases}.$$

If the DM rejects all projects, then the game enters the next period with new independent draws for each θ_i such that $i \neq n+1$, and continues indefinitely until a project is adopted. Projects are short-lived in that any project that has been rejected is lost and cannot be brought back. Finally, future payoffs are discounted by $\delta \in (0,1)$ and all players are expected utility maximizers.

Let each expert's message space be denoted by M, where $M = \mathbb{R}_+$. A strategy for expert i is then a sequence of functions $g_{i,s}$ that for each period s maps from the history of the game into ΔM, the set of all probability distributions over M. Formally, at period s the history of the game for expert i is $(\theta_{i,1}, ..., \theta_{i,s}) \times (m_{i,1}, ..., m_{i,s-1})$, where $m_{i,k}$ is the message expert i sent in period k. A strategy for the DM is a sequence of functions h_s that map from $\prod_{j=1}^{n}(m_{j,1}, ..., m_{j,s})$ into probability distributions over actions, $\Delta\{0, 1, 2, ..., n+1\}$. We look for a perfect Bayesian equilibrium that, unless otherwise stated, is stationary and non-babbling.

3 One expert

Initially we focus on the case of one expert and demonstrate that the existence of future periods can improve communication and payoffs. Since the DM takes a binary action of adopting the current project or not we will refer to any message that induces acceptance or rejection as a recommendation to accept or reject, respectively. We begin by supposing the game has just one period. But in this case the expert prefers all projects be adopted regardless of the state and so any message he sends will be uninformative. Accordingly, the DM accepts when the prior mean exceeds r and otherwise rejects. Thus,

Remark 1 *With only one expert and one period, persuasive communication cannot occur.*

Now suppose two periods exist and assume discount factor δ. Using backwards induction, in the second period if $0 \leq r \leq E[\theta]$ the DM will accept any project, the expected present value of which is $\delta E[\theta]$. This continuation value is enjoyed by the DM and the expert since both can benefit from a better second period project. Then in the first period the expert prefers adoption of only those projects exceeding $\delta E[\theta]$ while the DM wishes to only accept projects above $\max\{\delta E[\theta], r\}$. If $r \leq \delta E[\theta]$ incentives are fully aligned in the first period. If instead $r > \delta E[\theta]$, the outside option is always preferable to continuing for the DM and so the second period will not be reached. But then the expert attempts to induce acceptance of all first period projects and persuasive communication breaks down. We record these observations below.

Remark 2 *With one expert and two periods, persuasive communication can occur in the first period only if $r \leq \delta E[\theta]$. In this case, in any equilibrium the expert recommends acceptance when $\theta \geq \delta E[\theta]$ and rejection otherwise, while the DM follows all recommendations.*

When $r \leq \delta E[\theta]$ the existence of a second period improves outcomes. By having a future to look forward to, the expert is willing to recommend rejection of low quality projects in the first period since it is likely a better one can be obtained next period. This leads to higher expected payoffs, since $\int_0^\infty \max\{\delta E[\theta], \theta\}\, dF(\theta) > \int_0^\infty \theta\, dF(\theta)$. This last fact also shows that in a three-period game the continuation value from the two period subgame exceeds $\delta E[\theta]$, and thus so too would the threshold used in the first period. Extending this logic, it can be seen that the first-period threshold always increases in the total number of periods.

Finally we demonstrate that allowing the game to repeat indefinitely can further improve communication and payoffs. Initially consider $r = 0$ so that there is no divergence in incentives. There is no last period so we conjecture projects are adopted only if they exceed a threshold t. This is a familiar optimal stopping problem and the threshold must satisfy

$$\begin{aligned} t &= \delta \int_t^\infty \theta\, dF(\theta) \sum_{i=0}^\infty (\delta F(t))^i \quad \Longleftrightarrow \\ t &= \frac{\delta}{1-\delta F(t)} \int_t^\infty \theta\, dF(\theta). \end{aligned} \tag{1}$$

Lemma 1 *Equation (1) has a unique solution $\hat{t}$ which increases in δ, and the DM's payoff is single-peaked at $\hat{t}$.*

Proof See the appendix. ■

To show the adoption threshold is more stringent with infinitely many periods than two ($\hat{t} > \delta E[\theta]$) is straightforward. Now suppose $r > 0$, and we note the logic proceeds much the same as in the two period case. The expert prefers adoption only if the project exceeds $\hat{t}$ while the DM's preferred threshold is $\max\{\hat{t}, r\}$, and thus when $r \leq \hat{t}$ incentives are fully aligned. If instead $r > \hat{t}$, future periods have no effect since the DM prefers the outside option to continuing, the expert therefore attempts to induce acceptance of all projects, and persuasive communication breaks down. We summarize these observations below.

Remark 3 *With one expert and infinitely many periods, persuasive communication can occur only if $r \leq \hat{t}$ from Lemma 1. In this case, in any equilibrium in each period the expert recommends acceptance when $\theta > \hat{t}$ and rejection otherwise, while the DM follows all recommendations.*

Infinite repetition thus supports even higher payoffs than a single repetition. First, the expert will use a more conservative recommendation threshold because of the higher continuation value, which increases the expected payoff since $\int_0^\infty \max\{\hat{t}, \theta\}\, dF(\theta) > \int_0^\infty \max\{\delta E[\theta], \theta\}\, dF(\theta)$. Second, because of this increase in the threshold, persuasive communication can be supported for a larger range of outside options, a general property that is demonstrated in the example below.

Example 1 *Let $\theta \sim U[0,1]$ be i.i.d. across periods and $\delta = 0.8$. Table I below records payoffs and the values of the DM's outside option that support persuasive communication in a game with one, two, or infinitely many periods. The expected payoff shown for a one period game results from the babbling equilibrium while all other payoffs are for equilibria in which persuasion occurs.*

Table I: First period outcomes in Example 1

Number of periods	Persuasive communication is supported	Expected payoffs
1	$\emptyset$	0.5
2	$r \leq .4$	0.58
∞	$r \leq 0.5$	0.625

We conclude this section by commenting on the shared features of our one sender model with the comparative cheap talk literature (e.g., Chakraborty and Harbaugh, 2007 and 2010; Che, Dessein, and Kartik, 2013). To see this, suppose the game has reached period s and the expert must decide whether to recommend the adoption of the project whose value is θ_s.[1] A recommendation to adopt indicates the current project is better than what will likely arise in the future, while a recommendation to reject indicates the converse. That is, the expert makes a comparative cheap talk statement that ranks the two relevant dimensions of the state space: this period's project versus next period and beyond. However, in our model the expert does not have private information about *future* states, in contrast to static comparative cheap talk models in which the expert observes all dimensions of the state. It is for this reason, for example, that the pandering effect found in Che, Dessein, and Kartik (2013) is not present here.

In our model comparative statements can be informative due to an endogenous opportunity cost of lying. If the expert induces acceptance by falsely ranking the current project ahead of what can be expected in the future, then he himself suffers from the forgone benefit of continuing in the game. The continuation value depends on the discount factor δ and the number of periods the game might have. Lower δ increases the relative value of the present and so causes the expert to recommend adoption of the current project more often;

[1] Pursuant to the strategies in Remark 3, having reached period s implies the prior $s-1$ projects were below the threshold $\hat{t}$. Since the θ are i.i.d. any previously rejected project would never be returned to and so the decision reduces to recommending this period's project or not. This observation demonstrates that when restricting to stationary equilibria, the no recall assumption is without loss of generality.

that is, $\hat{t}$ decreases per Lemma 1. Thus when the present is more highly valued by the expert communication is supported for a smaller set of values of the DM's outside option. In the next section we explore another reason the present may be more highly valued, namely competition with other experts.

4 Competition among experts

Having seen that future periods help improve communication between a single expert and the DM, we now consider the effect of competition between experts to have their own favored projects adopted. Because expert i only observes θ_i and receives payoff 0 when project $j \neq i$ is adopted, if there is only one period all experts always recommend adoption of their project. However, when a second period exists an expert with a low realization may recommend rejection of his own project in the hopes that both next period will be reached and a better project will be realized and accepted then. We proceed by showing any equilibrium entails use of a threshold strategy and then consider equilibrium existence, selection, and comparative statics.

Proposition 1 (Equilibrium characterization) *In any symmetric equilibrium there is a threshold t^* such that projects $\theta \leq t^*$ are rejected while $\theta > t^*$ are recommended for acceptance. If one or more project is recommended the DM chooses one of these to adopt with equal probability.*

Proof We first prove a threshold is used. Given any strategy, the continuation value from inducing rejection is invariant to type while the expected payoff from inducing acceptance is increasing in type. Thus any message must be sent by a connected set of types. Next, project adoption can only be recommended for one set of pooled states. If not, then any type assigned to a message accepted with lower probability would defect to a message accepted with a higher probability.

Next, if a single expert prefers his project be rejected then the DM does *a fortiori* since the latter benefits when *any* expert's future project is above the threshold. Finally, given the experts employ a stationary threshold strategy it is a best response for the DM to accept a recommended project with probability $\frac{1}{j}$ when $j \geq 1$ projects are recommended, and reject when rejection is recommended by all experts. ■

Due to the continuation value of the game, an expert may prefer that his relatively low-quality project be rejected in the hope that he obtains a better project should next period be reached. This implies the DM would also prefer such a project be rejected since he is concerned with the more likely possibility that *any* expert receives a better project. In addition, waiting for a possibly better project next period in the single expert case only incurs the single expert a time delay cost while competing experts are also cognizant that the game may terminate this period, which we will show causes competing experts to use a lower threshold.

We now turn to equilibrium existence. In a symmetric equilibrium the threshold type must satisfy an indifference condition that equates the payoffs from recommending acceptance or rejection. By recommending acceptance an expert knows the game has ended: either his own project is selected, or a competing expert's project was recommended and accepted. Define $A(t) \equiv \sum_{i=0}^{n-1} \binom{n-1}{i} \frac{F(t)^{n-1-i}[1-F(t)]^i}{1+i}$, the probability an expert's project is chosen given he recommends it when all experts use threshold t. This probability depends on how likely no other experts recommend acceptance, exactly one does, exactly two do, and so on, where each project is accepted with equal probability if more than one is recommended.

An expert that recommends rejection continues in the game only when all other experts likewise have recommended rejection. In this case an expert may realize a type above the threshold next period and so win with probability $A(t)$, or else has the chance of a non-zero payoff if all other experts again have a type below the threshold. This argument repeats, and implies a continuation value from rejection which is equated to the expected payoff of recommending acceptance:

$$
\begin{aligned}
t\ A(t) &= \delta F(t)^{n-1}\left(A(t)\int_t^\infty \theta dF(\theta)+\delta F(t)^n\left(A(t)\int_t^\infty \theta dF(\theta)+\delta F(t)^n(...)\right)\right) \quad \Longleftrightarrow \\
t\ A(t) &= \sum_{\ell=0}^{\infty} A(t)\int_t^\infty \theta dF(\theta)\,\delta F(t)^{n-1}\left[\delta F(t)^n\right]^\ell \quad \Longleftrightarrow \\
t &= \frac{\delta F(t)^{n-1}}{1-\delta F(t)^n}\int_t^\infty \theta dF(\theta) \qquad (2)
\end{aligned}
$$

In the first line, the ellipses in the last parentheses repeats the expressions given in the big parentheses. The second line uses summation notation for this infinite sum, and the third line reduces the sum and cancels the common $A(t)$ term. Line 2 is a generalization of line 1 for $n > 1$ senders, and satisfying the sender constraints thus reduces to finding a fixed point of $G(t) \equiv \frac{\delta F(t)^{n-1}}{1-\delta F(t)^n}\int_t^\infty \theta\ dF(\theta)$. Given this, the following properties of $G(t)$ will be useful.

Lemma 2

(i). $G(t)$ is continuous, $G(0) = 0$, and $\lim_{t\to\infty} G(t) = 0$.

(ii). $G'(t)$ is continuous and $G'(0) = \begin{cases} \delta f(0)\,E[\theta] & \text{if } n = 2 \\ 0 & \text{if } n > 2 \end{cases}$.

(iii). Let $t_0 > 0$ be the largest fixed point of G. Then $G'(t_0) \leq 1$, and if θ is bounded above by $\overline{\theta}$ then $t_0 < \frac{\overline{\theta}}{n}$.

Proof See the appendix. ■

We are now ready to give necessary and sufficient conditions for the existence of a symmetric equilibrium. Let $\Pi(t)$ denote the DM's expected payoff in a symmetric equilibrium with threshold t. Finally, unless otherwise specified all references to fixed points of G will refer to non-zero fixed points.

Proposition 2 (Existence of equilibria) *A symmetric threshold t^* satisfies each expert's incentive compatibility constraint if and only if $G(t^*) = t^*$. At least one fixed point of G exists if $n = 2$ and $E[\theta] > 1/\delta f(0)$; otherwise, at least two fixed points exist if there is a t such that $G(t) > t$. The DM's constraints are satisfied if and only if $r \leq \delta\,\Pi(t^*)$.*

Proof From the discussion above, satisfying the sender constraints reduces to finding a fixed point of $G(t)$. By Lemma 2, $G(0) = 0$ and if $n > 2$ then $G'(0) = 0$. Since $\lim_{t\to\infty} G(t) = 0$, if there is a t such that $G(t) > t$ then there are at least two fixed points since G is continuous. In fact this argument only requires $G'(0) < 1$ and thus it applies when $n = 2$ and $E[\theta] < 1/\delta f(0)$, since by Lemma 2(ii) $G'(0) < 1$ in this case. If $n = 2$ and $E[\theta] > 1/\delta f(0)$ then $G'(0) > 1$ and thus $\lim_{t\to\infty} G(t) = 0$ implies at least one fixed point exists.

Regarding the DM's constraints, $r \leq E[\theta|\theta > t^*]$ is required to induce acceptance, and the DM will continue in the game only if $r \leq \delta\,\Pi(t^*)$. This latter condition implies the former since the DM's expected payoff is strictly less than $E[\theta|\theta > t^*]$, as an immediate success is not guaranteed (see Lemma 3 for an explicit calculation of $\Pi(t^*)$). ■

By inspection of $G(t) = \frac{\delta F(t)^{n-1}}{1-\delta F(t)^n} \int_t^\infty \theta\, dF(\theta)$ the condition that $G(t) > t$ occur is satisfied when, for example, $F(t)$ is large for small t but $E[\theta|\theta > t]$ is nonetheless high. Intuitively, the probability of low realizations must be high enough that an expert with a low outcome doesn't fear being preempted by a competing expert, but at the same time the expectation of a new project must be sufficiently high that the continuation value from proceeding to the next period is enticing enough.[2] When these conditions fail only a babbling equilibrium in which experts recommend all project types be adopted ($t = 0$) exists,[3] as depicted in Figure 1(a). This is in stark contrast to the single expert case, in which the existence of a non-trivial threshold was guaranteed for *any* distribution. In fact, if $\delta = 1$ a single expert would *never* recommend acceptance ($t = \infty$) while two competing experts may instead *always* do so ($t = 0$), or if $E[\theta] > 1/f(0)$ use a non-zero but finite threshold.

[2]The conditions on existence can also be interpreted as requiring the equilibrium threshold not be too close to 0. If the threshold were, an expert would never want to recommend rejection since the probability of continuing in the game would be too low relative to any expected gains that could be realized by delaying, and so recommending acceptance now with with some chance of being selected would be preferred.

[3]A babbling equilibrium always exists since $G(0) = 0$ by Lemma 2(i).

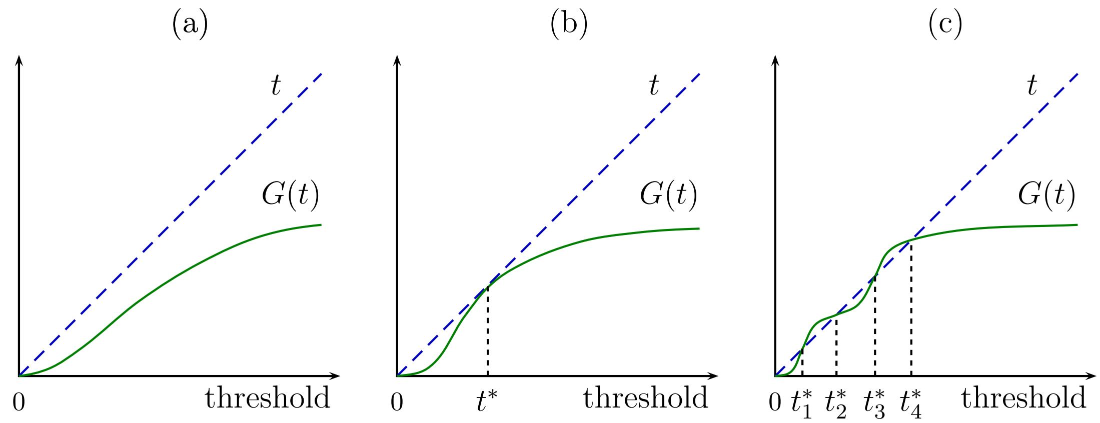

Figure 1: Each sender's incentive compatibility constraint is satisfied for fixed points of G. In panel (a) only a babbling equilibrium ($t = 0$) exists; in panel (b) there is a unique non-trivial equilibrium which is knife-edge; in panel (c) there are four non-trivial equilibrium thresholds.

Though informative communication is harder to sustain when experts compete, repetition of the stage game may help in a nuanced way. The DM does not directly observe the state and therefore does not benefit from a typical order statistic effect from, say, adding another period or increasing the discount factor in a decision problem. Instead the DM only infers the average quality of a recommended project given the experts' threshold, so that the communication strategy used in the future affects how valuable the future will be. For this reason n and δ have an indirect effect on payoffs and communication through the threshold. We explore such comparative statics further in a later subsection, but we first select an equilibrium.

Equilibrium selection

By Proposition 2 several non-trivial equilibria can exist, an example of which is depicted in Figure 1(c). In addition, since $G(0) = 0$ there is always a trivial equilibrium with $t = 0$ in which all projects are recommended regardless of their type. In Section 5 we show there are also equilibria in which some player(s) babble and are ignored by the DM while other players use non-zero thresholds. For the time being we restrict away from these cases, show that any equilibrium must therefore be symmetric, and select an equilibrium which is Pareto dominant.

Definition 1 *An expert is* ***influential*** *in an equilibrium if in each period there is a positive probability his recommended project is accepted, given the messages of the other experts.*

Proposition 3 *Any equilibrium in which each expert is influential is symmetric.*

Proof By the proof of Proposition 1 in any equilibrium an expert i uses a threshold strategy t_i. We claim each expert's best-response threshold is increasing in the other experts' thresholds and establish this by proving $\frac{\partial t_1}{\partial t_i} > 0$ for all $i \neq 1$. The indifference condition for threshold t_1, analogous to that in line 2, is

$$t_1 = \frac{\delta \ \Pi_{j=2}^{n} F\left(t_j\right)}{1 - \delta \ \Pi_{j=1}^{n} F\left(t_j\right)} \int_{t_1}^{\infty} \theta \, dF\left(\theta\right). \tag{3}$$

Let $t^* \equiv (t_1, ..., t_n)$ be a solution to line 3 and define $H\left(t_1, ...t_n\right) = \frac{\delta \ \Pi_{j=2}^{n} F(t_j)}{1-\delta \ \Pi_{j=1}^{n} F(t_j)} \int_{t_1}^{\infty} \theta \, dF\left(\theta\right) - t_1$. By the implicit function theorem $\frac{\partial t_1}{\partial t_i}\left(t^*\right) = -\frac{\frac{\partial H}{\partial t_i}(t^*)}{\frac{\partial H}{\partial t_1}(t^*)}$. We now show $\frac{\partial H}{\partial t_i}\left(t^*\right) > 0$ for $i \neq 1$.

$$\frac{\partial H}{\partial t_i}\left(t^*\right) = \delta \left(\frac{\left(1 - \delta \Pi_{j=1}^{n} F\left(t_j\right)\right) \ f\left(t_i\right) \ \Pi_{j=2,\neq i}^{n} F\left(t_j\right) + \left(\Pi_{j=2}^{n} F\left(t_j\right)\right) \ f\left(t_i\right) \ \delta \ \Pi_{j=2,\neq i}^{n} F\left(t_j\right)}{\left(1 - \delta \Pi_{j=1}^{n} F\left(t_j\right)\right)^2} \right) \int_{t_1}^{\infty} \theta \, dF\left(\theta\right)$$

and thus $\frac{\partial H}{\partial t_i}\left(t^*\right) > 0$ since both the numerator and denominator are positive. Finally, we show $\frac{\partial H}{\partial t_1}\left(t^*\right) < 0$ and thus conclude $\frac{\partial t_1}{\partial t_i}\left(t^*\right) > 0$.

$$\frac{\partial H}{\partial t_1}\left(t^*\right) = -t_1 f\left(t_1\right) \left(\frac{\delta \ \Pi_{j=2}^{n} F\left(t_j\right)}{1 - \delta \ \Pi_{j=1}^{n} F\left(t_j\right)} \right) + \left(\frac{-\left(\delta \ \Pi_{j=2}^{n} F\left(t_j\right)\right) \left(-f\left(t_1\right) \ \delta \ \Pi_{j=2}^{n} F\left(t_j\right)\right)}{\left(1 - \delta \ \Pi_{j=1}^{n} F\left(t_j\right)\right)^2} \right) \int_{t_1}^{\infty} \theta \, dF\left(\theta\right) - 1$$

and rearranging terms and substituting from line 3 gives $\frac{\partial H}{\partial t_1}\left(t^*\right) = -1 < 0$. Thus each expert's threshold is increasing each other expert's threshold.

Finally, toward a contradiction suppose there exists an asymmetric equilibrium in which each expert is influential. Then there exist experts i and j such that $t_i > t_j$. If these thresholds are part of an equilibrium then the symmetry of the players' type distributions implies there is also an otherwise identical equilibrium in which expert i uses threshold t_j and expert j uses threshold t_i, contradicting the fact that each best response threshold is increasing in other experts' thresholds. ■

Proposition 3 together with Proposition 1 go a long way towards characterizing the equilibrium set: any stationary equilibrium in which each expert is influential must be symmetric, and any symmetric equilibrium must conform to Proposition 1. We augment this characterization in Section 5 by discussing stationary equilibria in which some players are not influential. We now proceed by selecting a symmetric equilibrium and to this end we establish the following lemma.

Lemma 3 *The DM's expected payoff is* $\frac{1-F(t)^n}{1-\delta F(t)^n} E[\theta|\theta > t]$ *while each of the* n *experts' expected payoff is* $\frac{1}{n}$ *of this.*

Proof The DM's expected payoff with threshold t and discount factor δ is

$$\sum_{i=0}^{\infty} [\delta F(t)^n]^i (1 - F(t)^n) E[\theta|\theta > t] = \frac{1 - F(t)^n}{1 - \delta F(t)^n} E[\theta|\theta > t]$$

while each expert's expected payoff is

$$\sum_{i=0}^{\infty} [\delta F(t)^n]^i (1 - F(t)) A(t) E[\theta|\theta > t] = \frac{(1 - F(t)) A(t) E[\theta|\theta > t]}{1 - \delta F(t)^n}.$$

Eliminating common terms the result then follows since

$$\frac{1 - F(t)^n}{1 - F(t)} = \sum_{i=0}^{n-1} F(t)^i = n\ A(t).$$

■

Intuitively, since experts are *ex-ante* identical and use symmetric strategies the probability any particular expert's project is selected is $\frac{1}{n}$. In addition, as soon as the DM accepts an expert's project the game ends and that expert and the DM receive the same payoff, the expectation of which is $E[\theta|\theta > t^*]$. Thus among the set of equilibrium thresholds the DM

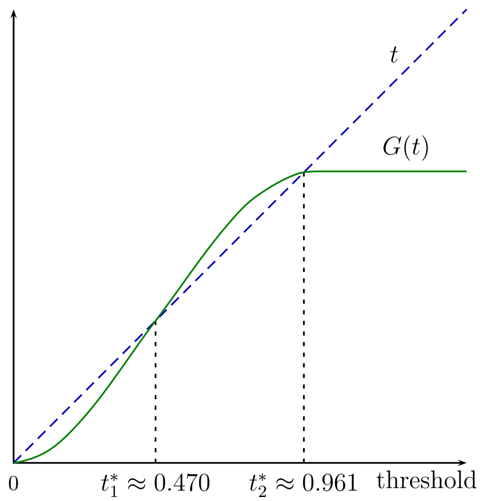

Figure 2: Determination of equilibrium thresholds for Example 2

and each expert share the same maximizer, a result we utilize below.

Proposition 4 *The equilibrium with the highest threshold is Pareto dominant.*

Proof A Pareto ranking follows from Lemma 3. To show the highest equilibrium threshold is Pareto dominant we note that (i) it suffices to track the DM's payoff; (ii) letting $F_n \sim \max\{\theta_1, ..., \theta_n\}$ in Lemma 1 there is a t_n^{DM} at which the DM's payoff is single-peaked, and t_n^{DM} is increasing in n since F_n first-order stochastically dominates F_m for $n > m$; and (iii) in any symmetric equilibrium with $n > 1$ the equilibrium threshold $t < t_1^{DM}$ (by Proposition 5(ii) below). ■

We henceforth focus on the Pareto dominant symmetric equilibrium. Next we present a stylized example in which multiple equilibria exist.

Example 2 *Let $\delta = 1$ and suppose the DM will adopt one project from among three experts whose projects are i.i.d. with pdf parameterized by $b \geq 1$ as follows:*

$$f(\theta; b) = \begin{cases} 1 - \theta & \text{if } \theta \in [0, 1] \\ 1 & \text{if } \theta \in [b, b + 0.5] \\ 0 & \text{otherwise} \end{cases}$$

and let $F(t;b)$ be the corresponding CDF.[4] *Thus $F(t;b)$ is invariant to b for $t \leq 1$ but $E[\theta|\theta > t]$ is increasing in b and so an equilibrium threshold exists for high enough b. Figure 2 graphs $G(t)$ for $b = 6.5$ and shows there are two thresholds that satisfy the sender constraints, $t_1^* \approx 0.470$ and $t_2^* \approx 0.961$. If the DM's outside option $r < \delta\,\Pi(t_1^*) \approx 5.412$ both thresholds constitute an equilibrium though t_2^* is Pareto dominant. If $r > \delta\,\Pi(t_2^*) \approx 6.741$ the DM will always reject while for intermediate r only t_2^* is an equilibrium threshold.*

5 Factors affecting the DM's payoff

In this section we explore how the equilibrium threshold responds to the rate of time discounting and the number of players, and how each affects the DM's payoff. Then using these results we show the DM's payoff may be highest in an asymmetric equilibrium in which some experts are not influential.

Comparative statics

Since each expert's recommendation is a comparison of his current project to the future, anything that increases the relative value of the future will induce experts to recommend adoption of their current project less often; that is, each expert will use a higher threshold. In turn the DM will infer a higher expected type for those projects that are recommended, leading to acceptance for a larger range of values of the outside option r. In this subsection we analyze two factors that increase the value of the future: less competition between experts, which implies a higher likelihood that future periods will be reached, and a higher discount factor.

Proposition 5 *In the Pareto dominant symmetric equilibrium,*

(i). the equilibrium threshold t^* increases in the discount factor δ, resulting in a higher payoff for the DM from both the direct effect of δ and the indirect effect of t^*; and

[4] Although f is not continuous and differentiable at every point of its domain it is for $\theta < 1$ and thus the properties of $G(t)$ previously established hold when $t < 1$.

(ii). the equilibrium threshold t^* decreases in the number of experts, n.

Proof See the appendix. ■

When the future is discounted less (higher δ) there is a direct effect that improves payoffs but also an indirect effect through the equilibrium threshold. With higher δ the value of waiting for a better project increases for the DM and each expert. This makes experts more willing to divulge information about states that are relatively low and improves payoffs since the DM also prefers to reject these low states. A similar logic regarding the threshold applies when there are fewer competing experts and thus a higher probability that the game continues.[5] However, the effect of more experts on the DM's payoff is less straightforward since the direct effect of receiving another draw from the distribution is offset by the indirect effect of the lower threshold this induces.

Expanding on our earlier notation, let $\Pi(t, n)$ be the DM's expected profit from n experts each using threshold t, and t_n^* be the symmetric equilibrium threshold with n experts. We decompose the two effects in the identity below:

$$\underbrace{\Pi(t_n^*, n) - \Pi(t_{n-1}^*, n-1)}_{\substack{\text{Effect of an} \\ \text{additional expert}}} = \underbrace{\Pi(t_n^*, n) - \Pi(t_{n-1}^*, n)}_{\substack{\text{Loss from} \\ \text{lower threshold}}} + \underbrace{\Pi(t_{n-1}^*, n) - \Pi(t_{n-1}^*, n-1)}_{\substack{\text{Gain from} \\ \text{quicker success}}}$$

The right-most term shows the gain to the DM from an additional expert given a fixed threshold, which arises from the increased likelihood that at least one expert's project exceeds the threshold and thus is adopted immediately. The greater is the rate of time-discounting (lower δ) the larger is the benefit from more quickly adopting a project. The other term in the decomposition captures the loss to the DM from a lower threshold, keeping fixed the number of experts. Letting $F_n \sim \max\{\theta_1, ..., \theta_n\}$ we can apply Lemma 1 to conclude there is

[5] An earlier version of the paper explored another means by which competition can decrease: allowing the DM to select $k < n$ projects to fund over the course of the game. For simplicity suppose that if an expert's project is chosen that expert is removed from the game in all subsequent periods should they arise, and assume the number of remaining projects to be selected is commonly known. Then higher k directly improves the DM's payoff but also indirectly does so by inducing a higher equilibrium threshold.

a threshold at which the DM's payoff is single-peaked. When $n > 1$ this threshold exceeds the equilibrium threshold and so $t_n^* < t_{n-1}^*$ (by Proposition 5(ii)) implies a lower payoff for the DM.

In summary, by consulting an additional expert the DM avoids time-delay costs by more quickly generating a successful project but harms communication which reduces the average quality of an accepted project. The net effect is unclear in general but depends on the rate of time discounting. By Lemma 3 when $\delta = 1$ the DM's payoff is $E[\theta|\theta > t^*]$, which decreases in n since t^* does, and so by the continuity of Π in δ this result remains true for δ sufficiently close to 1. In other words, when time-discounting is not too severe the gain from a quicker success is small and dominated by the loss from a lower threshold, and thus consulting more experts always harms the DM in this case. For lower values of δ the tradeoff must be explicitly calculated. However, this depends in part on the magnitude of $\frac{\partial^2 t^*}{\partial n^2}$ and is intractable. Example 3 below demonstrates that Π can increase in the number of experts over some range, though in general Π must eventually decrease with n as a babbling equilibrium is approached.

Proposition 6 *There exists a $\tilde{\delta} < 1$ such that the DM's payoff decreases in the number of experts when $\delta \geq \tilde{\delta}$. When $\delta < \tilde{\delta}$ there exists an $\tilde{n}$ such that the DM's payoff decreases in n when $n \geq \tilde{n}$.*

Proof See the appendix. ■

Example 3 *Let $\delta = 0.7$ and suppose $\theta_i \sim B\left(\frac{1}{5}, \frac{1}{5}\right)$ have a Beta distribution. Table II shows how the equilibrium threshold and the DM's payoff vary with the number of experts. In addition, it records the values of the DM's outside option for which a non-trivial equilibrium is supported. The DM's payoff increases from* 1 *to* 2 *experts and decreases thereafter. Numerical calculations show that for $\delta \geq \tilde{\delta} \approx 0.816$ the DM's payoff always decreases in n.*

Now suppose further $n = 1$ and $r = 0.6$. Since $r > 0.484$ only a trivial equilibrium exists in which the DM always takes his outside option and the expert babbles, resulting in payoffs of 0.6 *and* 0*, respectively. In contrast, if the DM's outside option were weak enough*

Table II

Number of experts, n	Equilibrium threshold, t^*	DM's payoff, Π	DM's outside option supporting $t^* > 0$
1	0.484	0.691	$r \in [0, 0.484]$
2	0.136	0.735	$r \in [0, 0.515]$
3	0.021	0.657	$r \in [0, 0.460]$
4	0.001	0.568	$r \in [0, 0.397]$
5 or more	0	0.5	$r \in [0, 0.35]$

($r \leq 0.484$) meaningful communication would be possible and result in an expected payoff of 0.691 *for both. The DM's inability to commit against using his outside option may therefore reduce his payoff, a result that holds generally. See the appendix for further discussion of equilibrium existence and properties for the symmetric Beta distribution.*

Asymmetric equilibria

Given the DM can benefit from consulting fewer experts we now turn to asymmetric equilibria in which not all experts are influential. We first examine asymmetric equilibria that are stationary. Next we consider the full set of equilibria and show that when time discounting is not too severe the DM's payoff is highest in a stationary equilibrium in which only one expert is consulted.

Any symmetric equilibrium with n experts can be changed into an asymmetric equilibrium as follows. Specify $i < n$ experts who will babble while the remaining $(n - i)$ experts play the equilibrium strategy that ensues in the symmetric case with $(n - i)$ experts. Given these strategies the DM will ignore the babbling experts and accept projects from the other experts as in the symmetric case. It is plain to see these strategies constitute mutual best responses. This observation together with Proposition 3 then provides a characterization of all stationary equilibria in which each expert is influential or babbles: the experts either use a symmetric threshold strategy as described in Proposition 1, or some subset of them do while the others babble (use threshold 0).[6] Thus all the equilibrium properties derived in the prior sections still apply in this setting, including existence (Proposition 2), selection (Proposition 4), and comparative statics (Propositions 5 and 6).

It then follows that when the hypothesis of Proposition 6 is satisfied, among stationary equilibria the DM prefers any in which the same single expert is consulted. In fact the proposition below shows this remains true when selecting from among the *entire* set of equilibria.

Proposition 7 *There exists $\bar{\delta} < 1$ such that when $\delta \geq \bar{\delta}$ the DM's expected payoff is highest among all equilibria in a stationary equilibrium in which only one expert is influential.*

Proof See the appendix. ■

This finding can be compared to a result in Li's (2016) model of sequential cheap talk. There two experts with an additive Crawford and Sobel (1982) style bias compete over time to have their own projects implemented, where only one expert is consulted ("active") in each period, and expert i does not observe j's project. The DM commits to a Markov decision rule that probabilistically determines which single expert will be active in each period, and this process continues until one project is adopted. Letting p denote the probability that the expert who was active this period remains active next period, Li parameterizes the amount of competition between experts across time, where $p = 1$ results in the same expert being consulted each period (no competition) and $p = 0$ results in the two experts alternating across periods (most competition).[7]

[6] To get an idea of what lies outside this set, consider the following example of a stationary equilibrium in which an expert is neither babbling nor influential. Suppose there are two experts and let $r = 0$. Conjecture the DM accepts a project from either expert if only one has recommended acceptance, rejects when both recommend rejection, but always accepts Expert 1's project when both experts recommend acceptance. Then Expert 1 uses a higher threshold than Expert 2 since the former has higher continuation value than the latter, but Expert 2 need not babble since he has some positive continuation value. Thus a stationary equilibrium can be supported where Expert 2 does not babble yet also is not influential since, given Expert 1's recommendation to accept, Expert 2's project is never adopted. I thank an anonymous referee for suggesting this example.

[7] This last case can be reproduced in our model without commitment in a non-stationary equilibrium. To see this, conjecture an equilibrium in which expert 1 is ignored only if the period is odd while expert 2 is ignored only if the period is even, and all other experts are always ignored. Given this, it is a best response for each expert to babble during the periods he is ignored and use an optimal threshold strategy otherwise.

Considering all $p \in [0,1]$ Li finds the best decision rule for the DM involves *no* alternation between experts at all but instead consulting the same expert each period ($p = 1$). This follows in his base model because competition over time between experts reduces the threshold each uses further below what the DM prefers, without any offsetting benefit. In the present paper increased competition in the form of consulting an additional expert each period also results in a loss by inducing greater exaggeration (a lower threshold), but this effect is in part offset by a gain in the form of a quicker success. Interestingly, a similar tradeoff is present in a generalization of Li's base model in which he endogenizes the search effort of the experts. Let s be the probability an expert receives a draw from the distribution; otherwise no project is obtained and the next period commences. Higher effort benefits the DM by increasing the chance a project will be generated, which then might exceed the threshold and be accepted. However, experts incur an increasing marginal cost to raise s and so may not exert enough search effort. In this setting Li finds that competition between experts across time induces a lower threshold that harms the DM, but this effect may be offset by the increased effort that an expert facing competition will exert. In other words, more competition (in the form of lower p) implies the quicker adoption of a lower quality project. Our results complement Li's in that we find the same basic tradeoff in a setting with competition both across time and within each period, and where the DM lacks commitment power.

6 Conclusion

When a decision maker consults multiple experts about which project to adopt, the information revealed by each expert will depend on both the amount of competition between experts and whether new projects arrive over time. In this paper we develop a model of competitive cheap talk in which n experts receive i.i.d. projects each period and vie to have their own project adopted by the decision maker through simultaneously reporting their type by cheap talk. Each expert is only informed of the quality of his own current project and only one project will ultimately be adopted due to resource constraints.

When experts compete we show that any symmetric equilibrium is characterized by a threshold above which states are recommended for adoption and below which will be rejected. Since the DM has the option to reject all experts' projects and proceed to the next period, recommendations can be viewed as comparative cheap talk about the current project versus what is expected in the future. Multiple symmetric equilibria may exist though we show the equilibrium with the highest threshold is Pareto dominant. One main finding is that in any symmetric equilibrium each expert's threshold is decreasing in the amount of competition and increasing in the value of future periods. The logic behind each result is the same: an expert is willing to divulge that his type is low and thus be rejected in the current period if the value from continuing in the game is high enough. More experts imply a higher probability the game will terminate this period and thus a lower continuation value, while a higher discount factor has the opposite effect.

The number of experts also affects payoffs. On the one hand, more experts induce more exaggeration and so lower the average quality of a recommended project, while on the other hand higher n implies a successful project will be adopted more quickly. When time discounting is not too severe, the former communication effect dominates the latter time effect and the DM is harmed by consulting more experts. For this reason asymmetric equilibria in which all but one expert babble and are ignored can be best for the DM.

Appendix: proofs of lemmas and propositions

Proof of Lemma 1

Letting $n = 1$ in Lemma 3, the DM's expected payoff using threshold t is $\frac{(1-F(t))E[\theta|\theta>t]}{1-\delta F(t)} = \frac{\int_t^\infty \theta \, dF(\theta)}{1-\delta F(t)}$. Differentiating with respect to t gives

$$\frac{-t\ f(t)}{1-\delta F(t)} + \frac{\delta\ f(t) \int_t^\infty \theta\ dF(\theta)}{(1-\delta F(t))^2}. \tag{4}$$

Setting line 4 equal to 0 and rearranging terms yields

$$\frac{\int_t^\infty \theta \; dF(\theta)}{1-\delta F(t)} = \frac{t}{\delta}, \tag{5}$$

which is equivalent to line 1. Differentiating line 4 with respect to t gives

$$\begin{aligned} & \frac{-\left(f(t)+t\; f'(t)\right)}{1-\delta F(t)} - \frac{-t\; f(t)\left(-\delta f(t)\right)}{\left(1-\delta F(t)\right)^2} \\ & + \frac{-t\; f(t)\,\delta f(t) + \delta f'(t)\int_t^\infty \theta \; dF(\theta)}{\left(1-\delta F(t)\right)^2} - \frac{2\int_t^\infty \theta \; dF(\theta)\;\; \delta f(t)\left(-\delta f(t)\right)}{\left(1-\delta F(t)\right)^3}. \end{aligned} \tag{6}$$

Substituting the right hand side of line 5 into the terms in line 6 gives the value of the second derivative when the first order condition is satisfied:

$$\begin{aligned} & \frac{-\left(f(t)+t\; f'(t)\right)}{1-\delta F(t)} - \frac{t\; f(t)^2\,\delta}{\left(1-\delta F(t)\right)^2} - \frac{t\; f(t)^2\,\delta}{\left(1-\delta F(t)\right)^2} + \frac{t\; f'(t)}{\left(1-\delta F(t)\right)} + \frac{2tf(t)^2\,\delta}{\left(1-\delta F(t)\right)^2} \\ = \;\; & \frac{-f(t)}{1-\delta F(t)} < 0. \end{aligned}$$

Thus if a critical point exists it is a maximum. Furthermore, since all critical points are maxima there is a unique maximum, and so the function is single-peaked at the maximizer. To prove existence of the maximizer, we use line 5 and observe $\frac{\delta \int_t^\infty \theta \; dF(\theta)}{1-\delta F(t)}$ indeed has a fixed point: the function is continuous, is positive for $t=0$, and approaches 0 as $t \to \infty$. Finally, by inspection the right hand side of line 1 increases in δ and thus by the implicit function theorem $\frac{\partial \hat{t}}{\partial \delta} > 0$.

Proof of Lemma 2

For part (i), the continuity of $G(t)$ follows from the continuity of $F(t)$ and $G(0) = 0$ by direct computation. Next, $\lim_{t\to\infty} G(t) = \frac{\delta}{1-\delta} \times 0 = 0$. For part (ii), $G'(t)$ calculates to

$$\frac{\partial}{\partial t}\left(\frac{\delta F(t)^{n-1}}{1-\delta F(t)^n}\right)\int_t^\infty \theta dF(\theta) + \frac{\delta F(t)^{n-1}}{1-\delta F(t)^n}\frac{\partial}{\partial t}\left(\int_t^\infty \theta dF(\theta)\right). \tag{7}$$

Next, $\frac{\partial}{\partial t}\left(\int_t^\infty \theta dF(\theta)\right) = -t\ f(t)$ and thus when $t = 0$ the second term in line 7 evaluates to 0 for $n \geq 2$. Next,

$$\frac{\partial}{\partial t}\left(\frac{\delta F(t)^{n-1}}{1-\delta F(t)^n}\right) = \frac{(1-\delta F(t)^n)(n-1)\delta F(t)^{n-2} f(t) + \delta F(t)^{n-1} n\delta F(t)^{n-1} f(t)}{(1-\delta F(t)^n)^2} \tag{8}$$

and this evaluates to 0 when $t = 0$ and $n > 2$ while if $n = 2$ it becomes

$$\frac{(1-\delta F(t)^n)(n-1)\delta F(t)^{n-2} f(t)}{(1-\delta F(t)^n)^2} = \frac{\delta f(0)}{1-\delta F(0)^2} = \delta f(0)$$

and the result follows by substituting this into line 7. Finally, $G'(t)$ is continuous by the continuity of f and lines 7 and 8.

Part (iii). Let t_0 be the largest t such that $G(t) = t$. Then the graph of G cannot cross the graph of t from below at t_0, since this implies $G(t_0 + \epsilon) > t_0 + \epsilon$ for sufficiently small $\epsilon > 0$ by the continuity of G. But then by part (i) of this lemma there must be another fixed point $t_1 > t_0$ since $G \to 0$ in the limit, a contradiction. Thus at t_0 the graph of $G(t)$ crosses t from above, or is just tangent to it. In the former case $G'(t_0) < 1$ while in the latter $G'(t_0) = 1$.

We now prove $t_0 < \frac{\overline{\theta}}{n}$ if θ is bounded above by $\overline{\theta}$. First, denote the upper bound of G by x and note $t_0 \leq x$. Next, by inspection $\frac{\partial G}{\partial \delta} > 0$ for all $t > 0$, and so it suffices to bound G by $\frac{\overline{\theta}}{n}$ for $\delta = 1$:

$$G(t; \delta = 1) = \frac{F(t)^{n-1}}{1-F(t)^n}\int_t^\infty \theta dF(\theta) = \left(\frac{F(t)^{n-1}}{\sum_{i=0}^{n-1} F(t)^i}\right) E[\theta | \theta \geq t]. \tag{9}$$

We claim line 9 increases in t. Since $E\left[\theta|\theta \geq t\right]$ obviously increases in t it suffices to show

$$\frac{\partial}{\partial t}\left(\frac{F\left(t\right)^{n-1}}{\sum_{i=0}^{n-1} F\left(t\right)^{i}}\right) = \frac{\left(n-1\right)F\left(t\right)^{n-2} f\left(t\right)\sum_{i=0}^{n-1} F\left(t\right)^{i} - F\left(t\right)^{n-1}\frac{\partial}{\partial t}\left(\sum_{i=0}^{n-1} F\left(t\right)^{i}\right)}{\left(\sum_{i=0}^{n-1} F\left(t\right)^{i}\right)^{2}} > 0.$$

This is true since the numerator is positive, which follows from

$$\begin{aligned}
\left(n-1\right)F\left(t\right)^{n-2} f\left(t\right)\sum_{i=0}^{n-1} F\left(t\right)^{i} \quad &> \quad F\left(t\right)^{n-1}\frac{\partial}{\partial t}\left(\sum_{i=0}^{n-1} F\left(t\right)^{i}\right) \iff \\
\sum_{i=0}^{n-1}\left(n-1\right)F\left(t\right)^{i} \quad &> \quad F\left(t\right)\left(\sum_{i=0}^{n-2}\left(1+i\right)F\left(t\right)^{i}\right),
\end{aligned}$$

where the equivalence is established by evaluating the derivative and dividing by $F\left(t\right)^{n-2}$ and the last inequality holds since the left hand side is larger than even the term in the parentheses on the right hand side. Finally, since $G\left(t;\delta=1\right)$ is increasing in t, taking the limit of line 9 as $t \to \overline{\theta}$ gives the result.

Proof of Proposition 5

Part (i). By inspection, $\frac{\partial G}{\partial \delta} > 0$. Next, by Lemma 2(iii) either $G'\left(t^*\right) < 1$ and so $\frac{\partial G}{\partial \delta} > 0$ and the implicit function theorem imply $\frac{\partial t^*}{\partial \delta} > 0$, or $G'\left(t^*\right) = 1$ and so $G\left(t^*\right)$ is tangent to t and thus by $\frac{\partial G}{\partial \delta} > 0$ higher δ gives rise to two fixed points, the greater of which exceeds t^*. Thus $\frac{\partial t^*}{\partial \delta} > 0$. The change in the DM's payoff with respect to δ can be decomposed into a direct and indirect effect:

$$\frac{\partial}{\partial \delta}\left(\frac{1-F\left(t^*\right)^{n}}{1-\delta F\left(t^*\right)^{n}}E\left[\theta|\theta > t^*\right]\right) + \frac{\partial}{\partial t^*}\left(\frac{1-F\left(t^*\right)^{n}}{1-\delta F\left(t^*\right)^{n}}E\left[\theta|\theta > t^*\right]\right)\frac{\partial t^*}{\partial \delta}.$$

The first term is positive by inspection while $\frac{\partial t^*}{\partial \delta} > 0$ was just proved. Finally, the DM's payoff increases in t by arguments (ii) and (iii) in the proof of Proposition 4.

Part (ii). By Lemma 2(iii), $G'\left(t^*\right) \leq 1$ and so it suffices to show that $G\left(t\right)$ is decreasing in n for all t. Since $G(t;n) = \frac{\delta F(t)^{n-1}}{1-\delta F(t)^{n}}\int_t^\infty \theta \; dF\left(\theta\right)$ the statement follows, as $F\left(t\right)^{n-1}$ and

$F(t)^n$ are decreasing in n for all $t > 0$.

Proof of Proposition 6

By Lemma 3, $\Pi(n,t) = \frac{1-F(t)^n}{1-\delta F(t)^n} E[\theta|\theta > t]$. When $\delta = 1$ this reduces to $E[\theta|\theta > t]$, which clearly decreases in t. Then, by Proposition 5(ii) we have t^* decreasing in n, and thus $\Pi(n,t^*)$ decreasing in n. Next, for any δ if $t_i^* = 0$ then $t_j^* = 0$ for all $j > i$. Additionally, observe that Π is continuous in δ and thus there exists a $\tilde{\delta} < 1$ such that the ordering $\Pi(1,t_1^*) \geq \Pi(2,t_2^*) \geq ...$ from $\delta = 1$ is preserved when $\delta \geq \tilde{\delta}$, where the inequality between any $\Pi(i,t_i^*)$ and $\Pi(i+1,t_{i+1}^*)$ is strict when $t_i^* > 0$ and holds with equality when $t_i^* = 0$.

To complete the proof of the first sentence of the proposition we note that $\delta\,\Pi(t^*)$, the upper bound of r for which an equilibrium is supported, is also decreasing in n. Without this, r could be such that for low n only a babbling equilibrium exists while for some locally higher n a non-trivial equilibrium leading to a higher payoff exists.[8]

To prove the last part of the proposition it suffices to show that $t^* \to 0$ as n increases, and $\Pi \to E[\theta]$ from above. The latter statement follows from the former and the fact that in any equilibrium the DM's payoff is bounded below by the babbling payoff $E[\theta]$. Thus it remains to show $\lim_{n\to\infty} t_n^* = 0$. First, if ever $t_i^* = 0$ the statement immediately follows since then $t_j^* = 0$ for all $j > i$. Otherwise there does not exist an i such that $t_i^* = 0$, and as argued in the proof of Proposition 5(ii), $G'(t^*) \leq 1$, and so for any t

$$\lim_{n\to\infty} G(t;n) = \lim_{n\to\infty} \int_t^\infty \theta dF(\theta) \frac{\delta F(t)^{n-1}}{1-\delta F(t)^n} = \int_t^\infty \theta dF(\theta) \lim_{n\to\infty} \frac{\delta F(t)^{n-1}}{1-\delta F(t)^n} = \int_t^\infty \theta dF(\theta) \times 0 = 0,$$

which then implies $\lim_{n\to\infty} t_n^* = 0$.

[8] For instance, in Example 3 an equilibrium with communication can be supported when $r \leq 0.484$ if $n = 1$ and for $r \leq 0.515$ if $n = 2$. Thus if $r = 0.51$ babbling ensues with one expert, giving the DM a payoff of 0.51, while a non-babbling equilibrium resulting in a payoff of 0.735 exists with two experts.

An example: the symmetric beta distribution

Let $\theta_i \sim B(\alpha, \beta)$ where we specify $\alpha = \beta > 0$ so that the distribution is symmetric. The Beta distribution has full support on $(0,1)$, and with our assumptions $E[\theta_i] = \frac{1}{2}$ while $Var[\theta_i] = \frac{1}{4(2\alpha+1)}$, so lowering α constitutes a mean-preserving spread.

Proposition 8 *An equilibrium exists if and only if $\alpha < \widehat{\alpha}_n$ for some $\widehat{\alpha}_n > 0$. Additionally, the equilibrium threshold is decreasing in α.*

Proof If an equilibrium threshold t^* exists it is implicitly defined by $H(t) = 0$, where $H(t) = G(t) - t$. By the implicit function theorem

$$\frac{\partial t}{\partial \alpha}(t^*) = -\frac{\frac{\partial H}{\partial \alpha}(t^*)}{\frac{\partial H}{\partial t}(t^*)},$$

and Lemma 2(iii) implies $\frac{\partial H}{\partial t}(t^*) < 0$ (except for the knife-edge case $G'(t^*) = 1$, where higher α implies the fixed point no longer exists). Thus it suffices to show $\frac{\partial H}{\partial \alpha}(t^*) < 0$, which is established if $\frac{\partial G}{\partial \alpha}(t^*) < 0$. Now, any fixed point $t^* < \frac{1}{2}$ by Lemma 2(iii). But then by the properties of the symmetric Beta distribution $\frac{\partial F}{\partial \alpha} < 0$ and $\frac{\partial E[\theta|\alpha;\theta>t]}{\partial \alpha} < 0$. Writing G as

$$\left(\frac{\delta F(t)^{n-1}(1-F(t))}{1-\delta F(t)^n}\right) E[\theta|\theta > t], \tag{10}$$

it suffices to show the term in the large parentheses decreases in α. Since the denominator is increasing in α it suffices to show that

$$\frac{\partial}{\partial \alpha}\left(\delta F(t)^{n-1}(1-F(t))\right) \;<\; 0 \iff \delta\,\frac{\partial F}{\partial \alpha} F(t)^{n-2}\Big((1-F(t))(n-1) - F(t)\Big) < 0,$$

which holds since $t^* < \frac{1}{2}$ implies $F(t^*) < \frac{1}{2}$ and $\frac{\partial F}{\partial \alpha} < 0$, and thus $(1-F(t))(n-1) - F(t) > 0$ since $n \geq 2$.

By Proposition 2, to establish existence of $t^* > 0$ it suffices to show that $G(t) > t$ is possible. For $t \in \left(0, \frac{1}{2}\right)$, $\lim_{\alpha\to 0} E[\theta|\theta > t] = 1$, $\lim_{\alpha \to 0} F(t) = \frac{1}{2}$, and thus using line 10 gives

$$\lim_{\alpha \to 0} G(t) = \frac{\delta \left(\frac{1}{2}\right)^{n-1} \left(\frac{1}{2}\right)}{1 - \delta \left(\frac{1}{2}\right)^{n}} \times 1 = \frac{\frac{\delta}{2^n}}{1 - \frac{\delta}{2^n}} > 0.$$

Thus for any n there is a t and an α_n depending on n such that $G(t) > t$, and furthermore since $\frac{\partial t}{\partial \alpha} < 0$ this remains true for $\alpha < \alpha_n$. ■

Notice that when $\alpha = 1$ the distribution is uniform, and letting $n = 2$ we find $G(t) = \frac{t}{1+t}\left(\frac{1+t}{2}\right) = \frac{t}{2} < t$ except for the trivial solution $t = 0$. Given Proposition 5 we can conclude that for any $n > 1$ and δ no equilibrium exists when $\alpha \geq 1$.

Proof of Proposition 7

We establish the result without time discounting and then by the continuity of the DM's payoff in δ the result obtains. When $\delta = 1$ the DM's payoff is determined solely by the expected value of the project he accepts, which is increasing in the threshold used in that period by the expert who recommended it. Free from competition, consulting the same single expert each period will result in the DM's first-best outcome. Furthermore, this outcome strictly dominates the payoff from any other equilibrium, since in any equilibrium in which more than one expert's project would be accepted by the DM the experts will use a lower threshold than if they were free from competition, owing to their lower continuation value. The only possible exception are equilibria in which the DM ignores all experts for finitely many periods, followed by the same single expert being consulted indefinitely thereafter. However, such equilibria are dominated by the candidate equilibrium when $\delta < 1$. Finally, an argument similar to that in the proof of Proposition 6 shows the set of r supporting a non-babbling equilibrium decreases in the number of non-babbling experts in each period and thus if babbling must occur when consulting the same single expert it must also occur in any other configuration of experts consulted over time.

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